



# ACOUSTICS 2012

## Modeling of the part-pedaling effect in the piano

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This study presents the theoretical modeling of the part-pedaling effect in the piano. Part-pedaling means a common use of the sustain pedal where the pedal is not fully depressed, but pressed somewhere between the two extremes. The model implies to consider the distributions of the string deflection and the string velocity along the string as functions created by the traveling nondispersive waves generated by the hammer impact and moving in both directions. The damper restricts the amplitude of the string deflection in the region of its position, and also suppresses the "up" velocity of the string. Such a nonlinear model gives possibility to calculate the spectrograms of the string vibration tone, and to compare the numerical calculations with the measured example tones of the string. To obtain the appropriate decay rates of the string vibrations, the damping factor at one termination of the string is induced. The modeling confirms that in the bass range the nonlinear amplitude limitation causes energy transfer from the lower partials to higher partials, which can excite missing modes during the damper-string interaction.

## 1 Introduction

This paper presents a physics-based model for simulating the damper-string interaction in the part-pedaling effect in the piano. Part-pedaling is a popular pedaling technique used for creating various artistic expressions. It means that the sustain pedal is not fully depressed, but pressed somewhere between the two extremes. The effect has been studied experimentally earlier [1] by analyzing recorded piano tones played with part-pedaling. The results showed that the use of the part-pedaling affects the timbre and decay characteristics of the tone, since the damper limits the amplitude of the string motion in a nonlinear fashion.

Physics-based sound synthesis is a popular approach for modeling musical instrument sounds [2]. In the case of string instruments, modeling of the string vibrations forms a basis for the synthesis. The most popular approaches are the digital waveguides [3] (DWG), the finite-difference scheme [4], and the modal synthesis approach [5]. Specifically, models for the sustain pedal synthesis that use the DWG technique [6, 7] and modal synthesis approach [8] have been presented. These methods simulate the effect of the sympathetically resonating string register in a real piano when the sustain pedal is completely pressed down. On the other hand, the effect of the nonlinear amplitude limitation present in the part-pedaling is not accounted for in these models.

However, the idea of modeling a collision between a vibrating string and a rigid obstacle in the context of physics-based sound synthesis has been presented earlier. Rank and Kubin [9] developed a DWG model for slapping technique on electric bass guitars that introduces nonlinear signal processing elements for limiting the amplitude of the string vibration. Krishnaswamy and Smith [10] proposed methods for simulating a collision between an ideal string and a rigid obstacle using both DWG modeling and finite-difference scheme.

This paper shows how the damper-string interaction can be simulated in a DWG model of a piano string, when part-pedaling is used. This can be divided into two stages. Firstly, mathematical modeling of the hammer-string interaction allows prediction of the the piano string motion [11, 12]. Secondly, this knowledge is used for appropriate simulation of the interaction of the vibrating string with a damper. When part-pedaling is used, the string motion is limited when the damper altitude, i.e., the depth of the sustain pedal, above the string is smaller than the transverse string deflection.

The numerical simulation of the hammer-string interaction is based on physical models of a piano hammer described in [13, 14, 15]. These models are based on the assumption that the woollen hammer felt is a microstructural material possessing history-dependent properties. The elastic and hereditary parameters of piano hammers were obtained

experimentally using a special piano hammer testing device that was developed and built in the Institute of Cybernetics at Tallinn University of Technology [15].

In this paper a number of simplifying assumptions regarding the string and string supports are introduced. Thus, the piano string is assumed to be an ideal flexible string, but the coupling of strings at the end supports is neglected, and the bridge motion is also ignored. We also assume that the left string termination (agraffe) is the ideal rigid support. The right string termination (bridge) we consider here as a rigid but not ideal support. To obtain the appropriate decay rates of the string vibrations, we introduce the reflection coefficient for the traveling waves reflecting from the bridge. The application of the proposed procedure in modeling of the part-pedaling in the piano clarifies the physics of this effect.

