Solitary Waves for Non-Destructive Testing Applications: Delayed Nonlinear Time Reversal Signal Processing Optimization

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Abstract

An original signal processing method called delayed Time Reversal - Nonlinear Elastic Wave Spectroscopy is introduced in the present paper. The method could be used to amplify signal in certain regions of the material under Non Destructive Testing. It allows to optimize and change the shape of the received focused wave in the material, either by making the focusing sharper by decreasing the side lobes or making it wider by modifying the actual focusing peak. It is also possible to use the focused signal as a delta-basis to construct a signal with arbitrary envelope or reduce the side lobes of the focused signal. These concepts are shown to work well in the simulations and the physical experiments. This signal processing method is particularly promising for nonlinear and solitary wave analysis, since it allows to create an interaction of sharp and solitary wave peaks just underneath the receiving transducer. Due to simple and accurate linear prediction of the received interaction signal, any differences of measurements and predictions could indicate the presence of nonlinearities.

Keywords: Delayed Time Reversal - Nonlinear Elastic Wave Spectroscopy, Carbon Fibre Reinforced Polymer, solitary waves, Non Destructive Testing, Finite Element Method

1. Introduction

The objective of the paper is to present the concept of delayed Time Reversal - Nonlinear Elastic Wave Spectroscopy (delayed TR-NEWS) [1]. It is an original method based on TR-NEWS method, used to obtain and modify focusing or convergence of ultrasonic waves in complex media.

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Original TR-NEWS signal processing procedure can be used to focus the wave energy under the receiving transducer or vibrometer of an ultrasonic Non-Destructive Testing (NDT) setup. TR-NEWS is a promising method for evaluation of complex, dispersive and nonlinear media because it relies on the internal reflections as virtual transducers [1] to focus the wave energy into a specific spot in a certain time, therefore taking advantage of the complex internal structure of the material [2]. Such focused wave has an improved signal-to-noise ratio, making it suitable for investigating dispersive, chaotic or highly attenuating media [3, 4].

During the last ten years there has been an increase of interest in using symmetry and similarity properties in the signal processing of nonlinear acoustics [5, 6, 7]. New signal processing methods have been developed and validated for NDT and harmonic imaging. Pulse Inversion (PI) techniques [8, 9], have been extended and generalized using Symmetry Analysis [10]. Coded excitations (for example a chirp signal) and signal processing is now considered as an efficient way for imaging the complexity in bio-materials. These methods improve the determination of nonlinear properties by using optimized excitations [11].

Recently there has been a considerable development of TR based NEWS methods using invariance with respect to TR and reciprocity, both in numerical and experimental aspects. These methods have been experimentally elaborated as the well-known TR-NEWS methods [12, 13, 14, 15, 16]. TR-NEWS fundamental experimental demonstrations [17] have been conducted with applications in the improvement of nonlinear scatterers identification using symbiosis of symmetry analysis (TR, reciprocity, chirp-coded PI, etc.) and NEWS methods. TR-NEWS based imaging continues to be developed, with new systems being designed to obtain better focusing and optimal images. New excitations are now under study in order to give to TR-NEWS methods the practicability needed for both the NDT and the medical imaging community [18].

A new direction in NDT is the use of solitary waves, as their important properties differ from linear waves and they are overall well-studied phenomenon [19]. Nonlinear effects depend mostly on the signal power and wave shape. This shape could then be analysed and compared to linear cases to detect the presence of nonlinearities.

In this paper the delayed TR-NEWS is numerically and experimentally validated for allowing to manipulate the focused wave shape of the ordinary TR-NEWS. It is a promising method for studying the nonlinear properties of materials in nonlinear acoustics and NDT of complex materials and composites. The paper will demonstrate the signal optimization potential of the delayed TR-NEWS method for NDT purposes using experiments in bi-layered aluminium and Carbon Fibre Reinforced Polymer (CFRP) and simulations in CFRP. It will be shown that the wave focused under the receiving transducer can be manipulated to have a different shape. The method will be shown to be useful for changing the extent of the material affected by focused ultrasonic wave, side lobe reduction and introduction of low-frequency signal into the medium by amplitude modulation. It is also possible to introduce a low-frequency wave by high frequency input by using delayed TR-NEWS as amplitude modulation of the low frequency signal. Additionally, it will be shown that the method is highly predictable in linear materials, and could be used to analyse nonlinear effects as deviations from the linear prediction.

Physical experiments were carried out on a bi-layered aluminium and a CFRP sample. The findings were studied further using Finite Element Method (FEM) simulations on a linearly elastic laminate model of CFRP block with stochastic layer thicknesses to examine the signal propagation and focusing inside a complex material. It is well known that layered periodical materials can be dispersive and therefore solitary waves can emerge in the presence of nonlinearities in such materials [20, 21].

The goal of this paper is to show that the delayed TR-NEWS procedure gives good results in numerical and physical experiments for examining complex materials. In Section 2 the test object and the experimental setup is described and the computational model is introduced. In Section 3 the signal processing steps of TR-NEWS and delayed TR-NEWS methods are explained. In Section 4 the results of numerical and physical experiments are presented. In Section 5 the conclusions and the possible practical uses of delayed TR-NEWS procedure are given.

2. Materials and methods

The delayed TR-NEWS signal processing method was initially validated in a bi-layered aluminium sample [22]. In the present paper the method was used to optimize the signal in a complex CFRP sample and the wave motion inside the CFRP was further studied using a FEM model. Having an agreement between the FEM model and physical experiments, it is possible to study the wave motion in more detail. While physical experiments give the actual measured values on the surface of the test object, the simulations allow to investigate the internal wavefield in the object.

The simulations and experiments must agree qualitatively. The quantitative agreement is less important because: i) We are studying how waves behave at the focusing; ii) Simulations are in 2D due to computational constraints while the real world is in 3D and this always produces differences in quantitative results; iii) The developed signal optimization must be robust enough to account for some variations in signal power; iv) In this work the simulation and experimental material is considered linearly elastic, rendering amplitude analysis unnecessary.

