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NUMERICAL SIMULATION OF ELASTIC WAVE DIFFRACTION AT EMBEDDED GRATINGS

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Summary. Results of numerical simulations of two-dimensional elastic wave propagation through gratings in a homogeneous medium are presented. The possible application of the self-imaging Talbot effect to the non-destructive testing is demonstrated.

1 INTRODUCTION

The invention of metamaterials with exotic properties that are unavailable in nature and the state-of-the-art fabrication tools demands a more accurate prediction of wave propagation in structured solids. Any substructure suggests a discontinuity in properties of the structured material. Diffraction of elastic waves at discontinuities results in the dynamic stress concentration. Compared to acoustic and electromagnetic scattering, the elasticity problem is more complicated because of the coexistence of compressional and shear waves that propagate at different speeds. For example, the elastic counterpart of the well-known Talbot effect in optics[1] was elaborated only recently[2]. The Talbot effect represents self-imaging of the wavefield transmitting through the periodic grating at a certain defined distance. The wave patterns formed by the self-imaging provide new possibilities for the non-destructive testing of materials.

Various numerical methods can be applied to computing of elastic wave propagation[3,4,5]. However, only few of them are stable and accurate at discontinuities. Therefore, numerical simulations are performed by the modification of the finite-volume wave-propagation algorithm[6], which provides the stable and high-order accurate solution of wave propagation problems in inhomogeneous solids.

2 PLANE STRAIN ELASTICITY

Numerical simulation of elastic wave propagation is based on the solution of equations of linear elasticity. Neglecting both geometrical and physical nonlinearities, we can write the bulk equations of homogeneous linear isotropic elasticity in the absence of body force as follows[7]:

$$\rho_0 \frac{\partial v_i}{\partial t} = \frac{\partial \sigma_{ij}}{\partial x_j}, \tag{1}$$

$$\frac{\partial \sigma_{ij}}{\partial t} = \lambda \frac{\partial v_k}{\partial x_k} \delta_{ij} + \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right), \tag{2}$$

where t is time, x_j are spatial coordinates, v_i are components of the velocity vector, σ_{ij} is the Cauchy stress tensor, ρ_0 is the density, λ and μ are the Lamé coefficients.

Consider a sample that is relatively thick along x_3, and where all applied forces are uniform in the x_3 direction. Since all derivatives with respect to x_3 vanish, all fields can be viewed as functions of x_1 and x_2 only. This situation is called plane strain. The corresponding displacement component (e.g., the component u_3 in the direction of x_3) vanishes and the others (u_1, u_2) are independent of that coordinate x_3; that is,

$$u_3 = 0, \quad u_i = u_i(x_1, x_2), \quad i = 1, 2. \tag{3}$$

It follows that the strain tensor components, ε_{ij} are

$$\varepsilon_{i3} = 0, \quad \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad i, j = 1, 2. \tag{4}$$

The stress components follow then

$$\sigma_{3i} = 0, \quad \sigma_{33} = \frac{E}{1-2\nu} \left(\frac{\nu}{1+\nu} \varepsilon_{ii} \right), \quad i = 1, 2. \tag{5}$$

$$\sigma_{ij} = \frac{E}{1+\nu} \left(\varepsilon_{ij} + \frac{\nu}{1-2\nu} \varepsilon_{kk} \delta_{ij} \right), \quad i, j, k = 1, 2, \tag{6}$$

where E is the Young's modulus, ν is the Poisson's ratio, δ_{ij} is the unit tensor.

Inversion of Eq. (6) yields an expression for the strains in terms of stresses:

$$\varepsilon_{ij} = \frac{1+\nu}{E} \left(\sigma_{ij} - \nu \sigma_{kk} \delta_{ij} \right), \quad i, j, k = 1, 2. \tag{7}$$

System of Eqs. (1)-(2), specialized to plane strain conditions by Eqs. (3)-(7) is solved numerically by means of the conservative finite-volume wave-propagation algorithm[5,8] modified for the application to front propagation[9,10,11]. The advantages of the wave-propagation algorithm are its stability up to the Courant number equal to unity, high-order accuracy, and energy conservation. The algorithm was successfully applied to wave propagation simulation in inhomogeneous solids[6].

