Proceedings of the Estonian Academy of Sciences,
2010, **59**, 2, 61–65
doi: 10.3176/proc.2010.2.01
Available online at www.eap.ee/proceedings

EDITORIAL

Highlights in the research into complexity of nonlinear waves

Jüri Engelbrecht, Arkadi Berezovski, and Tarmo Soomere

Centre for Nonlinear Studies, Institute of Cybernetics at Tallinn University of Technology, Akadeemia tee 21, 12618 Tallinn, Estonia; je@ioc.ee

Received 5 March 2010, accepted 17 March 2010

Abstract. We reflect highlights of studies into a variety of phenomena reflecting the complexity of underlying nonlinear processes in a selection of research disciplines in the Centre of Nonlinear Studies (CENS), presented in the International Conference on Complexity of Nonlinear Waves, 5–7 October 2009, Tallinn, Estonia. We emphasize the similarity of mathematical description of and potential synergy arising from complementary studies in general soliton science, wave propagation in microstructured and functionally graded materials, related inverse problems, issues of nondestructive testing, weak resonant interactions of water waves, wave transformation and run-up, soliton interactions in shallow water, and selected problems of passive scalar turbulence.

Key words: complexity, nonlinear waves, microstructured medium, nondestructive testing, surface waves, soliton interactions, turbulence.

INTRODUCTION

The theory of nonlinear complex systems is related to interdisciplinary studies which enable us to understand macroscopic phenomena via nonlinear interactions of constituents of a whole. The contemporary understandings of complex systems stem from the fundamental ideas of Prigogine (Prigogine and Stengers, 1984; Mainzer, 1997). Nowadays clear signatures of complex phenomena are known: multiplicity, emergence, nonlinearity, interaction of constituents and/or processes, etc. (Nicolis and Nicolis, 2007; Érdi, 2008). Mechanics is full of such examples, starting from studies of the movement of planets, movement of a pendulum, waves on a free surface of fluids and so on. Nonlinear dynamics has greatly influenced a dynamic view of the world not only in mechanics but also in physics in a broad sense, in chemistry, biology and lately also in physiology and social sciences (West, 1985; Strogatz, 1994; Scott 1999).

During the last decade the Centre for Nonlinear Studies (CENS) of the Institute of Cybernetics (IoC) at Tallinn University of Technology (TUT) has been active in many subfields of nonlinear dynamics. Launched in 1999 by the Department of Mechanics and Applied Mathematics of the IoC, the Biomedical Engineering Centre of TUT and the Chair of Geometry of the Institute of Pure Mathematics and the wave research group of the Estonian Marine Institute at the University of Tartu, CENS meets further challenges as a vibrant international centre. Its strength is built on a multidisciplinary approach, involving wave motion, fractality, biophysics, signal processing, etc. In this special issue of the *Proceedings of the Estonian Academy of Sciences* attention is focused on wave motion and general dynamic processes. To facilitate the understanding of the results, some earlier studies are briefly described. Out of the full range of research in CENS, which now involves also control theory and proactive systems (for *Annual Reports* see http://cens.ioc.ee), here we briefly summarize highlights of studies of deformation waves in solids, water waves, and fractal processes, which were the key topics of the International Conference on Complexity of Nonlinear Waves (CNLW), held on 5–7 October 2009 in Tallinn, Estonia.

WAVES IN SOLIDS

The concept of nonlinearity in mechanics is not new (cf., for example, the three-body system). In introducing nonlinearities into the governing equations of wave motion in solids, other effects of the same order must also be taken into account. The dispersive effects together with nonlinearity may lead to the emergence of solitary waves and the interaction of various fields may greatly influence the wave motion. The emergence of soliton trains is known from the analysis of the celebrated Korteweg–de Vries (KdV) equation. But it is not only the emergence of solitons which is of interest. The striking pattern of soliton trajectories emerging for the KdV solitons is described by Salupere et al. (2003) and the possibility of amplifying the 'hidden' solitons is shown by Engelbrecht and Salupere (2005).