2.1. Experiment configuration

The tests were conducted with TR-NEWS ultrasonic testing equipment which focuses the wave energy near the surface of the material under the receiving transducer. The initial validation was conducted on a bi-layered aluminium sample [22]. Thereafter the tests were performed on a CFRP block (composed of 144 layers of carbon fibre fabric). It was excited from its side with 70° shear wave transducer. The signal was received with a plane wave transducer on the top of the block (Fig. 1). The roles of the transducers are not changed during the

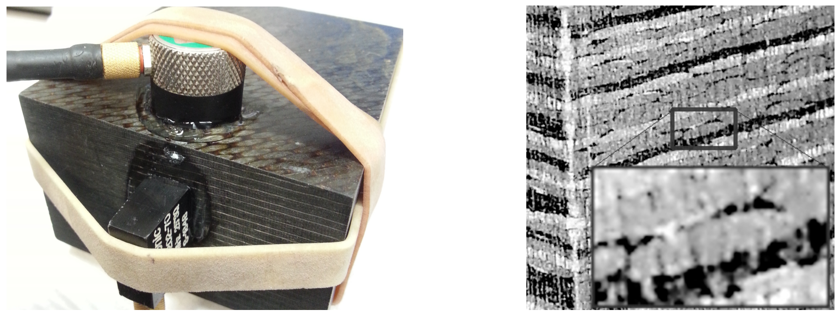


Figure 1: Carbon Fibre Reinforced Polymer (CFRP) block (left) and its structure (right). This test configuration was chosen arbitrarily in order to verify the practical feasibility of the method in an NDT test.

experiment as the focusing of an ultrasonic wave relies on the signal processing of TR-NEWS. This is a two-pass method where the receiving and transmitting transducers do not change their roles. In this sense the "Time Reversal" denotes the signal processing method which accounts for internal reflections of the material as virtual transducers in the material to use for focusing the wave in the second pass of the wave transmission. The placement of the transducers is otherwise not important: they could be placed arbitrarily in NDT investigation, they do not have to be in line with each other, but the configuration must remain fixed during the TR-NEWS procedure.

The test equipment consisted of:

Preamplifier Juvitek TRA-02 (0.02 - 5 MHz) connected to a computer,

Amplifier ENI model A150 (55 dB at 0.3-35 MHz),

Shear wave transducer Technisonic ABFP-0202-70 (2.25 MHz),

Longitudinal wave transducer Panametrics V155 (5 MHz).

2.2. Finite Element Model

CFRP is composed of fabric woven from yarns of fibre impregnated with epoxy. The cross-section of the yarns have elliptical shape (Fig. 1) and the material has inclusions of pure epoxy between yarns, so a wave propagating through the material would encounter yarns (fibres with epoxy) and areas of pure epoxy. A simple laminate model was used for FEM: the material was considered to be homogeneous, generally anisotropic, but linearly elastic where each layer had its own elasticity properties according to the carbon fibre cloth orientation. The thicknesses of the individual layers were chosen from a function of random variables according to the probability of the wave encountering these thicknesses in packed ellipse configuration of the real material (Figs. 1 and 2). Dispersion could be expected to arise due to the discontinuity of the material properties. In this way the layered model, which is often used in studying of nonlinear waves in complex materials [20, 21], is extended by making the layers have stochastic thicknesses. The stochastic thicknesses take into account the normal distribution of the ellipse sizes and the uniform distribution of geometric probability which describes where along the ellipse width the wave passes through the ellipse (Fig. 2).

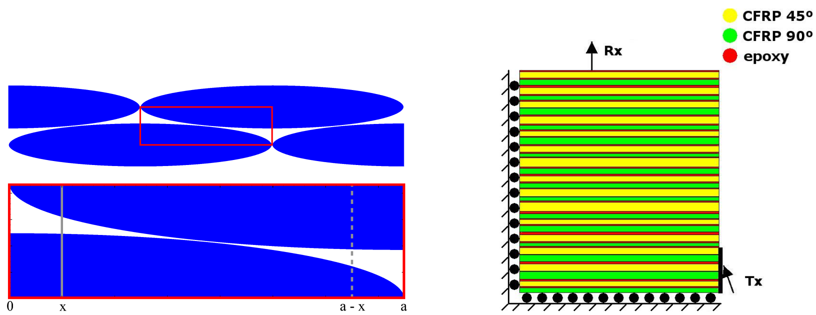


Figure 2: The material consisting of ellipsoidal yarns in tight packing (left) is modelled as a material with stochastic thicknesses of layers (right) due to different possible wave paths encountering different thicknesses of CFRP yarn and epoxy

The material in the simulation was modelled as laminate model of three different kinds of layers with different mechanical properties [23]:

- pure epoxy layer as an isotropic material: $E = 3.7$ GPa, $\nu = 0.4$, $\rho = 1200$ kg/m^3,
- composite with fabric at 0/90° orientation as transversely isotropic material: $E_1 = E_2 = 70$ GPa, $G_{12} = 5$ GPa, $\nu_{12} = 0.1$, $\rho = 1600$ kg/m^3,
- composite with fabric at 45°/45° orientation as transversely isotropic material: $E_1 = E_2 = 20$ GPa, $G_{12} = 30$ GPa, $\nu_{12} = 0.74$, $\rho = 1600$ kg/m^3.

The thickness of each separate composite yarn layer was found with

$$f_C(x) = \frac{b}{a}\left(\sqrt{a^2 - x^2} + \sqrt{2ax - x^2}\right), \tag{1}$$

and the thickness of its corresponding pure epoxy layer with

$$f_E(x) = \sqrt{3}b - f_C(x). \tag{2}$$

where $a = \mathcal{N}(\mu_a, \sigma_a^2)$ and $b = \mathcal{N}(\mu_b, \sigma_b^2)$ are normal random variables for the semi-axes of the ellipses found from the distribution of carbon fibre ellipses in the real material (Fig. 1). Therefore the computational model reflected the actual material due to the randomness of the layer thicknesses being in accordance with the carbon fibre ellipses in actual CFRP block. Uniform random variable $x = \mathcal{U}(0, a)$ is a function of geometric probability which describes where along the ellipse semi-major axis the wave passes through the ellipse (in the direction of its minor axis) (Fig. 2). The laminate model was constructed by finding the semi-major axis length a (where mean $\mu_a = 0.750$ mm, dispersion $\sigma_a = 0.130$ mm) and semi-minor axis length b (where mean $\mu_b = 0.130$ mm, dispersion

$\sigma_b = 0.025$ mm) and wave traversing location x is a random variable for a pair of layers (composite and epoxy). The composite fabric orientation (90° or 45°) alternated for each pair (Fig. 2). The model consisted of 50 such pairs which were generated and stacked together. The thicknesses of composite fabric and epoxy layers were found from Eqs. (1) and (2). As a practical aspect, the thickness of an epoxy layer f_E could not be too small or it prohibited the generation of a good FEM mesh.