## 2 String and hammer models

In this paper it is assumed that the piano string is an ideal (flexible) string. The displacement  $y(x, t)$  of such a string obeys the simple wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}. \quad (1)$$

As in [11], we have the system of equations describing the hammer-string interaction

$$\frac{dz}{dt} = -\frac{2T}{cm} g(t) + V, \quad (2)$$

$$\frac{dg}{dt} = \frac{c}{2T} F(t), \quad (3)$$

where  $g(t)$  is the outgoing wave created by the hammer strike,  $c$  is the speed of a transverse nondispersive wave traveling along the string;  $F(t)$  is the acting force,  $T$  is the string tension;  $m$ ,  $z(t)$ , and  $V$  are the hammer mass, the hammer displacement, and the hammer velocity, respectively. The hammer felt compression is determined by  $u(t) = z(t) - y(0, t)$ . Function  $y(0, t)$  describes the string deflection at the contact point  $x = 0$ , and is given by [12]

$$y(0, t) = g(t) + 2 \sum_{i=1}^{\infty} g\left(t - \frac{2iL}{c}\right) - \sum_{i=0}^{\infty} g\left[t - \frac{2(i+a)L}{c}\right] - \sum_{i=0}^{\infty} g\left[t - \frac{2(i+b)L}{c}\right]. \quad (4)$$

It is assumed that the string of length  $L$  extends from  $x = -aL$  on the left to  $x = bL = (1-a)L$ . Parameter  $a = l/L$  is the fractional length of the string to striking point. The initial

conditions at the moment when the hammer first contacts the string, are taken as  $g(0) = z(0) = 0$ , and  $dz(0)/dt = V$ .

The physical sense of Eq. (4) is simple. It means that the deflection of the string at the contact point is determined by the traveling waves moving in both directions along the string and reflecting back from the string supports. Here the index of summation  $i$  simply denotes the number of rereflections.

The experimental testing of piano hammers demonstrates that all hammers have a hysteretic type of force-compression characteristics. A main feature of hammers is that the slope of the force-compression characteristics is strongly dependent on the rate of loading. It was shown that nonlinear hysteretic models can describe the dynamic behavior of the hammer felt [13, 14, 15]. These models are based on assumption that the hammer felt made of wool is a microstructural material possessing history-dependent properties. Such a physical substance is called either a hereditary material or a material with memory.

According to a four-parameter hereditary model of the hammer presented in [13, 14], the nonlinear force  $F(t)$  exerted by the hammer is related to the felt compression  $u(t)$  by the following expression

$$F(u(t)) = F_0 \left[ u^p(t) - \frac{\varepsilon}{\tau} \int_0^t u^p(\xi) \exp\left(-\frac{\xi-t}{\tau}\right) d\xi \right]. \quad (5)$$

Here the instantaneous hammer stiffness  $F_0$  and compliance nonlinearity exponent  $p$  are the elastic parameters of the felt, and  $\varepsilon$  and  $\tau$  are the hereditary parameters.

The parameters of the hammers in this model were obtained experimentally by measuring a whole hammer set of recently produced unvoiced *Abel* hammers. The results of this experiment and continuous variation in hammer parameters across the compass of the piano are presented in [15].

The hammer-string interaction is simulated using the basic formulae presented above. The tone G3 ( $f_0 = 196.4$  Hz; the note and the hammer number  $N = 35$ ) was chosen for modeling. The string length  $L = 878$  mm, the actual distance of the striking point from nearest string end  $l = 111$  mm, the string tension  $T = 847$  N.

The four-parameter hammer model (Eq. (5)) is explored, and the values of hammer parameters are computed by formulae presented in [15]. These parameter values used are  $F_0 = 122220$  N/mm $^p$ ;  $p = 4.225$ ;  $\varepsilon = 0.9925$ ;  $\tau = 2.13$   $\mu$ s, and the hammer mass  $m = 8.7$  g.

