Thermoelastic waves in microstructured solids: dual internal variables approach

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A coupled system of thermoelasticity equations including an evolution equation for an internal variable in addition to the traditional equations of motion and heat conduction is solved numerically. The internal variable is interpreted as a microtemperature or, in other words, as a temperature fluctuation due to the microstructure. The results of computation show that besides the usual diffusion of the macrotemperature in course of time, the wave-type behavior of temperature is observed because of coupling effects between microtemperature, stress, and macrotemperature. Although the observed effect of the microstructure is small, it exists in the case of realistic values of material parameters. The formulated model includes coupling parameters in addition to material properties. The ranges of these parameters are established for the prescribed materials and their influence on the wave-like temperature behavior is analyzed.

Keywords: Thermoelasticity, Microstructured solids, Dual internal variables, Coupled waves.

1. INTRODUCTION

The classical continuum theory is based on the assumption of the homogeneity of materials. Clearly this assumption is a constraint in reflecting the real properties of many materials, especially when dynamic loading is considered. The early studies by Mindlin [1] and Eringen and Suhubi [2] have paved the way to develop mathematical models for inhomogeneous microstructured materials. The problem is becoming very important due to the wide application of alloys, composites, functionally graded materials, etc., in contemporary technology. Generalized continuum theories extend conventional continuum mechanics for incorporating intrinsic microstructural effects into the mechanical behaviour of materials [3–5, etc.]. A leading concept is to separate macro- and microstructure in continua and to formulate the conservation laws for both structures separately [1, 4]. Recently, an approach is proposed to use internal variables [6] into modelling of microstructure. Internal variables are supposed to compensate for our lack of a precise description of a microstructure and describe the influence of internal structures as internal fields [7]. Internal variables are usually taken to be responsible for dissipative processes and must satisfy the second law of thermodynamics (see [6]). The concept of dual internal variables [8] is a generalization from a single internal variable and permits to model also conservative processes. In this way, for example, it is possible to represent the Mindlin micromorphic theory [1] also by using the concept of internal variables and the material formulation of continuum mechanics [9].

The heat conduction phenomenon, starting already from Biot and Fourier, is based on phenomenological models using the theory of continua. Classical theories of heat conduction neglect the microstructure of a material (see, for example [10]). The theory of thermoelasticity is based on coupling between heat conduction and elastic deformation and follows also the assumption of homogeneity. Even intuitively it is understood that the existence of a microstructure should influence the heat conduction and also the propagation of thermoelastic waves. However, as noted by Tamma and Zhou [10]: "The heat transport mechanisms in materials with nonhomogeneous inner structures is clearly not understood to date". Much attention has been paid to the applicability of the Fourier law for describing the propagation of thermoelastic waves – see overviews by Joseph and Preziosi [11], Chandrasekharaiah [12], Tamma and Zhou [10], and monographs by Hetnarski and Reza Eslami [13], Ignaczak and Ostoja-Starzewski [14], Straughan [15], where the modified Cattaneo, Jeffreys, and other models are analysed. These modified models predict the existence of thermal waves but the question about the relaxation time(s) is still not answered with the full physical rigour. It has been shown that given the estimates for the relaxation time, the wave characteristics in modified models are effected at high frequencies [16]. It should be stressed that the modified models (including the two-temperature model by Chen and Gurtin [17]) are also based on the assumption of the homogeneity of the material.

Introducing internal variables into the description of thermoelastic waves opens a new, physically clear way to describe thermal effects in microstructured materials. Even if we consider temperature as an internal variable and displacements as observable variables, a simple example shows that in this case a discontinuity of temperature may exist provided the gradient of the initial excitation is large enough [18]. Based on the concept of dual internal variables [8], a consistent theory of thermoelasticity is built [19]. In

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addition to the coupling of elastic and thermal effects on the macrolevel, this theory includes also the coupling between the macro- and microlevels and on the microlevel. In this case, the wave-like behaviour of microtemperature (fluctuation of temperature in microstructural elements) is possible without proposing the finite speed of thermal waves on the macrolevel. This idea was recently reported by Berezovski and Engelbrecht [20] and the numerical scheme for such calculations presented by A. Berezovski and M. Berezovski [21].