2.2.1. Equations

The material in the simulation was assumed to be linearly elastic. The following variational problem was solved [24]:

$$\int_V \rho \ddot{u} \delta u dV + \int_V \sigma_{ij} \delta \varepsilon_{ij} dV - \int_\Gamma f \cdot \delta u dS = 0, \tag{3}$$

where u is displacement, ρ is material density, σ_{ij} is stress, ε_{ij} is strain and f is traction on surface. Newmark's method was used for time stepping ($\Delta t_\mathrm{s} = 5 \cdot 10^{-8}$ s). The constant average acceleration variant of the Newmark's scheme has following relations

$$\begin{cases} \dot{u}_{s+1} = \dot{u}_s + \Delta t_\mathrm{s} \frac{\ddot{u}_s + \ddot{u}_{s+1}}{2}, \\ u_{s+1} = u_s + \Delta t_\mathrm{s} \dot{u}_s + (\Delta t_\mathrm{s})^2 \frac{\ddot{u}_s + \ddot{u}_{s+1}}{4}. \end{cases}$$

From these equations one can express acceleration

$$\ddot{u}_{s+1} = \frac{4}{(\Delta t_\mathrm{s})^2} u_{s+1} - \ddot{u}_s - \frac{4}{\Delta t_\mathrm{s}} \dot{u}_s - \frac{4}{(\Delta t_\mathrm{s})^2} u_s,$$

and substitute $\ddot{u} = \ddot{u}_{s+1}$ into Eq. (3). After that the equation for solving the wave motion problem in a layer becomes as follows:

$$\int_V \left[\rho \left(\frac{4}{(\Delta t_\mathrm{s})^2} u_{s+1} - \ddot{u}_s - \frac{4}{\Delta t_\mathrm{s}} \dot{u}_s - \frac{4}{(\Delta t_\mathrm{s})^2} u_s \right) \delta u + \sigma_{ij} \delta \varepsilon_{ij} \right] dV = \int_\Gamma t \cdot \delta u dS.$$

This FEM variational model was solved using FEniCS libraries [25].

2.2.2. Boundary conditions

The following boundary conditions (Fig. 2, right) were used in the FEM simulation:

- Excitation signal was introduced by Dirichlet boundary condition on the right hand side of bottom 7 pairs of layers. The excitation was introduced in 70° angle upwards and lasted for 60 μs.
- Dirichlet boundary condition of zero displacement $u_1 = 0$ on left and $u_2 = 0$ on bottom boundary.

In physical experiments the displacement component u_2 was recorded at the top of the sample, at $x_1 = 5$ mm from the left side of the material (Fig. 2) (equivalent for the receiver placement in the experiment, 9 mm from the free right hand side in Fig. 1). Since the displacement component u_2 was used for signal processing, it was also the component exhibiting TR-NEWS focusing and the only component shown in the results.

The signal input and output correspond to the physical experiment shown in Fig. 1. There were some differences between simulation and experiment parameters due to computational considerations. Notable differences were that the simulation considered only the material between and near the transducers, not the whole CFRP object, so the wave field had less space to travel and reverberate from. Also the simulation did not include losses. The simulation used 60 μs signal length while the physical experiment had 1280 μs signal length. Obviously, the simulation was in 2D, unlike the physical experiment.

3. Delayed TR-NEWS signal processing

This section describes the signal processing steps of delayed TR-NEWS used identically in simulations and physical experiments. Both the delayed and the original TR-NEWS methods are two-pass techniques for use in NDT experiments. The first pass is used to gather information about internal reflections (virtual transducers) in material. The second pass transmits an excitation which uses these internal reflections to focus the wave under receiving transducer. Both passes are conducted in the same direction without replacing the transmitting and receiving transducers with each other. The first steps of the delayed TR-NEWS coincide with the known TR-NEWS steps. The TR-NEWS procedure consists of the following steps [1](outlined in Figs. 3 and 4):

1. The first transmission pass involves transmitting a chirp-coded excitation $c(t)$ through the medium

$$c(t) = A \sin\left(\psi(t)\right),$$

   where $\psi(t)$ is linearly changing instantaneous phase. In this work, a sweep from 0 to 2 MHz was used.

2. Simultaneously the chirp-coded coda response $y(t)$ with a time duration T is recorded

$$y(t,T) = h(t) * c(t) = \int_{\mathbb{R}} h(t-t',T)c(t')dt',$$

   where $h(t-t',T)$ is regarded as impulse response of the medium. Here the asterisk denotes convolution and $y(t,T)$ the direct response from the receiving transducer when transmitting the chirp excitation $c(t)$ through the medium.

3. Next the information about the internal reflections $\Gamma(t)$ is found by cross-correlating the received response $y(t, T)$ with the sent chirp-coded excitation $c(t)$. This is computed for some time period Δt,

$$\Gamma(t) = \int_{\Delta t} y(t - t', T)c(t')dt' \simeq h(t) * c(t) * c(T - t, T). \tag{4}$$

Here $h(t)*c(t)*c(T-t, T)$ is a pseudo-impulse response . It is proportional to the impulse response $h(t)$ if using linear chirp excitation for $c(t)$ because then $\Gamma_c(t) = c(t) * c(T - t) = \delta(t - T)$. Therefore the actual correlation $\Gamma(t) \sim h(t)$ contains information about the wave propagation paths in complex media. Time reversing the correlation $\Gamma(t)$ from the previous step results in $\Gamma(T - t)$ used as a new input signal. This time-reversed $\Gamma(T - t)$ is shown in Figs. 3(3) and 4(3).