The simulation of the hammer-string interaction was provided by solving the system of equations Eq. (2), Eq. (3) for initial hammer velocity  $V = 3$  m/s, and for one string per note. For this purpose the acting mass of a hammer is defined as being the total hammer mass divided by the number of strings per note  $n = 3$ . Thus the hammer mass used is  $m = 2.9$  g. As a result of simulation we can find the force history  $F(t)$  and the time dependence of the outgoing wave  $g(t)$  created by the hammer strike.

### 3 String-damper interaction model

The proposed model of the string-damper interaction is based on the knowledge of the outgoing wave function  $g(t)$  created by the hammer strike. It is evident that Eq. (1) may be satisfied by combination of simple nondispersive waves  $g_1(t - x/c)$  and  $g_2(t + x/c)$  moving in either directions along

the string from the point  $x = 0$  where the string makes contact with the hammer. At this point  $g_1(t) = g_2(t) = g(t)$ . These two waves  $g_1$  and  $g_2$  are simply translations of the outgoing wave  $g(t)$  from the point  $x = 0$  to the other segment of the string, and their amplitudes are always positive, because  $g(t) > 0$ . These two waves are being reflected from each end of the string, and create the pair of reflected waves  $g_3(t - x/c)$  and  $g_4(t + x/c)$ . The amplitude of these waves is always negative. The scheme of waves propagation along the string is shown in Figure 1.

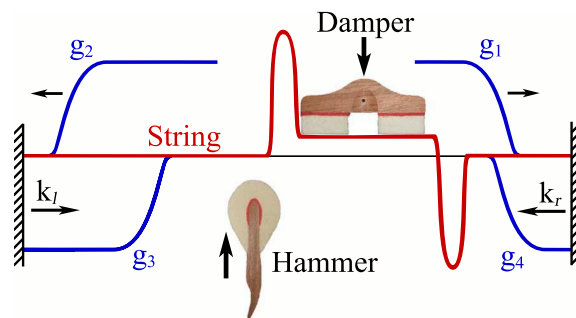


Figure 1: Scheme of string-damper interaction in the waveguide model. Functions  $g$  are the traveling waves;  $k_l$  and  $k_r$  are the reflection coefficients.

At the left end of the string the reflected wave  $g_3(t) = -k_l g_2(t)$ , and at the right end of the string the reflected wave  $g_4(t) = -k_r g_1(t)$ . Here  $k_l$  and  $k_r$  are the left and right reflection coefficients, which are introduced to obtain the appropriate decay rates of the string vibrations.

The physical interpretation of the functions  $g_3$  and  $g_4$  shows what we should use for their values: they exist only because the outgoing wave  $g$  at some earlier time has been reflected from the string ends. According to our model, the string deflection  $y(x, t)$  (shown in Figure 1 by red line) at any point  $x$  and at any time  $t$  is simply the resulting sum of waveforms  $g$  moving in both directions:

$$y(x, t) = g_1(t - x/c) + g_2(t + x/c) + g_3(t - x/c) + g_4(t + x/c). \quad (6)$$

A computing method that realizes the calculation of the string deflection determined by Eq. (6) is based on a digital delay-line procedure. The numerical application of this method is best explained by Hall in Appendix A [11].

The position of the damper at any moment is fixed by two points on the  $x$  axes,  $D_1$  and  $D_2$ , which denote the coordinates of the left and right side of the damper, and the altitude  $H$  of the damper above the string. At each time step of calculations and at each point of the string we control a value of the string deflection calculated according to Eq. (6). If the string deflection at the moment  $t_i$  at the left point  $D_1$  of the damper is greater than  $H$ , we reduce the amplitude of the positive wave  $g_1(t_i - D_1/c)$  traveling above the string in the right direction to obtain  $y(D_1, t_i) = H$ . If the string deflection at the moment  $t_j$  at the right point  $D_2$  of the damper is greater than  $H$ , we reduce the amplitude of the positive wave  $g_2(t_j + D_2/c)$  traveling above the string in the left direction to obtain  $y(D_2, t_j) = H$ . Therefore, at any moment the amplitude of the string deflection under the damper is less than the altitude of the damper above the string. Thus we may simulate the part-pedaling effect in the piano by using the traveling-wave model of the string-damper interaction.