In this paper we focus on coupling effects and leave aside the non-dissipative and fully dissipative cases analyzed in [19]. In Section 2, the corresponding one-dimensional mathematical model for thermoelastic waves in microstructured solids is presented by using the concept of dual internal variables. The attention is paid to thermodynamic fluxes and forces and to the interpretation of internal variables. Section 3 describes the statement of the problem for thermoelastic plane wave propagation an a half space specifying governing equations and coupling parameters. The numerical results shown in Section 4 are obtained by means of the finite-volume algorithm presented earlier in [21]. Finally, in Section 5, the discussion about the novelty of modelling and results is presented and conclusions are written separately in Section 6.

2. ONE-DIMENSIONAL THERMOELASTICITY WITH DUAL INTERNAL VARIABLES

The most suitable framework for the generalization of continuum theory by weakly nonlocal dual internal variables enriched by an extra entropy flux is the material formulation of thermomechanics [22]. We present this framework on the example of one-dimensional thermoelasticity. The full 3D presentation is given in [19].

The one-dimensional motion of thermoelastic conductors of heat is governed by standard local balance laws for linear momentum and energy (no body forces)

$$(\rho_0 v)_t - \sigma_x = 0, \tag{1}$$

$$\left(\frac{1}{2}\rho_0 v^2 + E\right)_t - (\sigma v - Q)_x = 0, \tag{2}$$

and by the second law of thermodynamics

$$S_t + \left(\frac{Q}{\theta} + J\right)_x \geq 0. \tag{3}$$

Here σ is the one-dimensional stress, v is the particle velocity, ρ_0 is the matter density in the reference configuration, Q is the heat flux, E is the internal energy, S is the entropy, θ is temperature, J is the extra entropy flux, subscripts denote derivatives.

The canonical energy equation is derived from Eq. (2) by introducing the free energy per unit volume $W := E - S\theta$ and taking into account balance of linear momentum (1)

$$(S\theta)_t + Q_x = h^{int}, \quad h^{int} := \sigma\varepsilon_t - W_t. \tag{4}$$

Multiplying Eq. (1) by u_x we then check that Eq. (1) yields the following canonical balance of material momentum (cf. [22])

$$P_t - b_x = f^{int} + f^{inh}, \tag{5}$$

where the material momentum P, the material Eshelby stress b, the material inhomogeneity force f^{inh}, and the material internal force f^{int} are defined by [22]

$$P := -\rho_0 u_t u_x, \quad b := -\left(\frac{1}{2}\rho_0 v^2 - W + \sigma\varepsilon\right), \tag{6}$$

$$f^{inh} := \left(\frac{1}{2}v^2\right)(\rho_0)_x - W_x|_{expl}\,, \tag{7}$$

$$f^{int} := \sigma u_{xx} - W_x|_{impl}\,. \tag{8}$$

In the case of non-zero extra entropy flux, the second law of thermodynamics gives

$$-(W_t + S\theta_t) + \sigma\varepsilon_t + (\theta J)_x - \left(\frac{Q}{\theta} + J\right)\theta_x \geq 0, \tag{9}$$

where $\varepsilon = u_x$ is the one-dimensional strain measure.

2.1. Dual internal variables

Now we suppose that the free energy depends on the internal variables φ, ψ and their space derivatives $W = \overline{W}(u_x, \theta, \varphi, \varphi_x, \psi, \psi_x)$. Then the constitutive equations follow

$$\begin{aligned} &\sigma := \frac{\partial\overline{W}}{\partial u_x}, \quad S := -\frac{\partial\overline{W}}{\partial\theta}, \quad \tau := -\frac{\partial\overline{W}}{\partial\varphi}, \\ &\eta := -\frac{\partial\overline{W}}{\partial\varphi_x}, \quad \xi := -\frac{\partial\overline{W}}{\partial\psi}, \quad \zeta := -\frac{\partial\overline{W}}{\partial\psi_x}. \end{aligned} \tag{10}$$

Taking into account constitutive relations (10), we can represent the source term in canonical energy equation (4) as follows:

$$\begin{aligned} h^{int} &= S\theta_t + \tau\varphi_t + \eta\varphi_{xt} + \xi\psi_t + \zeta\psi_{xt} = \\ &= S\theta_t + (\tau - \eta_x)\varphi_t + (\eta\varphi_t)_x + \\ &+ (\xi - \zeta_x)\psi_t + (\zeta\psi_t)_x. \end{aligned} \tag{11}$$