4. The second pass of the transmission involves re-propagating the new excitation $\Gamma(T - t)$ in the same configuration as the initial chirp, yielding the received signal

$$y_{TR}(t, T) = \Gamma(T - t) * h(t) \sim \delta(t - T), \tag{5}$$

where $y_{TR} \sim \delta(t-T)$ is now the focused signal under receiving transducer where the focusing takes place at time T. This is because $\Gamma(t)$ contains information about the internal reflections of the complex media. Transmitting its time reversed version $\Gamma(T-t)$ eliminates these reflection delays by the time signal reaches the receiver, resulting in the focused signal y_{TR} (Eq. (5)).

The test configuration must remain constant during all of these steps, otherwise the focusing is lost. The roles of the transmitting and the receiving transducers are never exchanged. Time reversal in this sense is conducted purely in the signal processing: it will use the internal reflections to create focusing under the receiving transducer in the second pass of the ultrasonic transmission. This concludes the signal processing steps for the TR-NEWS procedure which is known and published [1, 3].

The subject of this paper is the delayed TR-NEWS signal processing method, which is based on the TR-NEWS, so it uses the same initial steps. Its added value over the TR-NEWS is the possibility of changing the focused wave by considering a single y_{TR} signal as a new basis which can be used to build arbitrary wave shapes at the focusing. This is done by time-delaying and superimposing n time-reversed correlation $\Gamma(T - t)$ signals [1] (Fig. 5 left column)

$$\Gamma_s(T - t) = \sum_{i=0}^{n} a_i \Gamma(T - t + \tau_i) = \sum_{i=0}^{n} a_i \Gamma(T - t + i\Delta\tau), \tag{6}$$

where a_i is the i-th amplitude coefficient and τ_i the i-th time delay. In case of a uniform time delay the $\Delta\tau$ is the time delay between the samples. Upon propagating this $\Gamma_s(t - T)$ through the media according to the last step of TR-NEWS, a delayed scaled shape of signal at the focusing point can be created

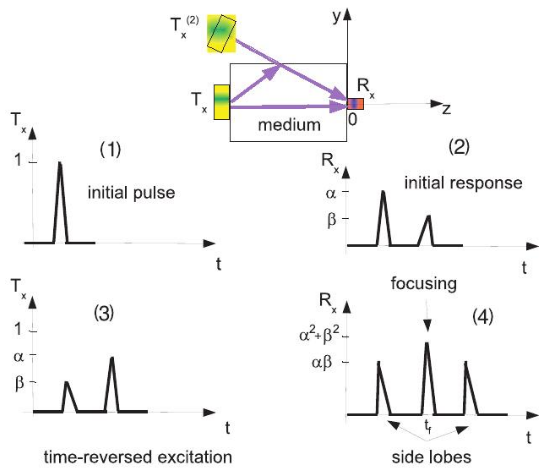


Figure 3: Schematic process of TR-NEWS with the virtual transducer concept. (1) The initial broadband excitation $T_x(t)$ propagates in a medium. (2) Additional echoes coming from interfaces and scatterers in its response R_x could be associated to a virtual source $T_x^{(2)}$. (3) Applying reciprocity and TR process to R_x. (4) The time reversed new excitation $T_x = R_x(-t)$ produces a new response R_x (the TR-NEWS coda $y_{TR}(t)$) with a spatio-temporal focusing at $z = 0; y = 0; t = t_f$ and symmetric side lobes with respect to the focusing.

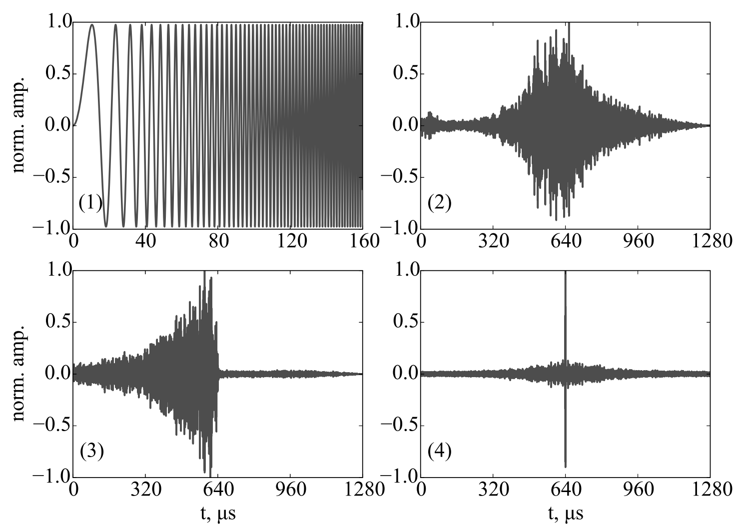


Figure 4: Chirp-coded TR-NEWS signal processing steps in bi-layered aluminium experiment: (1) chirp excitation; (2) output recorded at Rx; (3) cross-correlation between input and output; (4) focusing resulting from transmitting the time-reversed cross-correlation as a new ultrasonic input.

(Fig. 5 right column). Various optimizations are possible using the delayed TR-NEWS scheme, for example amplitude modulation, signal improvement and side lobe reduction. It is possible to counteract the side lobes of a single focusing pulse by its time-delayed and scaled versions, according to the scheme in Fig. 6.

It is possible to predict the result of the delayed TR-NEWS focusing in a linear material (Fig. 5 right column):

$$y_{dTR}(t) = \left[\sum_i a_i \Gamma_c \left(T - t + \tau_i\right)\right] * h(t) \overset{\text{linearity}}{=\!=\!=} \\ = \sum_i a_i \Gamma_c \left(T - t + \tau_i\right) * h(t) = \sum_i a_i y_{TR}(t - \tau_i). \quad (7)$$

The purpose of the linear prediction of the delayed TR-NEWS result is twofold. Firstly it can be used to find optimal parameters for the delayed TR-NEWS experiment, using the original focusing peak y_{TR}. Secondly it could be possible to analyse the differences between the predicted and actual measured delayed TR-NEWS result. The difference could indicate the magnitude of nonlinearity, because the prediction relies on the applicability of linear superposition and is found to be quite accurate in experiments with linear material.