## 4 Results and analysis

With the model described in Section 3 it is possible to compute the string vibrations in a desired point of the string. Thus, it is possible to simulate the damper-string interaction in a part-pedaling situation and compare the results with recorded waveforms described in [1]. Figure 2 shows the waveforms of the recorded tone G3 ( $f_0 = 196.4$  Hz) played without the sustain pedal, with part-pedaling, and with full sustain pedal, and Figure 3 shows the waveforms of the corresponding simulated tones.

The simulation parameters for the string and the damper were chosen based on the signal analysis of the recorded tones, and they are described in Section 2. The damper covers the range 113-184 mm measured from agraffe. The left end of the string is assumed absolutely rigid, thus the value of the left reflection coefficient  $k_l = 1$ . To obtain the similar decay rate of the string vibrations the value of the right reflection coefficient was chosen  $k_r = 0.995$ .

Figure 3(a) shows the string vibrations after interaction with the damper, which at the moment  $t = 1$  s falls down to the altitude 0.005 mm above the string (without the sustain pedal); Figure 3(b) shows the string vibrations after interaction with the damper, which at the moment  $t = 1$  s falls down to the altitude 0.1 mm above the string (part-pedaling); Figure 3(c) shows the string vibrations without the interaction with the damper (full sustain pedal) at the point of the string, which is in vicinity of right side of the damper.

The nonlinear effect caused by the damper-string interaction was studied through spectrograms. Figure 4(a), 4(b), and 4(c) show the spectrograms of the recorded tone G3 ( $f_0 = 196.4$  Hz) played without the sustain pedal, with part-pedaling, and with the sustain pedal, respectively. These spectrograms correspond to the sound waveforms in Figures 2(a), 2(b), and 2(c).

Figures 5(a), 5(b), and 5(c) show the spectrograms of the simulated tone G3 ( $f_0 = 196.4$  Hz) without the sustain pedal, with part-pedaling, and with full sustain pedal, respectively. The corresponding waveforms are presented in Figures 3(a), 3(b), and 3(c). All spectrograms were computed using a Chebyshev window of length 200 ms with a 150 ms overlap.

By comparing Figures 4 and 5 several observations can be made. First of all, the effect of the damper is very similar in Figures 4(a) and 5(a), as well in Figures 4(b) and 5(b), which implies that the model presents well the effect of the shaking damper when it comes in contact with the vibrating string; this is visible as "spreading" in the vicinity of the horizontal lines that correspond the partials in Figures 5(a) and 5(b). In Figure 5(a) it can be seen that after the damper-string interaction also the level of the background noise is lower compared to the beginning of the tone. The frequencies that correspond to the partials have some energy left; the present model does not take into account the frequency-depending damping of the partials.

Figure 5(b), which presents the spectrogram of the simulated tone with part-pedaling shows that after the damper-string interaction the levels of the partials are slightly lower compared to the full pedal case (see Figure 5(c)). Additionally, the eighth partial gains energy after the interaction, like in the case of the recorded tone (see Figure 4(b)). Thus, with the suggested model it is possible to imitate the energy transfer from the lower partials to the higher partials, a phenomenon which is present in part-pedaling [1].