Defining thermal and intrinsic parts of the internal heat source,

$$h^{th} := S\theta_t, \quad \widetilde{h}^{intr} := (\tau - \eta_x)\varphi_t + (\xi - \zeta_x)\psi_t, \tag{12}$$

we arrive at another form of the canonical energy balance

$$(S\theta)_t + \widetilde{Q}_x = h^{th} + \widetilde{h}^{intr}, \tag{13}$$

with the appropriate modification of heat flux

$$\widetilde{Q} = Q - \eta\varphi_t - \zeta\psi_t. \tag{14}$$

Similarly, calculating the internal force

$$\begin{aligned} f^{int} &= S\theta_x + \tau\varphi_x + \eta\varphi_{xx} + \xi\psi_x + \zeta\psi_{xx} = \\ &= S\theta_x + (\tau-\eta_x)\varphi_x + (\eta\varphi_x)_x + \\ &+ (\xi-\zeta_x)\psi_x + (\zeta\psi_x)_x, \end{aligned} \tag{15}$$

and defining of thermal and intrinsic parts of the internal force

$$f^{th} := S\theta_x, \quad \widetilde{f}^{intr} := (\tau-\eta_x)\varphi_x + (\xi-\zeta_x)\psi_x, \tag{16}$$

we can represent the canonical equation of momentum as

$$P_t - \widetilde{b}_x = f^{th} + \widetilde{f}^{intr}, \tag{17}$$

with the appropriate modification of the Eshelby stress

$$\widetilde{b} = -\left(\frac{1}{2}\rho_0 v^2 - W + \sigma\varepsilon - \eta\varphi_x - \zeta\psi_x\right). \tag{18}$$

Comparing modified heat flux (14) with the expression in parenthesis in the last term of dissipation inequality (9), we conclude that the extra entropy flux should be chosen as

$$J = -\theta^{-1}\eta\varphi_t - \theta^{-1}\zeta\psi_t. \tag{19}$$

The latter means that dissipation inequality (9) reduces to

$$\Phi = (\tau-\eta_x)\varphi_t + (\xi-\zeta_x)\psi_t - \left(\frac{Q-\eta\varphi_t-\zeta\psi_t}{\theta}\right)\theta_x \geq 0. \tag{20}$$

If we assume that the intrinsic dissipation is independent of temperature gradient, then we are forced to modify the Fourier law

$$Q - \eta\varphi_t - \zeta\psi_t = -k\theta_x, \tag{21}$$

to satisfy the thermal part of the dissipation inequality. Here $k > 0$ is the thermal conductivity.

The remaining intrinsic part of dissipation inequality (20) is nothing else but a linear combination of products of thermodynamic fluxes and forces (Table 1). It is straightforward to point out the simplest solution of the intrinsic part of the dissipation inequality assuming linear relationships between the thermodynamic fluxes and their multipliers, the thermodynamic forces

TABLE I: Thermodynamic fluxes and forces.

	Internal 1	Internal 2
Fluxes	φ_t	ψ_t
Forces	$\tau-\eta_x$	$\xi-\zeta_x$

$$\varphi_t = R_{11}(\tau-\eta_x) + R_{12}(\xi-\zeta_x), \tag{22}$$

$$\psi_t = R_{21}(\tau-\eta_x) + R_{22}(\xi-\zeta_x). \tag{23}$$

Equations (22) and (23) are evolution equations for dual internal variables φ and ψ that close the thermoelasticity theory.

2.2. Interpretation of internal variables

Coefficients in the right hand side of Eqs. (22)-(23) constitute the conductivity matrix $\mathbf{R}$

$$\mathbf{R} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}. \tag{24}$$

These coefficients may depend on state variables. For simplicity, we consider here constant coefficients.

The interpretation of internal variables relates to properties of the conductivity matrix. The conductivity matrix can be symmetric, if Onsager symmetry relations are valid between its coefficients, or antisymmetric in the case of Casimir relations. Using the fact that internal variables are not specified yet, we may assume that a symmetric conductivity matrix is represented in its diagonal form

$$\mathbf{R} = \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix}, \tag{25}$$

where r_1 and r_2 are real and distinct eigenvalues of the matrix $\mathbf{R}$. In this case, the intrinsic part of the dissipation inequality (20) is a quadratic form

$$\Phi = r_1(\tau-\eta_x)^2 + r_2(\xi-\zeta_x)^2 \geq 0, \tag{26}$$

and its positive definiteness is provided by the non-negativity of the eigenvalues. The absence of coupling between internal variables takes the introduction of the second one superfluous. It can be omitted without the loss of generality. The situation is reduced to the theory of single internal variable of a dissipative nature [23].