3 DIFFRACTION GRATING

As it was shown recently[2], the Talbot effect can be observed also in the case of elastic waves. The corresponding simulations, however, were performed for the case of perfectly rigid gratings. To extend the results onto fully elastic case, we have chosen high contrast material properties

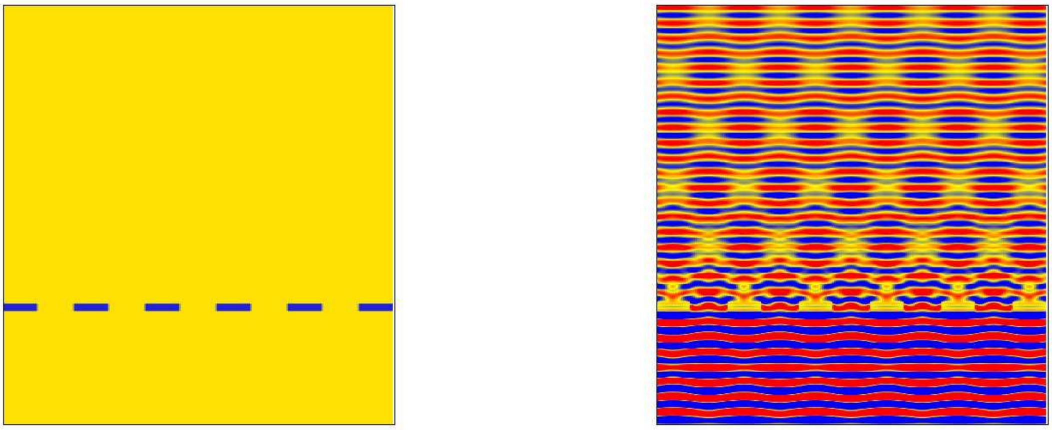

Figure 1: Single grating (left) and wave pattern due to diffraction (right). Wavelength is equal to the width of slits.

for the carrier material and the grating material. Namely, the properties of the carrier material ρ=8900 kg/m^3, c_p=6040 m/s, c_s=3000 m/s correspond to a metal like Nickel, and grating properties correspond to those for Lucite: ρ=1100 kg/m^3, c_p=2610 m/s, c_s=1140 m/s.

As one can see from Fig. 1, the periodic wave pattern is formed though not as sharp as in the case of perfectly rigid grating. To be able to apply the elastic Talbot effect for interferometry, we need to introduce a second grating placed on the Talbot distance, as it proposed for X-Ray Talbot interferometry[12]. The next step is to embed an inclusion (defect) inside the material and see how this defect changes the wave pattern. This situation is presented in Fig. 2.

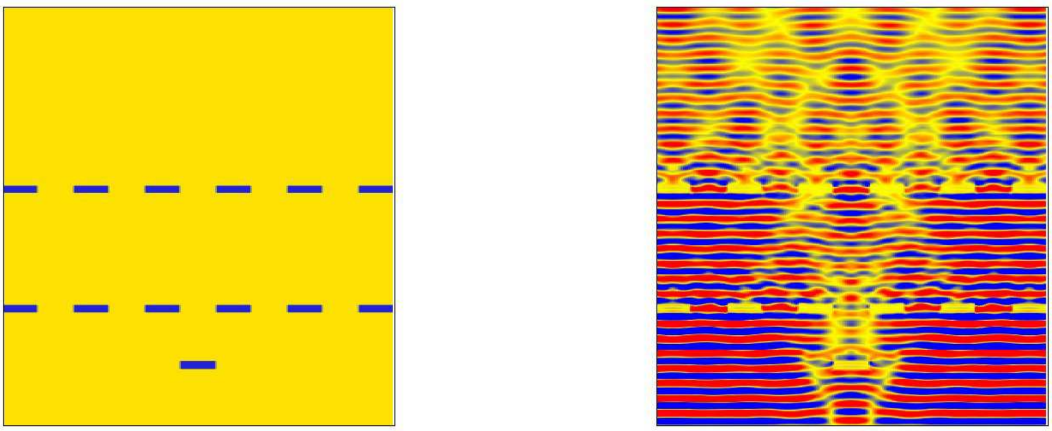

Figure 2: Double grating with an inclusion (left) and wave pattern due to diffraction (right). Wavelength is equal to the width of slits.

The distortion of the wave pattern is significant. It takes a hope to see the image of the defect mirrored by the Talbot self-imaging property. The location of the image of the inclusion can be easily seen after the second grating as a bright area in the center of the upper part of the picture.

4 CONCLUSIONS

1. Numerical simulations show that elastic waves form a periodic pattern after a grating embedded into a material.
2. The distortion of the pattern by a defect in the material can localize the position of the defect.

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