Nonlinear interaction of ultrasonic waves with pre-stress fields or inhomogeneous distribution of material properties opens new possibilities for nondestructive evaluation of material properties or residual stresses (Braunbrück and Ravasoo, 2005; Ravasoo, 2007). Proper modelling of microstructured materials draws attention to hierarchies of waves where dispersive effects are described over multiple scales (Engelbrecht et al., 2005). Such full models (two-wave models in the 1D case) permit better analysis of the formation and interaction of solitary waves than one-wave evolution equations (Salupere et al., 2008). They also open new ways to solve the inverse problems in order to determine the properties of microstructure (Janno and Engelbrecht, 2005, 2008). The models of microstructured solids are analysed from various viewpoints (Berezovski et al., 2009), including the concept of internal variables which weaves all the models into the thermodynamic framework. On the other hand, the thermodynamically consistent numerical algorithms are derived for solving the wave problems (Berezovski et al., 2008). Such algorithms enable solution of various problems, including the propagation of fronts and cracks and waves in functionally graded materials (Berezovski et al., 2003). For example, for waves in a material where the microstructure is formed of piecewise homogeneous layers, the role of nonlinearity is crucial in order to match experimentally measured results (Berezovski et al., 2006).

SIMILARITY OF WAVES IN FLUIDS AND SOLIDS

Waves on the water surface form a fascinating medium of studies of complexity and associated emerging features. The relevant phenomena are familiar to everybody and can be easily visualized and verified against simple experiments. This environment, however, is one of the few fields of physical oceanography that has substantially contributed to cutting edge fundamental research. For example, Hasselmann's theory of resonant interactions of surface waves (Hasselmann, 1962) preceded by a few years the discovery of solitons and served as one of the first rigorously derived Boltzmann-type equations. Recently, surface soliton interactions in deep ocean have initiated a completely new field of research concerning optical rogue waves (Solli et al., 2007).

Studies of water waves in CENS focus on wave–wave and soliton interaction, anomalies of wave fields, ship wakes, and extreme waves. The essence of all these studies is the possibility of energy exchange between different components of multi-componential wave fields. This feature becomes evident in a range of different phenomena, from slow changes in the spectral composition of complex wave fields to almost explosive formation of unexpectedly high and steep wave humps in nonlinear interactions of solitons.

From the viewpoint of complexity, a striking feature for waves in solids and waves in fluids is the conceptual similarity of models which stresses the interaction between the constituents. Leaving aside the standard balance of momentum as a basis for equations of motion in continua, there is a possibility of interpreting the interactive forces and/or fields in a similar way for both cases. In solids, Maugin (1993) has shown that on the material manifold, the governing wave equation based on the balance of the canonical (material) momentum reads (see Engelbrecht, 2010, eq. (1)):

$$\left.\frac{\partial \mathbf{P}}{\partial t}\right|_{\mathbf{X}} + \mathrm{Div}_R \mathbf{b} = \mathbf{f}^{\mathrm{inh}} + \mathbf{f}^{\mathrm{ext}} + \mathbf{f}^{\mathrm{int}}, \tag{1}$$

where $\mathbf{P}$ is the material momentum (pseudo-momentum), $\mathbf{b}$ is the material Eshelby stress, and $\mathbf{f}^{\mathrm{inh}}$, $\mathbf{f}^{\mathrm{ext}}$, $\mathbf{f}^{\mathrm{int}}$ are the material inhomogeneity force, the material external (body) force, and the material internal force, respectively.

For water surface waves, the slow energy exchange within resonance quartets is described by the so-called kinetic equation (Hasselmann, 1962; see also Onorato et al., 2009), which today is the core of spectral wave prediction models:

$$\frac{\partial N_1}{\partial t} + \nabla \cdot (\mathbf{c}_{\mathrm{g}} N_1) = S_{\mathrm{NL}} + S_{\mathrm{diss}} + S_{\mathrm{In}}. \tag{2}$$

Here $N(\mathbf{k})$ is the wave action spectral density, $\mathbf{c}_{\mathrm{g}}$ is the group velocity, the so-called interaction integral

$$S_{\mathrm{NL}} = \int |T_{1234}|^2 [N_3 N_4 (N_1 + N_2) - N_1 N_2 (N_3 + N_4)] \times \delta^2(\Delta\mathbf{k})\delta(\Delta\omega)\, d\mathbf{k}_{234},$$

usually characteristic of the Boltzmann equation (where it describes collisions between particles), integrates the contribution from nonlinear interactions into changes in the wave fields, T_{1234} is the interaction coefficient, $\Delta\mathbf{k} = \mathbf{0}$, $\Delta\omega = 0$ are the resonance conditions for the wave vectors and angular frequencies, respectively, $S_{\mathrm{diss}} = -\gamma_D(\mathbf{k})N(\mathbf{k})$ reflects dissipation of wave energy due to different reasons and $S_{\mathrm{In}} = \beta(\mathbf{k})N(\mathbf{k})$ expresses the wind input to the wave systems.