We are interested in coupling, therefore, we have to look for a more specific case with the conductivity matrix of the form

$$\mathbf{R} = \begin{pmatrix} 0 & R_{12} \\ -R_{12} & R_{22} \end{pmatrix}. \tag{27}$$

The conductivity matrix is antisymmetric, but with a contribution to the entropy production. The intrinsic part of the dissipation inequality (20)

$$\Phi = R_{22}(\xi-\zeta_x)^2 \geq 0, \tag{28}$$

is satisfied by the non-negativity of the coefficient R_{22}, but internal variables cannot be considered relating to a microdeformation due to dissipation. As shown in [19], they can be interpreted as the microtemperature.

3. STATEMENT OF THE PROBLEM

To be more specific, let us prescribe explicitly the dependence of the free energy on state variables. We use a quadratic free energy function like in [19]

$$W = \frac{1}{2}(\lambda + 2\mu)\, u_x^2 - \frac{\rho_0 c_p}{2\theta_0}(\theta - \theta_0)^2 + \\ + m(\theta - \theta_0)\, u_x + A\varphi_x u_x + \frac{1}{2}C\varphi_x^2 + \frac{1}{2}D\psi^2. \tag{29}$$

Here $u_x = \varepsilon$ is the one-dimensional strain measure, c_p is the heat capacity, the thermoelastic coefficient m is related to the dilatation coefficient α and the Lamé coefficients λ and μ by $m = -\alpha(3\lambda + 2\mu)$, θ_0 is the reference temperature, A, C, and D are material parameters, subscripts denote derivatives. As one can see, only contributions of gradients of the primary internal variable and of the second internal variable itself are included here. Constitutive relations (10) determine the macrostress σ

$$\sigma := \frac{\partial \overline{W}}{\partial u_x} = (\lambda + 2\mu)\, u_x + m(\theta - \theta_0) + A\varphi_x, \tag{30}$$

the microstress η

$$\eta := -\frac{\partial \overline{W}}{\partial \varphi_x} = -C\varphi_x - Au_x, \tag{31}$$

zero interactive internal force τ, since free energy does not depend explicitly on φ,

$$\tau := -\frac{\partial \overline{W}}{\partial \varphi} = 0, \tag{32}$$

and auxilary quantities related to the second internal variable

$$\zeta = -\frac{\partial \overline{W}}{\partial \psi_x} = 0, \quad \xi = -\frac{\partial \overline{W}}{\partial \psi} = -D\psi, \tag{33}$$

correspondingly.

According to evolution equations (22)-(23) and accounting the form of the conductivity matrix (27), we have

$$\varphi_t = R_{12}(\xi - \zeta_x), \\ \psi_t = -R_{12}(\tau - \eta_x) + R_{22}(\xi - \zeta_x). \tag{34}$$

Due to Eq. (33) evolution equation for the primary internal variable $(34)_1$ is reduced to

$$\varphi_t = -RD\psi, \tag{35}$$

and Eq. $(34)_2$ becomes (denoting $I = 1/R_{12}^2 D$)

$$I\varphi_{tt} + \frac{R_{22}}{R_{12}^2}\varphi_t = C\varphi_{xx} + Au_{xx}, \tag{36}$$

which is a Cattaneo-Vernotte-type hyperbolic equation [11] for the primary internal variable φ.

Correspondingly, energy conservation equation (4) determine the heat conduction equation

$$\rho_0 c_p\, \theta_t - (k\theta_x)_x = m\theta_0 u_{xt} + \frac{R_{22}}{R_{12}^2}\varphi_t^2, \tag{37}$$

which is influenced by a source term depending on the internal variable φ. Equation of motion (1) obtains a source term as well

$$\rho_0 u_{tt} = (\lambda + 2\mu)\, u_{xx} + m\theta_x + A\varphi_{xx}, \tag{38}$$

due to definition of macrostress (30). Therefore, all three governing equations (36) - (38) are coupled. This coupling can induce wave-like propagation for macrotemperature [20, 21].

