INTERACTION OF DEFORMATION WAVES WITH INTERNAL STRUCTURES IN SOLIDS

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Summary Wave propagation in solids is strongly influenced by accompanying physical effects. The existence of the microstructure (internal structure) of solids leads to dispersive and/or diffraction effects. Two cases must be distinguished: (i) disordered and (ii) regular microstructures. In the first case one-scale and two-scale models are analysed and it is shown that the dispersion together with nonlinearities may cause the emergence of solitary waves. The existence of the negative group velocity is demonstrated in the two-scale model. A typical regular microstructure is an embedded grating made of a different material. The diffraction pattern emerging after a wave passes through a grating is analysed assuming that the matrix and the grating both are elastic. The possible energy localization in such a process is demonstrated.

MICROSTRUCTURE MODELS

Basic concepts

There is a growing interest to microstructured materials in contemporary engineering. The main reason for the wide usage of such materials is related to possible enhancing the properties of materials under complicated loading conditions and to possible design of materials using the emerging physical effects. Here the attention is paid to modelling of wave motion in microstructured materials. In principle, the microstructures might be irregularly distributed (functionally graded materials, alloys, crystallites, etc) or regularly localized (isolated inclusions, gratings, composites). In the first case the dispersive effects are of importance, in the second case the diffraction is the leading process. Figure 1 shows these cases schematically.

(a) (b)

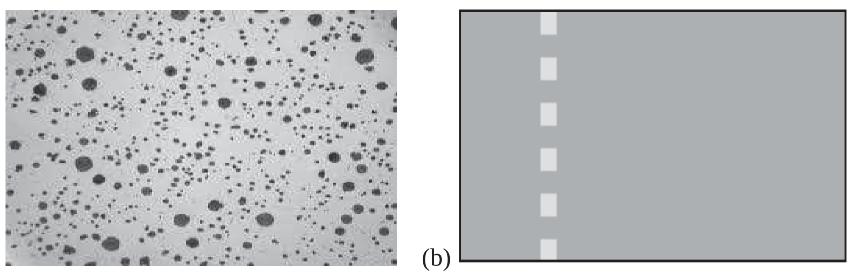

Figure 1: (a) Disordered microstructure, (b) Diffraction grating.

The influence of the disordered microstructure on wave propagation in solids is modelled best if (i) microinertia is taken into account, and (ii) the corresponding dispersion relation includes both acoustic (in-phase) and optical (out-of-phase) branches. In the case of regular internal structure, the wave field can be simulated accurately by solving the elastodynamic equations for the matrix and the scatterers respectively.

Disordered microstructure(s)

The mathematical models for wave motion are derived using the concept of dual internal variables which describes the embedded microstructure(s) as internal fields [1]. A typical one-scale 1D wave equation is the following:

$$u_{tt} = c_1^2 u_{xx} + A \left(u_{tt} - c_2^2 u_{xx}\right)_{xx} + B \left(u_{tt} - c_3^2 u_{xx}\right)_{tt}, \tag{1}$$

with material parameters A and B and velocities c_1, c_2 and c_3. Here the role of dispersion is reflected by higher order derivatives. This basic model can be generalized to describe nonlinear effects and the two-scale situation (a scale within scale). The respective characteristics of waves are analysed including the deriving of governing nondimensional parameters [2]. The two-scale model demonstrates also the existence of the negative group velocity due to the pre-resonant situation of internal modes of close frequencies. The conditions for existence of the negative group velocity are derived [3]. A possibility for the emergence of the negative group velocity exists also for waves in the wool felt. The similar modelling ideas are also implemented for deriving the governing wave equations of biomembranes. In this case the nonlinearities are expressed in terms of displacements rather than in deformations like in solids. This difference affects also the emergence of solitary waves.

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Localized microstructure(s)

The main attention is focused to analysing the influence of properties of gratings on wave motion. In optics, the gratings are considered as rigid, in case of solid mechanics both the matrix and the grating are elastic. In the latter case, wave motion is studied numerically in the plain strain setting. Standard 2-D wave equation is solved by means of a finite volume numerical scheme. The interaction of waves after passing through a periodically ordered elastic grating leads to a self-imaging Talbot effect for the wave length equal or close to the grating size [4]. The effect of energy localization is observed at the vicinity of the grating (see Figure 2).

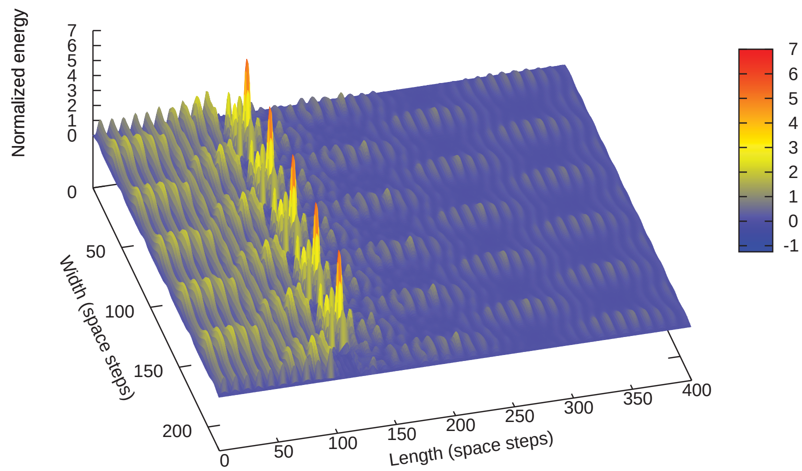

Figure 2: Energy localization in the vicinity of an elastic grating

The optical, acoustic and elastic diffraction processes are compared and differences explained. The observed difference between acoustic and elastic cases is unavoidable and should be taken into account in the topological optimization of composites for the controlling of elastic waves.

CONCLUSIONS

The mechanisms of interaction of waves with the microstructures (internal structure) in solids are analysed. The coupling of the macro- and microstructure leads to changes in velocities and wave profiles. The influence of the disordered microstructure leads to strong dispersive effects including the possible negative group velocity. Such modelling opens vistas for dispersion engineering of materials with specific properties. Numerical simulations of the elastic wave diffraction on elastic gratings show both similarities and differences with the case of a rigid grating where waves are not transmitted through inclusions. The localized microstructure may cause energy localization in emerging wave fields.

Acknowledgements. This research was supported by the European Union through the European Regional Development Fund and the Estonian Research Council (PUT 434).

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