In Eqs (1) and (2) the forcing term (the r.h.s.) for the linear part (the l.h.s.) of the governing equation reflects the interaction features associated with changes in certain properties of the counterparts. This way of description of processes is generic and universal almost everywhere in our world when nonlinearity gives birth to situations where the whole has new features compared to simple addition of the counterparts. True, the variables are different but the idea is the same: an action is driven by a combination of (possibly a continuum of) several forces which describe the complicated nature of constituents or processes. From the viewpoint of complexity science, this is essential for emerging macroprocesses.

The part of the research, based of which CENS was formed, focuses on the theory of double resonance in a sister system of resonantly interacting Rossby waves (Soomere, 1993) which is governed by a three-wave kinetic equation. This is a fascinating case when energy exchange between certain wave components is substantially (by up to two orders of magnitude) faster than for the rest of the waves. Similar effects were addressed in the CNLW conference by S. Badulin for the classical Hasselmann's equation applied to spectra that considerably deviated from the equilibrium solutions to this equation (Soomere, 2001).

WATER WAVES AND TURBULENCE

The associated almost explosive (occurring within the time scale comparable to a few or a few dozens of wave periods) energy exchange, transformation of the wave shape or the pattern of wave crests is typical of nonlinear processes on the water surface. It becomes evident in a variety of situations, from propagation of a single wave over a certain generic type of coastal profiles (Didenkulova et al., 2009) to crossing of highly nonlinear waves (Peterson and van Groesen, 2000). The fundamental importance of the wave shape has become clearly evident in studies into the changes in waves in the coastal zone (Didenkulova, 2009). Didenkulova et al. (2006) demonstrate that even small deviations of the *shape* of the incident wave from the ideal one may increase the run-up height several times. This effect may be even more drastic in non-plane geometry (Didenkulova et al. 2009).

The most spectacular phenomena occur during nonlinear interactions of shallow-water solitons. These are one of the few processes that can lead to the emergence of new structures merging the energy and momentum of the counterparts. A fascinating and somewhat unexpected feature of such interactions is that many of their properties can be studied analytically by means of classical calculus. Soliton interactions constitute a fundamental feature of soliton science because the definition of a soliton involves its resilience with respect to interactions with similar structures. Rapid progress in understanding the details of such interactions and the first attempts to use them in practical applications, however, started only a decade ago, largely driven by studies in the CENS (Peterson and van Groesen, 2000).

A drastic increase in surface elevation in interactions of shallow-water solitons (which was theoretically known in the 1970s in the context of Mach reflection and generalized to the case of interacting solitons of arbitrary amplitude by Peterson et al. (2003) and Soomere (2004)) is accompanied by an even larger increase in the slope of wave fronts (Soomere and Engelbrecht, 2005, 2006). This effect is the core of a new mechanism for the formation of long-living rogue (freak or giant) waves in shallow water (Kharif et al., 2009). Their sudden appearance is of paramount importance and a generic source of danger for navigation in shallow channels and certain areas of strong currents, and specifically in coastal and offshore engineering. These effects may be particularly pronounced in the case of ship wakes that often approach seawalls or breakwaters from other directions than wind waves do (Soomere, 2007). The contribution of CENS into the study of soliton interactions was recognized by an invitation to summarize the developments for a recent encyclopaedia of complexity and systems science (Soomere, 2009).