To verify the propagation of expected thermal waves in a realistic situation, we solve governing equations (36) - (38) in the case of plane wave motion in a thermoelastic half-space by means of the wave propagation algorithm explained in detail in [21]. It is assumed that the half-space is initially at rest. A heat pulse is applied at the traction free boundary plane for the first 60 time steps following the rule

$$\bar{\theta}(0,t) = \frac{1}{2}\left(1 + cos\left(\frac{\pi(t - 30\Delta t)}{30}\right)\right). \tag{39}$$

The time history of the thermal loading is shown in Fig. 1.

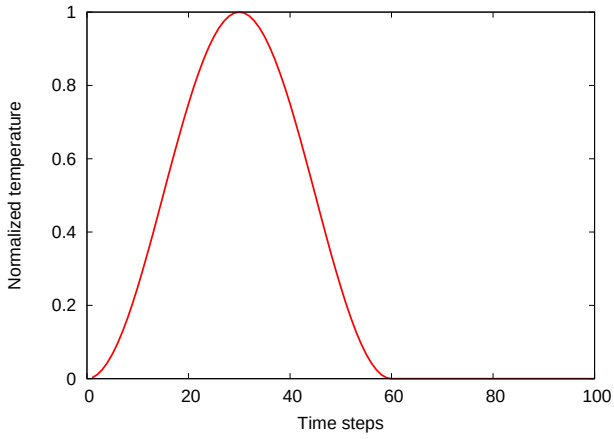

FIG. 1: Time history of thermal loading.

There are three kinds of parameters to be prescribed for the performance of calculations: material parameters, scale parameters, and coupling parameters. Material parameters

for the carrier medium are chosen similar to that for silicon [Si] [24]:
The macroscopic density, ρ_0, is equal to 2390 kg/m^3, the Lamé coefficients $\lambda = 48.3$ GPa, and $\mu = 61.5$ GPa, the heat capacity, $c_p = 800$ J/(kg K), the reference temperature, $\theta_0 = 300$ K, the thermal coductivity, $k = 149$ W/(m K), the thermal expansion coefficient, $\alpha = 2.610^{-6}$ 1/K.

It is assumed that the microstructure is formed by copper particles embedded randomly into silicon. Material parameters of copper are [25]:
The macroscopic density, ρ_0, is equal to 8960 kg/m^3, the Lamé coefficients $\lambda = 101.5$ GPa, and $\mu = 47.75$ GPa, the heat capacity, $c_p = 386$ J/(kg K), the reference temperature, $\theta_0 = 300$ K, the thermal coductivity, $k = 401$ W/(m K), the thermal expansion coefficient, $\alpha = 16.510^{-6}$ 1/K.

The scale of excitation, U_0, is chosen as 6% of the length of the computational domain, L, so that

$$\frac{U_0}{L} = 0.06. \tag{40}$$

The scale of the microstructure, l, is even less

$$\frac{l}{L} = 0.002. \tag{41}$$

Coupling parameters used in calculations are

$$\frac{R_{22}}{R_{12}^2}, \quad A, \quad \text{and} \quad C, \tag{42}$$

because the parameter D is related to the microinertia measure I which can be identified with the density of copper. We will study the influence of the values of the coupling parameters on the propagation of thermoelastic waves in the microstructured material.

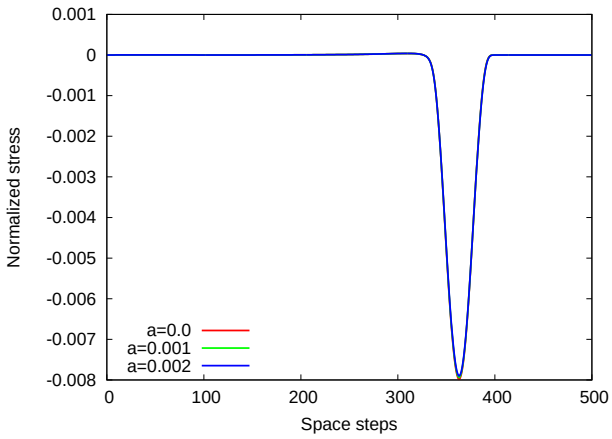

FIG. 2: Stress distribution at 400 time steps for different values of the normalized model parameter $a = A/\rho_0 c_0^2$. ($C/\rho_0 c_0^2 = 1.0$, $R_{22}l/R_{12}^2\rho_0 c_0 = 0.5$.)

