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Wave propagation in heterogeneous materials with secondary substructure

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ABSTRACT

We study numerically the influence of the presence of a complex internal structure of laminates consisting of layers of different properties on the dynamic response of a material. The influence of the mutual position of different layers is demonstrated by example of double periodic laminates.

Key Words: wave propagation; solids with microstructure; numerical simulation;

INTRODUCTION

The behavior of many materials of engineering interest is often influenced by an existing or emergent microstructure. In general, the components of such a microstructure have different material properties, resulting in a macroscopic material behavior like in highly anisotropic and inhomogeneous materials.

Due to the complex structure of such media, wave propagation is accompanied by reflection, refraction, diffraction and scattering phenomena that are difficult to quantify [1]. Small-scale changes in a heterogeneous material's microstructure can have major effects in its macro-scale behavior [2]. Modeling macroscopic mechanical properties of materials in relation to their substructure is a long standing problem in materials science [3].

In order to construct an appropriate model of response of a material with more refined internal structures, the first step is the understanding of the behavior of the material with at least two different substructures.

An important outcome from numerical simulations in solids with double substructure [4] was that not only the presence of the double substructure, but also its internal geometry and the mutual position of different layers are significant.

The aim of this paper is to investigate the influence of the mutual position of double internal structure on the dynamic response of heterogeneous material. For this purpose, we use numerical simulations of one-dimensional wave propagation in layered materials with several compositions of two substructures.

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Numerical simulations are performed by means of a finite-volume numerical scheme modifying the wave-propagation algorithm [5] by introduction of excess quantities instead of numerical fluxes, which simplifies the solution of the Riemann problem at boundaries between computational cells at each time step [6].

RESULTS OF NUMERICAL SIMULATIONS

The simplest example of heterogeneous media is a periodic laminate composed by layers with different properties. The one-dimensional wave propagation in linear elasticity is governed by the conservation of linear momentum [7]

$$\rho(x)\frac{\partial v}{\partial t} - \frac{\partial \sigma}{\partial x} = 0 \qquad (1)$$

and the kinematic compatibility condition

$$\frac{\partial \varepsilon}{\partial t} - \frac{\partial v}{\partial x} = 0. \qquad (2)$$

Here t is time, x is space variable, the particle velocity $v = u_t$ is the time derivative of the displacement u, the one-dimensional strain $\varepsilon = u_x$ is the space derivative of the displacement, σ is the Cauchy stress, and ρ is the material density.

The compatibility condition (2) follows immediately from the definitions of the strain and the particle velocity. Equations (1) and (2) contain three unknowns: v, σ and ε. The closure of this system is achieved by a constitutive relation, which in the simplest case is the Hooke's law

$$\sigma = \rho(x)c^2(x)\varepsilon, \qquad (3)$$

where $c(x)$ is the corresponding longitudinal wave velocity. The indicated explicit dependence on the location in space x means that the medium is materially inhomogeneous. The resulting system of equations (1) - (3) is solved numerically.

To investigate the influence of two substructures on the material behavior, the propagation of a pulse in a one-dimensional medium, which can be interpreted as an elastic bar, is considered. This bar is assumed homogeneous except of a region of length l in the middle of the bar, which contains periodically alternating layers of size d (Fig. 1). Total length of the bar L is equal to $5000\,\Delta x$. We set the length l of the inhomogeneous domain equal to $900\,\Delta x$ for all numerical simulations (Δx is the constant space step used in simulations). The density and the longitudinal velocity in the bar are chosen as $\rho = 4510\ kg/m^3$ and $c = 5240\ m/s$, respectively.

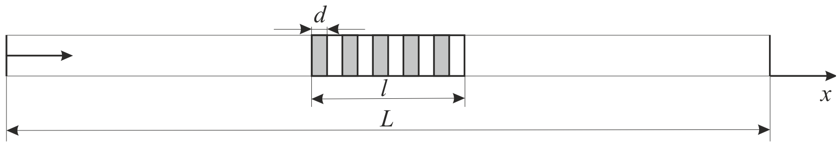

Figure 1. Geometry of the problem

We consider equal number of layers of each material in different substructure compositions within the inhomogeneous domain (Fig. 2, 4). We assume that the material of the bar itself is the hardest and of the highest density, and call it "hard" material in what follows. The substructure may contain two different materials. The one with the lowest density and longitudinal velocity ($\rho = 2703\ kg/m^3$ and $c = 5020\ m/s$, respectively) we call "soft" and the one with the intermediate parameters ($\rho = 3603\,kg/m^3$ and $c = 5100\ m/s$) we call "intermediate" material. The scale of substructure - the size of each sublayer - is set equal to $30\Delta x$.

The shape of the pulse is formed by an excitation of the strain at the left boundary for a limited time period $(0 < t < \lambda\Delta t)$

$$u_x(0,t) = 1 + \cos\left(2\pi\left(t - \frac{\lambda}{2}\Delta t\right)/\lambda\right),$$

where λ is the pulse length. After that the strain is equal to zero. At the right boundary we apply non-reflective boundary conditions. The arrow at the left end of the bar in Fig. 1 shows the direction of the pulse propagation. Numerical simulations were performed for lengths of a pulse $\lambda = 30\Delta x$ for each analyzed substructure composition.

The results of propagation of the pulse through different compositions of the substructure are compared against the behavior of the reference pulse, the propagation of which is calculated for a homogeneous bar of the "hard" material. The resulting pulse is recorded at 4600 time steps. We compare two different compositions of double substructure with equal numbers of layers of each material (Fig. 3, 5).