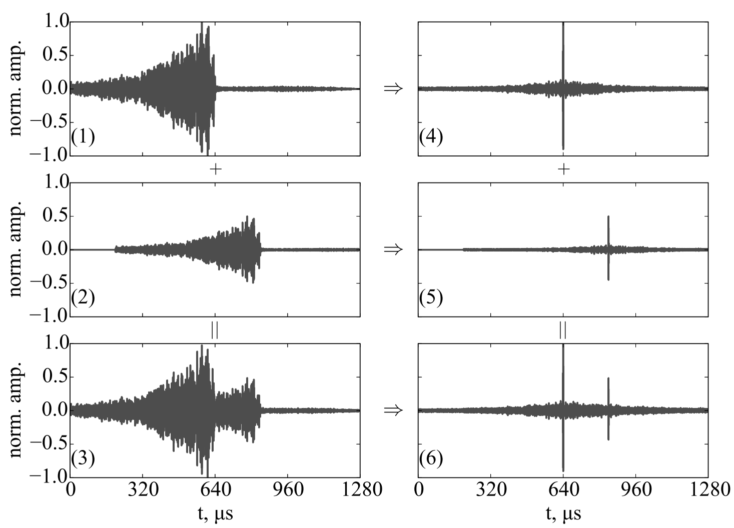


Figure 5: Delayed TR-NEWS signal processing steps conducted in bi-layered aluminium, starting from the cross-correlation step (left column) and prediction of linear superposition of waves (right column): (1) cross-correlation (Eq. (4)); (2) delayed and scaled cross-correlation; (3) linear superposition of two cross-correlations which becomes the new excitation; (4) focusing (Eq. (5)); (5) delayed and scaled focusing; (6) Linear superposition of the two focusing peaks.

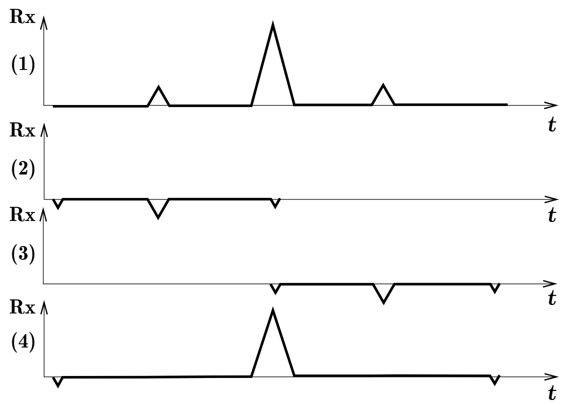


Figure 6: Scheme of the side lobe reduction using delayed TR-NEWS: (1) initial y_{TR} focusing with side lobes (Eq. (5)); (2) and (3) scaled and shifted focusing for eliminating side lobes; (4) resulting y_{dTR} from adding together signals (1)-(3) has reduced side lobes but slightly reduced main peak.

4. Results and discussion

The physical experiments were carried out on bi-layered aluminium and CFRP samples, the simulations only on layered FEM model of CFRP. Firstly the CFRP simulation and experiment results are compared to establish a link between them. Due to computational considerations, the signal length of the simulation is 60 μs and in experiment it is 1280 μs. The TR-NEWS focusing y_{TR} produces the focusing peak in the middle of the signal, therefore this midpoint is taken as point of reference as "time since focusing" to compare the simulation and experiment results. The amplitudes are normalized to the maximum value of absolute values in line plots. Both the simulation and the experiments use the same chirp $c(t)$ frequency range from 0 to 2 MHz. As much as possible, all simulation and experiment parameters are taken to be identical. Particularly the time delay values are the same when comparing delayed TR-NEWS simulation with experiment.

4.1. Validation of simulations for delayed TR-NEWS

Firstly, the simulations are compared to the experiments for CFRP for single TR-NEWS focusing peak y_{TR}. The close-up of the peak is shown in Fig. 7. A good agreement between the period of the focusing peak in simulation and experiment can be seen. Also, the approximate levels of noise in simulation and experiment are similar.

Secondly, the simulation and experiment is compared in case of delayed TR-NEWS signal processing where five delays are taken with relative amplitudes $a_i = \{0.1, 0.2, 0.3, 0.4, 0.5\}$, and a delay $\Delta\tau = 1$ μs between them (Eq. (6)). The results of the corresponding simulation and experiment are shown in Fig. 8. Again, there is a qualitative correspondence between the simulation and the experiment.

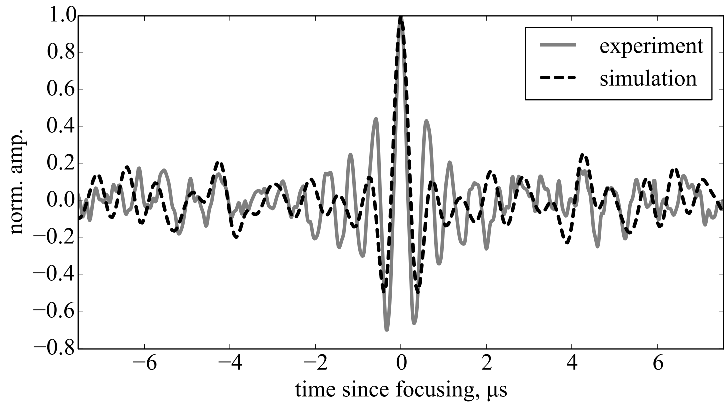


Figure 7: TR-NEWS wave focusing in CFRP, comparison of the TR-NEWS simulation with the physical experiment