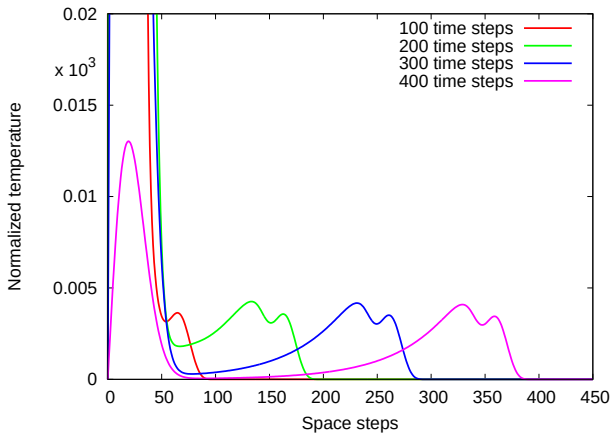

FIG. 3: Distribution of macrotemperature at different time instants for values of model parameters $a = A/\rho_0 c_0^2 = 0.02$, $C/\rho_0 c_0^2 = 1.0$, $R_{22}l/R_{12}^2\rho_0 c_0 = 0.2$.

4. NUMERICAL RESULTS

All calculations were performed by means of the finite-volume numerical scheme [21] using the value of the Courant number 0.98. This scheme is a modification of the previously reported conservative finite-volume algorithm [26, 27] adapted for microstructure modeling. It belongs to a broad class of finite-volume methods for thermomechanical problems [28–30]

To exclude the direct influence of stress field on the macrotemperature, it was assumed that the velocity gradient in Eq. (37) is negligible. First, massive diagnostic calculations determined the range of model parameters where the numerical scheme is stable:

$$0 \le \frac{R_{22}l}{R_{12}^2\rho_0 c_0} \le 4, \quad 0 \le \frac{A}{\rho_0 c_0^2} \le 0.02, \quad 0 \le \frac{C}{\rho_0 c_0^2} \le 2, \tag{43}$$

Then it was recognized that the influence of microstructure on the amplitude of the stress wave is small. It is illustrated in Fig. 2, where the difference in the corresponding curves can be seen only under a large zoom. The corresponding influence of microstructure on the macrotemperature is small as well, but it demonstrates a wave-like nature as it can be seen in Fig. 3. Without coupling, these wave-like temperature profiles do not appear [21].

In spite of the small amplitude of the microstructure influence, it is possible to analyze its variation depending on the values of coupling parameters. The parameter A determines the magnitude of the coupling between the equation of motion and the evolution equation for microtemperature. If this parameter is zero, there is no coupling and the variation of the microtemperature is absent (Fig. 4).

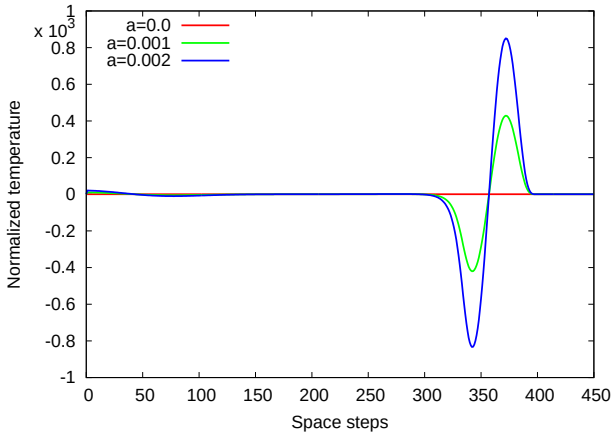

FIG. 4: Microtemperature variation at 400 time steps for different values of the normalized model parameter $a = A/\rho_0 c_0^2$. (Values of C and R_{22}/R_{12}^2 are fixed.)

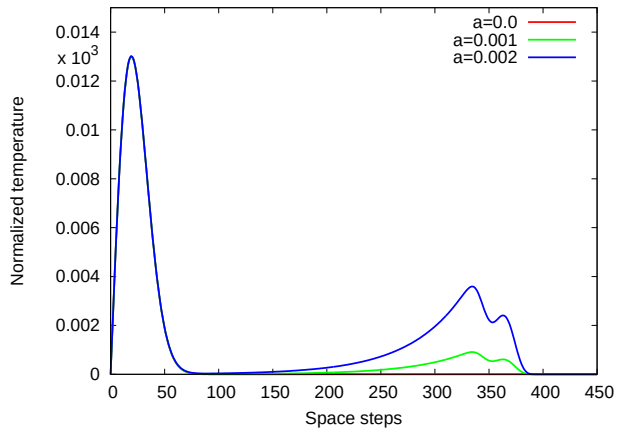

FIG. 5: Macrotemperature variation at 400 time steps for different values of the normalized model parameter $a = A/\rho_0 c_0^2$. (Values of C and R_{22}/R_{12}^2 are fixed.)