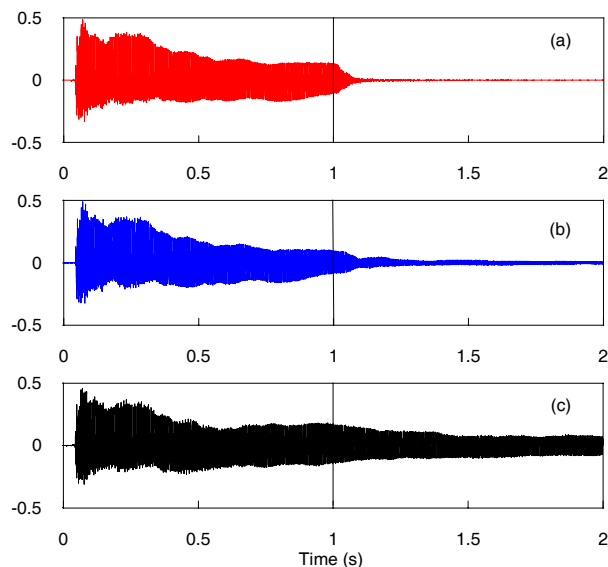


Figure 2: Recorded sound waveforms of the piano tone G3 ( $f_0 = 196.4$  Hz) played (a) without the sustain pedal, (b) with part-pedaling, and (c) with full sustain pedal. The vertical lines show the time instant when the key is released.

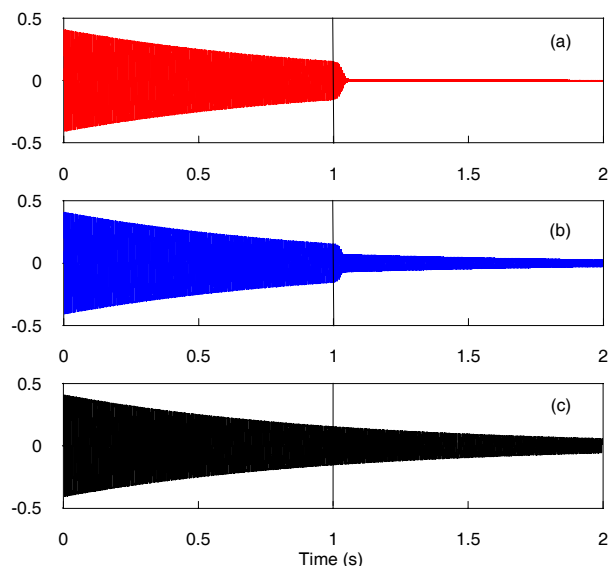


Figure 3: Normalized string vibration of the simulated piano tone G3 ( $f_0 = 196.4$  Hz) played (a) without the pedal, (b) with part-pedaling, and (c) with full sustain pedal.

The effect of the vibrating string register in the case of the recorded tones is clearly visible in Figure 4 as noise especially in the beginning of the tone. Since the current part-pedaling model does not contain the sympathetically resonating string register, it is natural that this effect is not present in Figure 5, which shows the spectrograms of the simulated tones.

## 5 Conclusions

This paper presented a method for simulating the nonlinear limitation of string vibrations caused by the interaction between the damper and the string in the context of the physics-based sound synthesis of the piano. Specifically, it was shown that the proposed approach is suitable of simulating the nonlinear effects of the part-pedaling technique.