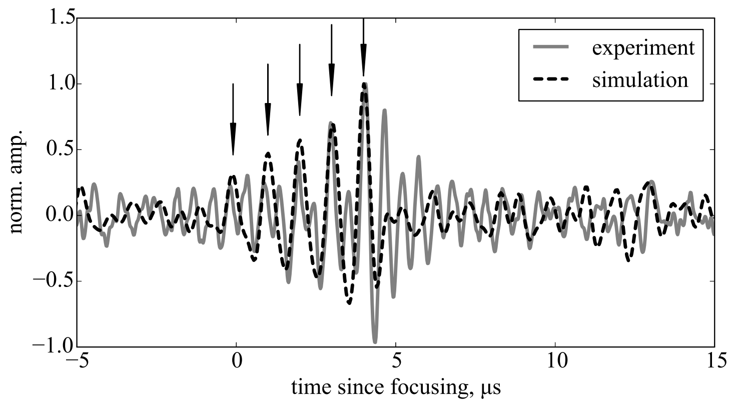


Figure 8: Comparison of the delayed TR-NEWS CFRP physical experiment and simulation. The five peaks are delayed with intervals $\Delta\tau = 1$ μs and have relative amplitudes $a_i = \{0.1, 0.2, 0.3, 0.4, 0.5\}$

4.2. Optimization of the focusing peak

Having just one additional delayed focusing peak in delayed TR-NEWS ($a_i = \{1,1\}$ and $\Delta\tau = 0.25$ μs in Eq. 6), the width of the focusing is modified and the side lobes decreased. The preliminary verification has been performed on bi-layered aluminium sample, and later also in CFRP sample and simulations. The effect of such delay (compared to the original TR-NEWS focusing) on the side lobes of the bi-layered aluminium can seen in Fig. 9. Much of the side lobe has been decreased at the expense of some added noise outside the focusing region. The same experiment and simulation has been conducted in CFRP, using coincidentally the same value $\Delta\tau = 0.25$ μs. In CFRP experiment and simulation (Fig. 10), the improvement is not as pronounced as in case of bi-layered aluminium: the decrease of oscillations adjacent to main focusing peak is smaller.

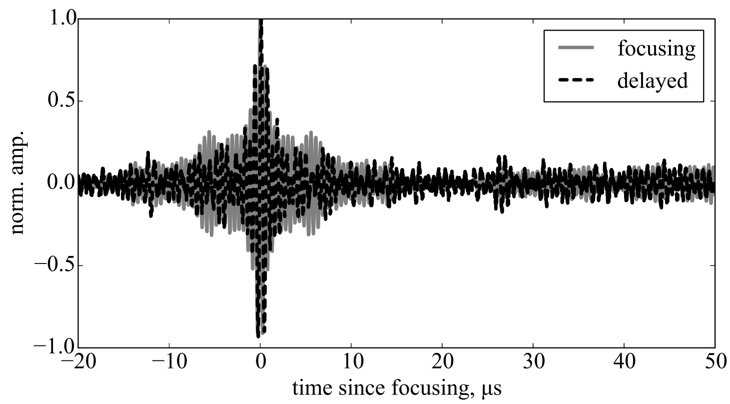


Figure 9: Improvement of focusing in bi-layered aluminium due to delayed TR-NEWS experiment with delay $\Delta\tau$ =0.25 μs and $a_i = \{1,1\}$

In CFRP such 0.25 μs delay mainly modifies the width of the focusing peak itself, making it wider, as can be seen from experiments and simulations in CFRP in Fig. 10. This property could be used to modify the spatial extent of the higher amplitude focusing. In these experiments this increase of the width of focused wave is small (Fig. 11) but different frequencies in different materials could yield better results in widening the area affected by high-amplitude focusing. The reduction of the side lobe, while noticeable, is small. In the CFRP simulation the focusing amplitude is also lowered from $A_{\max} = 2.27$ to $A_{\max} = 2.07$ due to the delayed TR-NEWS widening of the focusing. The maximum amplitude of the input excitation was always $A = 1$.

In all of these experiments and simulations, the delayed TR-NEWS results have been in good accordance with their linear superposition predictions (Eq. (7)), allowing to fine-tune the delay parameters before conducting the experiment. The small difference between the prediction and the experiment in bi-layered aluminium with $a_i = \{1,1\}$, $\Delta\tau = 0.25$ μs can be seen in Fig. 12. It

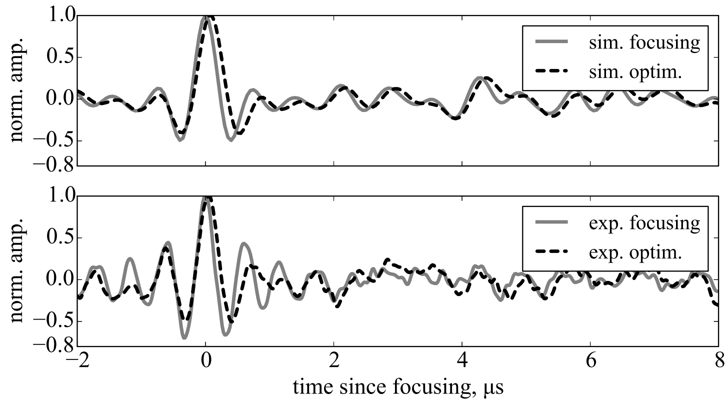
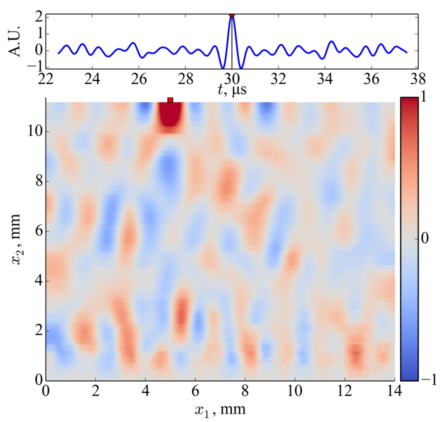
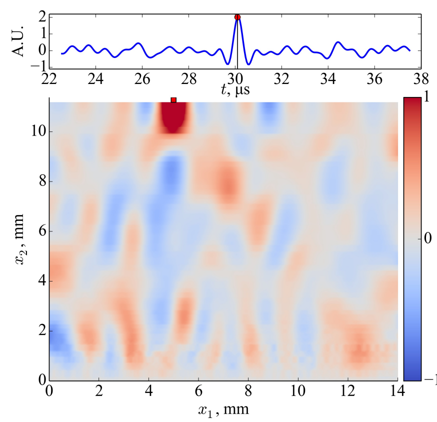