Here values of parameters C and R_{22}/R_{12}^2 were chosen as $c = C/\rho_0 c_0^2 = 1.0$ and $r = R_{22} l/R_{12}^2 \rho_0 c_0 = 0.5$, respectively. The higher the value of the parameter A, the higher amplitude of the microtemperature is achieved. The same is true for the variation of the macrotemperature, as it is seen in Fig. 5. It is clearly seen that there is practically no contribution of the microtemperature close to the boundary. Fluctuations of the microtemperature are induced by the stress and they are localized in the zone with non-zero stress values. In its turn, the macrotemperature is affected by the microtemperature due to the nonlinear term in the right hand side of Eq. (37).

A similar trend is observed for the variation of the material parameter C, both for the microtemperature (Fig. 6) and for the macrotemperature (Fig. 7). This material parameter plays no role in coupling and influences the magnitude of the microtemperature. The fixed values of parameters A and R_{22}/R_{12}^2 in Figs. 6 and 7 were chosen as $a = A/\rho_0 c_0^2 = 0.02$ and $r = R_{22} l/R_{12}^2 \rho_0 c_0 = 0.2$, respectively.

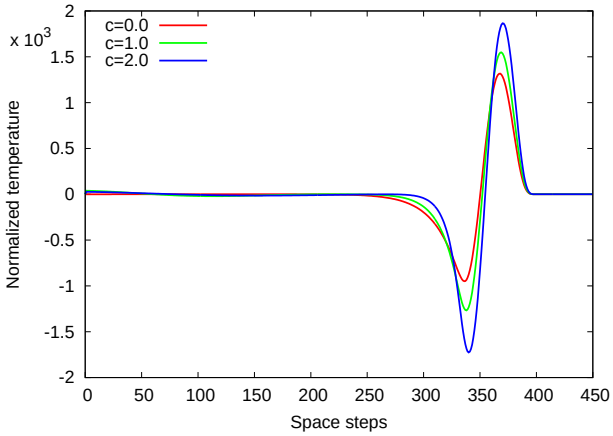

FIG. 6: Microtemperature variation at 400 time steps for different values of the normalized model parameter $c = C/\rho_0 c_0^2$. (Values of A and R_{22}/R_{12}^2 are fixed.)

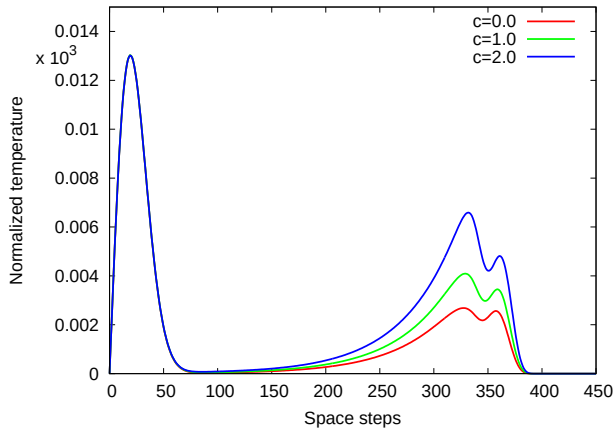

FIG. 7: Macrotemperature variation at 400 time steps for different values of the normalized model parameter $c = C/\rho_0 c_0^2$. (Values of A and R_{22}/R_{12}^2 are fixed.)

The coupling between micro- and macrotemperatures is provided by the ratio of the coefficients of the conductivity matrix R_{22}/R_{12}^2. However, its variation shows the reverse trend: maximal amplitudes of the microtemperature and its influence on the macrotemperature are observed for small values of this ratio (Figs. 8 and 9). This is because of the appearance of this parameter at the damping term in the left hand side of evolution equation of microtemperature (36). Parameters A and C in Figs. 8 and 9 had values satisfying $a = A/\rho_0 c_0^2 = 0.02$ and $c = C/\rho_0 c_0^2 = 1.0$, respectively.