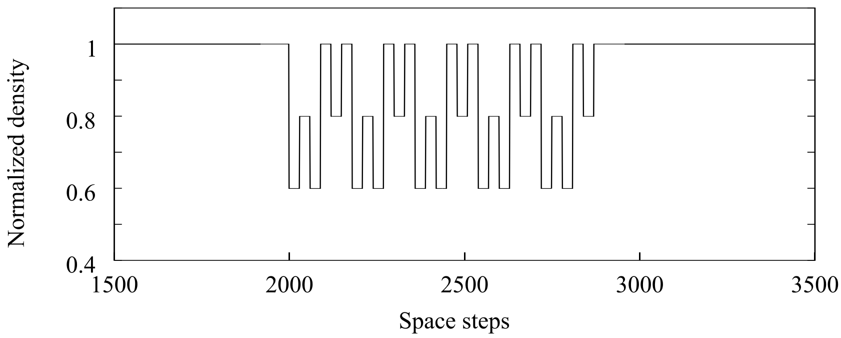

Figure 2. Double substructure composition I

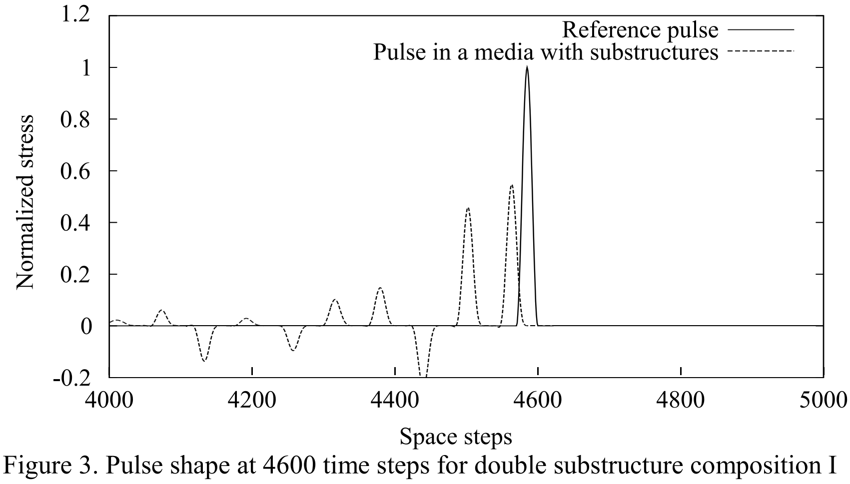

Figure 3. Pulse shape at 4600 time steps for double substructure composition I

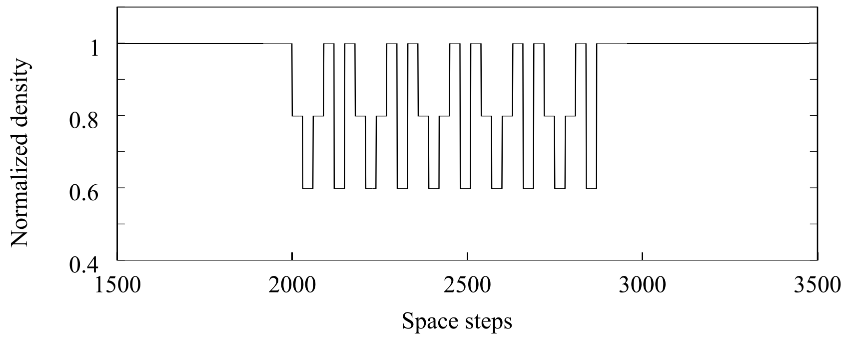

Figure 4. Double substructure composition II

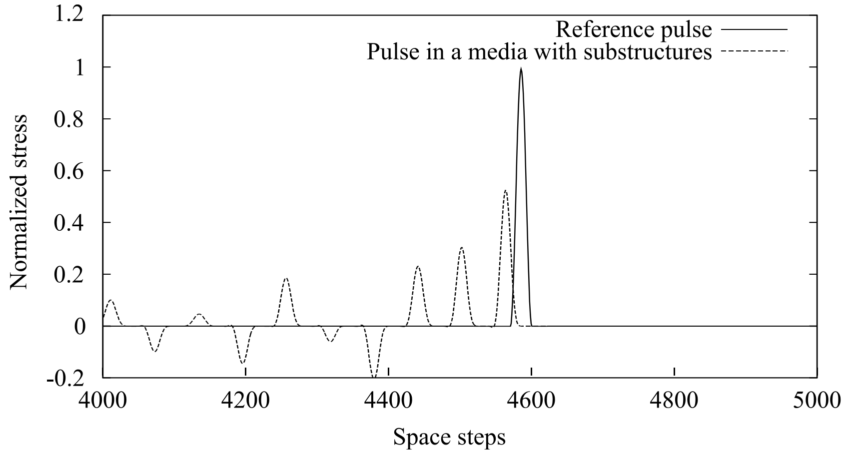

Figure5. Pulse shape at 4600 time steps for double substructure composition II

As it can be seen, the resulting form of transmitted pulses is dependent on the mutual position of layers. The presented results of numerical simulations confirm that the influence of mutual position of layers in the double substructure is very important and needs to be taken to account in the construction of an appropriate model of dynamic response of heterogeneous materials.

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