Figure 10: Comparison of delayed TR-NEWS in simulation (upper subfigure) and experiment (lower subfigure) with 0.25 μs delay, comparing the widening of the pulse



(a) TR-NEWS focusing



(b) Delayed TR-NEWS focusing

Figure 11: Extent of the material affected by high amplitude ($A_{\max} > 1$) focused wave in case of TR-NEWS focusing and delayed TR-NEWS focusing for $\Delta\tau = 0.25$ μs, $a_i = \{1, 1\}$. Unnormalized results are shown with focusing maximum amplitude $A_{\max} > 2.0$. Additional video files are available in the supplementary materials.

is clear that the difference between the prediction and the experiment is close to zero at the crests and troughs of the signal and it has extremal values between them due to slight phase differences between the prediction and the actual measurement.

4.3. Wave interaction at the focusing point

The simulations are important because they show what happens inside the material. It could be possible to use the delayed TR-NEWS for examining of nonlinear materials supporting solitary waves and to create an interaction of high amplitude waves right at the receiving transducer. The simulation results confirm that at the focusing under the receiver is indeed a constructive interaction of several waves and not just a passing of one wave (Fig. 13). The proposed nonlinear analysis method would rely on delayed TR-NEWS to pump the medium full of energy to be focused near the receiver and use time delays to slightly offset the focusing to investigate the interaction of the waves near the receiver. For an experiment of undamaged CFRP material such delayed signals are shown in Fig. 14. It could be possible to detect nonlinear effects (which can be caused by material defects [26]) in the focusing region by comparing the measured results to the prediction that is based on linear superposition assumption (Eq. (7)). Interaction of solitons (instead of linear waves) could also be detected, for example, by phase difference due to nonlinear interaction [27].

4.4. Arbitrary wave envelope generation

Experiments with bi-layered aluminium sample [22] and CFRP sample show that delayed TR-NEWS method can be used to generate an arbitrary wave en-

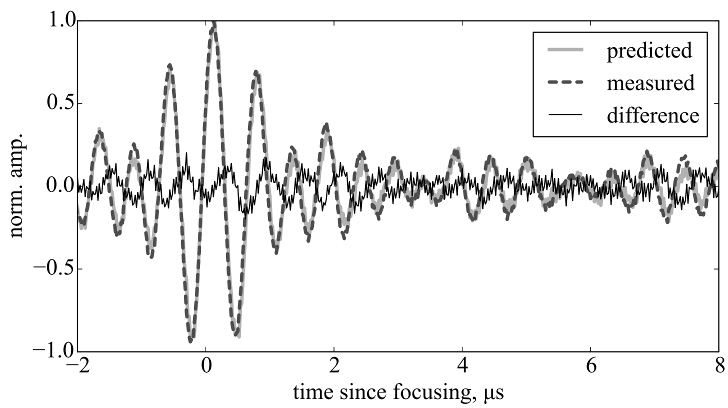


Figure 12: Prediction from delayed TR-NEWS signal processing ($\Delta\tau$ =0.25 μs) versus the results from actual experiment in bi-layered aluminium sample

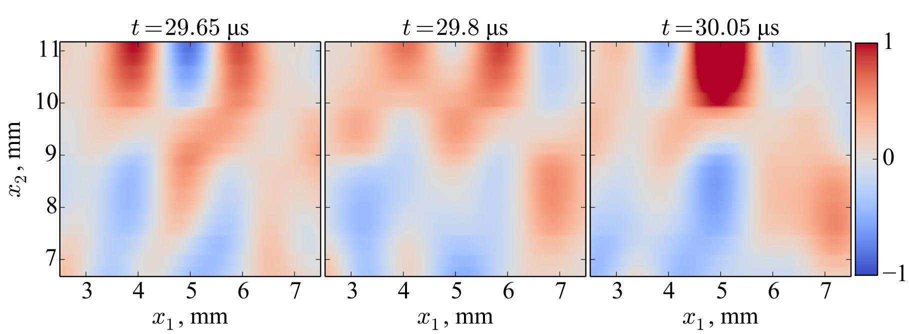


Figure 13: Several separate waves arriving and interacting in CFRP simulation of the focusing

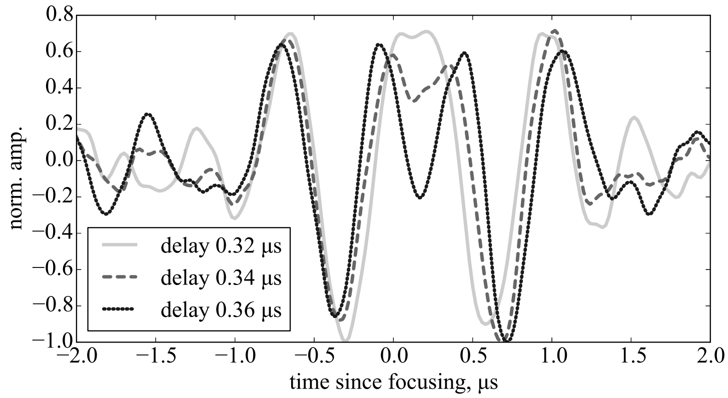


Figure 14: Delayed TR-NEWS focusing manipulations in CFRP experiment with 3 different delay values, showing different moments of converging wave interaction at the focusing moment

velope at the focusing point by taking the individual delta signal $y_{TR} \sim \delta(t-T)$ as a basis for amplitude modulation (Fig. 15). Such amplitude modulation could be used in a material to generate a low-frequency component in the focusing point (receiver) of the test setup.