Further increase in the complexity of motions and their interactions becomes evident in turbulence studies. The kinetic theory (also known as the theory of wave turbulence) and strongly nonlinear interactions of a few wavelike counterparts may be treated as simple special cases of turbulence that generally constitute an extremely complex system of continuously interacting virtual components with rapidly varying properties. In fluid dynamics, turbulence has a particular role in explaining many phenomena like flows, currents, diffusive fields, etc. and certainly a vast range of applications. However, what is more important in the context of complexity studies, is that turbulence is extremely well suited for investigating and understanding the generic laws of complex systems. Indeed, the physical building blocks of turbulent systems are, as a rule, very simple; yet, there are a large number of qualitatively different scenarios, in which turbulence can evolve, depending on the specific features of the system.

One of the foci of the studies at CENS is to understand passive scalar turbulence. In the case of smooth velocity fields (e.g. fluid flows below the Kolmogorov scale and geostrophic flows), the evolution of tracer blobs leads to multifractal dissipation fields (Kalda, 2000). Next, the behaviour of tracer fields in compressible flows (such as the surface flows at the free-slip surface of the turbulent fluid) depends on the stickiness of tracer particles and leads to clusterization (formation of tracer patches) as shown by Kalda (2007). Further, stationary mixing in non-smooth velocity fields (e.g. in the case of hydrodynamic turbulence, above the Kolmogorov scale), the tracer fields evolve into everywhere – discontinuous fields, characterized by a spectrum of fractal discontinuity fronts (Kalda and Morozenko, 2008). Although the list of examples could be much longer, it shows already now the complexity and richness of turbulence in general, and that of turbulent mixing, in particular.

CENS: FOSTERING COMPLEXITY RESEARCH

In recent years there has been an increased interest in advanced mathematical models and computational methods of solving wave problems, which cross the borders of specific applications. Despite an extreme variety of physical appearances of wave phenomena, different problems share many mathematical models and numerical methods. CENS has supported the idea of interdisciplinarity through all studies, contracts, and cooperative projects. The main goal of recent cooperation in 2005–2009, related to the Marie Curie Transfer of Knowledge Project 'Collaboration of Estonian and Norwegian Centres within Mathematics for Applications' (CENS–CMA) with two partners: CENS and the Centre of Mathematics for Applications (CMA) of the University of Oslo, was to reach synergy between various fields. The importance of applied mathematics in the modelling of wave motion has been highlighted by Quak and Soomere (2009).

Actually, the ideas of synergy have been fostered by CENS throughout its existence. The communication between various fields means that a breakthrough in one discipline may trigger similar successful studies in other areas. For example, the studies of soliton interactions (Peterson and van Groesen, 2000) have generated intensive research of freak waves (Soomere and Engelbrecht, 2005, 2006). The analysis of nonlinear waves in the 1990s initiated studies in biophysics and cell energetics which were not exposed at this conference. Nevertheless, it should be mentioned that the ideas of mathematical modelling and theory of continua (the concept of internal variables) have been successfully used and generalized by, e.g., Vendelin et al. (2004, 2007) within the framework of a fast developing discipline called systems biology (Saks, 2007).

An important step in understanding and formulation of synergy emerging from parallel and complementary treatment of problems in complexity in different research fields was the CNLW Conference (5–7 October 2009) held in Tallinn, Estonia. Its purpose was to foster research into theoretical, computational, and applied aspects of nonlinear wave phenomena through promoting transfer of competence over the existing borders of classical research disciplines for solids and fluids. Its central outcome and an important message to the scientific community is deep similarity of processes over a wide range of physical and biological phenomena – a feature that becomes evident only frequently because the relevant parties meet not so often.

This special issue of the *Proceedings* includes papers on analysis of complicated mathematical models, innovative ideas of computing, and novel applications. The conference gathered together 40 participants from 13 countries. Among 32 presentations at the conference, it is possible to distinguish the following areas: (i) complexity of nonlinear waves in solids and fluids including solitons and discontinuities; (ii) multiscale phenomena in heterogeneous media; (iii) propagation, interaction, properties, and statistics of ocean waves; (iv) numerical simulation of nonlinear wave propagation. Within this issue of the *Proceedings*, 18 papers are collected.

The conference marked 10 years of activities of CENS. The organizers would like to thank the Institute of Cybernetics at TUT, the Estonian Academy of Sciences, the FP6, Marie Curie Research and Training Network SEAMOCS (MRTN-CT-2005-019374) 'Applied stochastic models for ocean engineering, climate and safe transportation', the FP7 Future and Emerging Technologies network GSD 'Global System Dynamics and Policies' and the Frens Conference Services for the support.