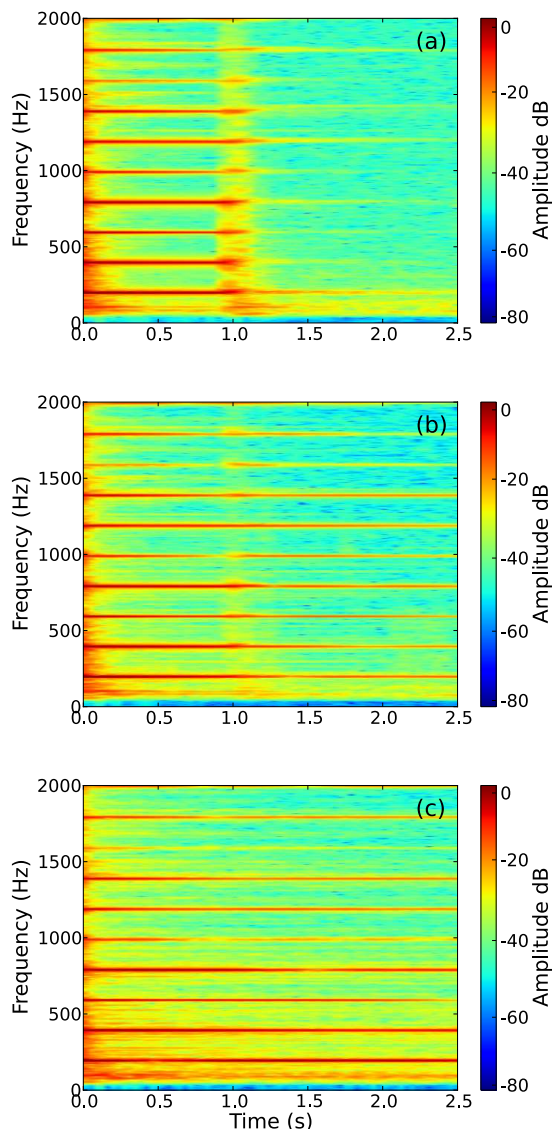


Figure 4: Spectrograms of recorded tones G3 ( $f_0 = 196.4$  Hz) corresponding to the sound waveforms in Fig. 2(a,b,c).

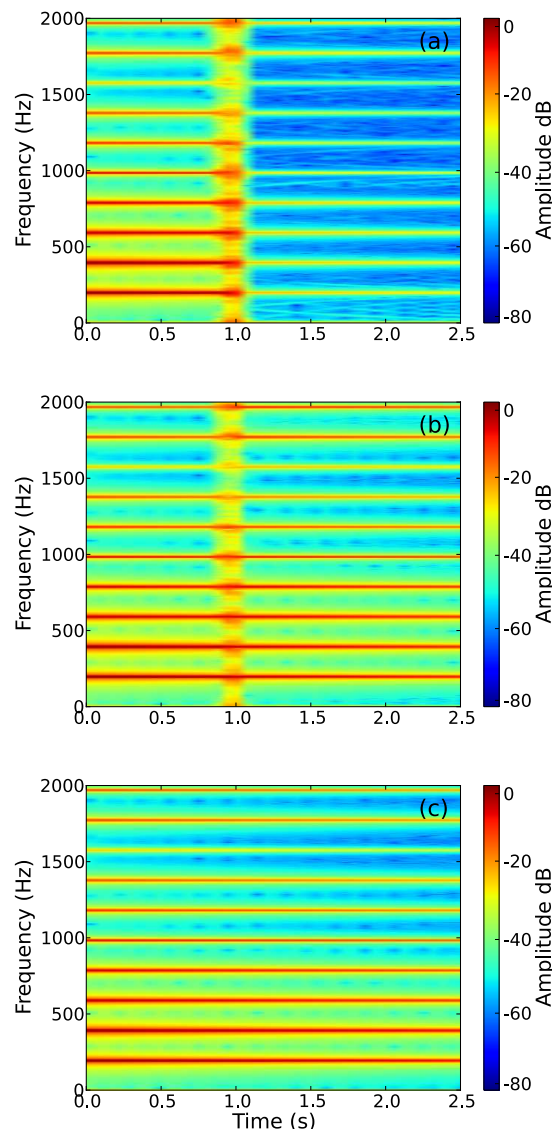


Figure 5: Spectrograms of simulated tones G3 ( $f_0 = 196.4$  Hz) corresponding to the string vibrations in Fig. 3(a,b,c).

The developed theoretical model was compared with experimental results shown in [1] and it was concluded that the proposed technique was able to produce the main effect of the nonlinear amplitude limitation caused by the damper limiting the amplitude of the string vibration.

Future research will concentrate on combining the results presented in this paper and the model of the effect of the whole piano string register, which is resonating sympathetically when the sustain pedal is used. This phenomenon affects the sound, since the vibrational energy is spreading among all strings that are coupled through the bridge.

## Acknowledgments

The authors are grateful to Dr. A. Askenfelt and piano technician T. Persson for collaboration in earlier stages of this research. Many thanks to D. Kartofelev for his assistance in preparing the figures. This research was supported by the European Union through the European Regional Development Fund.

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