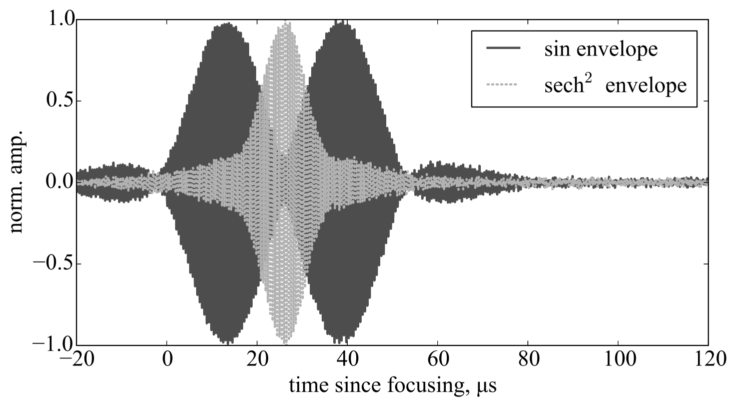


Figure 15: Delayed TR-NEWS in CFRP for creating an envelope of special shape at the focusing

4.5. Side lobe reduction

The side lobe reduction by using two delayed peaks of $a_i = \{1, 1\}$ and $\Delta\tau = 0.25$ μs was effective in bi-layered aluminium. In CFRP experiment, the side lobe reduction is most effective when performed directly by using delayed TR-NEWS on the side lobes according to scheme in Fig. 6. The process is as follows: the time difference between the side lobe and the main peak of the original focused signal is taken as delay τ_i and the amplitude of the side lobe as negative amplitude $-a_i$. This process is repeated for the other side lobe. Thereafter the Eq. (6) is used to sum these signals together to reduce the side lobes. In physical experiments the side lobe reduction is effective because the frequency content of the peak and the side lobes match well (Fig. 16).

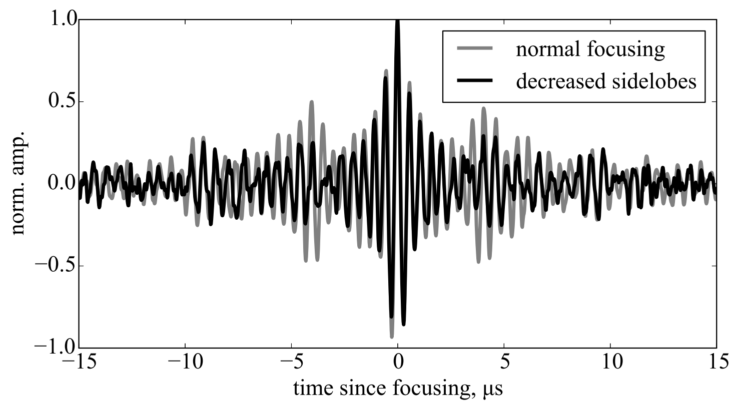


Figure 16: Results of a delayed TR-NEWS focusing experiment with the CFRP sample: due to using of two time-delayed and scaled copies of the original signal the size of the side lobes can be reduced significantly compared to normal focusing

5. Conclusions

It has been shown that the results of simulations and experiments match well. Therefore one can conclude that the simulation shows the nature of the wave field at the focusing point inside the material. By using the simulations and the experiments together it is possible to conclude several findings about the delayed TR-NEWS signal processing optimization method which can be important in NDT, especially if the focusing takes place in a region where the nonlinear properties of the material enable emergence of solitary waves above some threshold of wave energy.

It was shown that the delayed TR-NEWS can be used to modify and widen the focusing peak y_{TR}. This could be used to enlarge the region affected by the focusing. Solitary waves are evolution waves, which need to propagate in the nonlinear environment for some length to have an appreciable change of wave shape. Using the delayed TR-NEWS method, the high-amplitude focusing area could be widened to an extent necessary to see a wave shape change thanks to the propagation of the signal through a region of the material with nonlinear effects or a region of a defect. In bi-layered aluminium, the widening of the focusing was conducted by a two-pulse delay $\Delta\tau = 0.25$ μs with equal amplitudes $a_i = \{1, 1\}$, which also decreased the side lobes. In case of CFRP the effect of decreasing the side lobes by this particular time-delay was smaller. It was found that the reduction of the side lobes in CFRP is more effective by applying the delayed TR-NEWS impulses directly on the side lobes by using additional smaller waves tailored to arrive at the focusing point at the same time and magnitude as the side lobes but with an opposite amplitude sign.

It was shown in simulations that the focusing is the result of an interaction between several waves. It was also shown in the experiments that the delayed TR-NEWS method allows to slightly offset these interactions to capture different wave interaction moments. In case of linear waves there is a small difference between linear superposition prediction and actual interactions. However, in the presence of nonlinear effects at the focusing point, the actual focusing shape should differ from the linear superposition. Any differences could then be used to detect local sources of nonlinearities at the focusing point. Alternatively, pulse inversion could be used for detection of nonlinear effects.

The delayed TR-NEWS method allows to use the single focused peak as a basis of amplitude modulation to create wave envelopes of arbitrary shapes in the focusing region. This could be used to introduce a low-frequency component into the material by using a high-frequency transmitter.

Future studies will include the analysis of local nonlinear effects in the focusing region using the proposed delayed TR-NEWS signal processing method and performing corresponding FEM simulations. The latter can be used for prediction of the size of defects in the material.

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7. Vitae

Martin Lints is early-stage researcher and a PhD student at Institute of Cybernetics at Tallinn University of Technology, working on a *co-tutelle* thesis with INSA CVL at Blois, France. He is working on the subject of "Applications of solitary waves in Non Destructive Testing and medical imaging". He uses simulations and TR-NEWS experiments to find practical applications of nonlinear waves in complex media, under the supervision of Serge Dos Santos and Andrus Salupere.

Serge Dos Santos is Associate Professor (Hab. Dir. Rech.) at INSA Centre Val de Loire (INSA CVL), Blois, France. His research projects include ultrasonic characterisation of materials using nonlinear acoustics, nonlinear NDT of complex materials and biomedical applications using advanced signal processing methods.

Andrus Salupere is director of Institute of Cybernetics at Tallinn University of Technology. His research fields range from continuum mechanics to nonlinear waves, mechanical properties of materials and numerical experiments. Main work in recent years has been on the subject of deformation waves in microstructured materials, on the propagation and interaction of different kinds of waves.

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