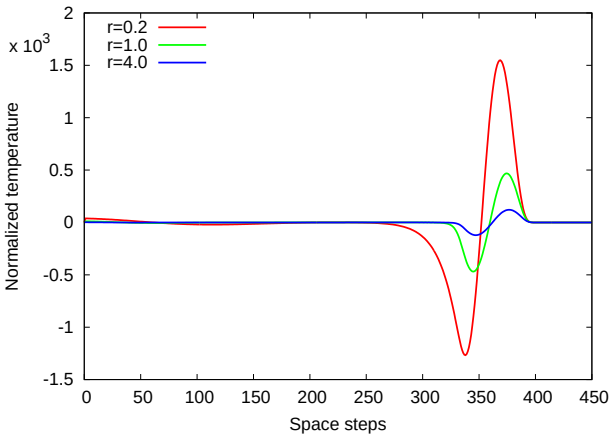

FIG. 8: Microtemperature variation at 400 time steps for different values of the normalized model parameter $r = R_{22}l/R_{12}^2\rho_0 c_0$. (Values of A and C are fixed.)

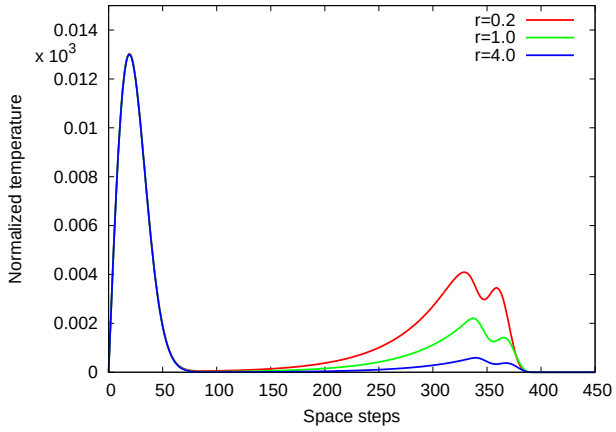

FIG. 9: Macrotemperature variation at 400 time steps for different values of the normalized model parameter $r = R_{22}l/R_{12}^2\rho_0 c_0$. (Values of A and C are fixed.)

5. DISCUSSION

Mathematical model (36) - (38) represents a coupled system of thermoelasticity equations where besides the traditional equations of motion and heat conduction, an evolution equation for the internal variable φ is included. This internal variable is interpreted as a microtemperature or, in other words, as a temperature fluctuation due to the microstructure.

The results of computation show that besides the usual diffusion of the macrotemperature in course of time, the wave-type behavior of temperature is seen (Figs. 3, 5, 7, 9). This is possible because of coupling effects between microtemperature (notice that Eq. (36) has a hyperbolic operator), stress, and macrotemperature. The reason why the microtemperature is governed by a hyperbolic equation is based on the inclusion of time rates of internal variables into the modified heat flux (Eq. 14).

Although the observed effect of the microstructure is small, it exists in the case of realistic values of material parameters. This effect can be amplified by a choice of suitable materials or even by a design of corresponding artificial materials.

The formulated model includes coupling parameters in addition to material parameters. Given the present form of free energy (29) and evolution equations for internal variables (34), we have three coupling parameters. The ranges of these parameters are established for the prescribed materials (see condition (43)), and their influence on the wave-like temperature behavior is analyzed. To exclude the direct influence of stress on the temperature field, the velocity gradient is neglected.

6. CONCLUSIONS

To sum up, it is shown that due to the existence of a microstructure (inhomogeneity of a material), the heat wave in a solid which accompanies the stress wave, may cause temperature fluctuations well ahead of the usual thermal diffusion process.

This paper has a clear novelty in comparison with previously reported results:

- The material formulation is clearly explained (omitted in earlier studies);
- The theoretical part includes detailed analysis of h^{int} and f^{int}, which are crucial for the phenomenon;
- The real material parameters are used and ranges of coupling parameters are established;
- The study is focused on the influence of the variation of coupling parameters, which open doors for experiments.

Clearly the problem is worth to further studies, especially the values of coupling parameters should be estimated either from experiments or from mesoscopic calculations. However, the model and numerical simulation described above cast more light on the extremely interesting phenomena.

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