REFERENCES

Berezovski, A., Engelbrecht, J., and Maugin, G. A. 2003. Numerical simulation of two-dimensional wave propagation in functionally graded materials. *Eur. J. Mech. A Solids*, **22**, 257–265.

Berezovski, A., Berezovski, M., and Engelbrecht, J. 2006. Numerical simulation of nonlinear elastic wave propagation in piecewise homogeneous media. *Mater. Sci. Eng.*, **A418**, 364–369.

Berezovski, A., Engelbrecht, J., and Maugin, G. A. 2008. *Numerical Simulation of Waves and Fronts in Inhomogeneous Solids*. World Scientific, Singapore.

Berezovski, A., Engelbrecht, J., and Maugin, G. A. 2009. Internal variables and generalized continuum theories. In *IUTAM Symposium on Scaling in Solid Mechanics* (Borodich, F. M., ed.). Springer, Heidelberg, 69–80.

Braunbrück, A. and Ravasoo, A. 2005. Application of counterpropagating nonlinear waves to material characterization. *Acta Mechanica*, **174**, 51–61.

Didenkulova, I. 2009. New trends in the analytical theory of long sea wave run-up. In *Applied Wave Mathematics: Selected Topics in Solids, Fluids, and Mathematical Methods* (Quak, E. and Soomere, T., eds). Springer, 265–296.

Didenkulova, I. I., Zahibo, N., Kurkin, A. A., Levin, B. V., Pelinovsky, E. N., and Soomere, T. 2006. Run-up of nonlinearly deformed waves on a coast. *Dokl. Earth Sci.*, **411**, 8, 1241–1243.

Didenkulova, I., Pelinovsky, E. N., and Soomere, T. 2009. Long surface wave dynamics along a convex bottom. *J. Geophys. Res. – Oceans*, **114**, C07006.

Engelbrecht, J. 2010. Nonlinear wave motion and complexity. *Proc. Estonian Acad. Sci.*, **59**, 66–71.

Engelbrecht, J. and Salupere, A. 2005. On the problem of periodicity and hidden solitons for the KdV model. *Chaos*, **15**, 015114.

Engelbrecht, J., Berezovski, A., Pastrone, F., and Braun, M. 2005. Waves in microstructured materials and dispersion. *Phil. Mag.*, **85**, 4127–4141.

Érdi, P. 2008. *Complexity Explained*. Springer, Berlin.

Hasselmann, K. 1962. On the nonlinear energy transfer in a gravity-wave spectrum. Part 1. General theory. *J. Fluid Mech.*, **12**, 481–500.

Janno, J. and Engelbrecht, J. 2005. An inverse solitary wave problem related to microstructured materials. *Inverse Problems*, **21**, 2019–2034.

Janno, J. and Engelbrecht, J. 2008. Inverse problems related to a coupled system of microstructure. *Inverse Problems*, **24**, 045017.

Kalda, J. 2000. Simple model of intermittent passive scalar turbulence. *Phys. Rev. Lett.*, **84**, 471–474.

Kalda, J. 2007. Sticky particles in compressible flows: aggregation and Richardson's law. *Phys. Rev. Lett.*, **98**, 064501.

Kalda, J. and Morozenko, A. 2008. Turbulent mixing: the roots of intermittency. *New J. Phys.*, **10**, 093003.

Kharif, Ch., Pelinovsky, E., and Slunyaev, A. 2009. *Rogue Waves in the Ocean*. Springer.

Mainzer, K. 1997. *Thinking in Complexity*. Springer, Berlin.

Maugin, G. A. 1993. *Material Inhomogeneities in Elasticity*. Chapman and Hall, London.

Nicolis, G. and Nicolis, C. 2007. *Foundations of Complex Systems*. World Scientific, New Jersey.

Onorato, M., Osborne, A. R., Janssen, P. A. E. M., and Resio, D. 2009. Four-wave resonant interactions in the classical quadratic Boussinesq equations. *J. Fluid Mech.*, **618**, 263–277.

Peterson, P. and van Groesen, E. 2000. A direct and inverse problem for wave crests modelled by interactions of two solitons. *Physica D*, **141**, 316–332.

Peterson, P., Soomere, T., Engelbrecht, J., and van Groesen, E. 2003. Interaction soliton as a possible model for extreme waves in shallow water. *Nonlinear Process. Geophys.*, **10**, 503–510.

Prigogine, I. and Stengers, I. 1984. *Order out of Chaos*. Heinemann, London.

Quak, E. and Soomere, T. (eds). 2009. *Applied Wave Mathematics: Selected Topics in Solids, Fluids, and Mathematical Methods.* Springer.

Ravasoo, A. 2007. Non-linear interaction of waves in prestressed material. *Int. J. Non-Linear Mech.*, **42**, 1162–1169.

Saks, V. (ed.). 2007. *Molecular System Biology: Energy for Life*. Wiley-VCH, Wertheim.

Salupere, A., Engelbrecht, J., and Peterson, P. 2003. On the long-time behaviour of soliton ensembles. *Math. Comp. Simul.*, **62**, 137–147.

Salupere, A., Tamm, K., and Engelbrecht, J. 2008. Numerical simulation of interaction of solitary deformation waves in microstructured solids. *Int. J. Non-Linear Mech.*, **43**, 201–208.

Scott, A. 1999. *Nonlinear Science, Emergence and Dynamics of Coherent Structures*. Oxford University Press.

Solli, D. R., Ropers, C., Koonath, P., and Jalali, B. 2007. Optical rogue waves. *Nature*, **450**(7172), 1054–1057.

Soomere, T. 1993. Double resonance and kinetic equation for Rossby waves. *Ann. Geophys.*, **11**, 90–98.

Soomere, T. 2001. New insight into classical equilibrium solutions of kinetic equations. *J. Nonlinear Sci.*, **11**, 305–320.

Soomere, T. 2004. Interaction of Kadomtsev–Petviashvili solitons with unequal amplitudes. *Phys. Lett. A*, **332**, 74–81.

Soomere, T. 2007. Nonlinear components of ship wake waves. *Appl. Mech. Rev.*, **60**, 120–138.

Soomere, T. 2009. Solitons interactions. In *Encyclopedia of Complexity and Systems Science* (Meyers, R. A., ed.), vol. 9, Springer, 8479–8504.

Soomere, T. and Engelbrecht, J. 2005. Extreme elevations and slopes of interacting solitons in shallow water. *Wave Motion*, **41**, 179–192.

Soomere, T. and Engelbrecht, J. 2006. Weakly two-dimensional interaction of solitons in shallow water. *Eur. J. Mech. B Fluids*, **25**, 636–648.

Strogatz, S. H. 1994. *Nonlinear Dynamics and Chaos*. Addison-Wesley, Reading, Mass.

Vendelin, M., Lemba, M., and Saks, V. 2004. Analysis of functional coupling: mitochondrial creatine kinase and adenine nucleotide translocase. *Biophys. J.*, **87**, 696–713.

Vendelin, M., Saks, V., and Engelbrecht, J. 2007. Principles of mathematical modelling and in silico studies of integrated cellular energetics. In *Molecular System Biology: Energy for Life*. Wiley-VCH, Wertheim, 407–433.

West, B. J. 1985. *An Essay on the Importance of Being Nonlinear*. Springer, Berlin.

Keerukate mittelineaarsete lainete uuringute teetähised

Jüri Engelbrecht, Arkadi Berezovski ja Tarmo Soomere

On antud lühiülevaade Mittelineaarsete Protsesside Analüüsi Keskuse viimase kümnendi olulistest edusammudest keerukate lainesüsteemide käsitlemisel rahvusvahelise konverentsi "Keerukad mittelineaarsed lainesüsteemid" valguses. On näidatud, kuidas erinevates valdkondades sarnaste meetoditega tehtud uuringud on viinud uute ideede ja valdkondadevahelise sünergiani. On toodud näiteid solitonide teooriast, lainelevi ülesannetest mikrostruktuuriga materjalides ja nendega seotud pöördülesannetest, mittepurustava katsetamise vallast, pinnalainete vastasmõju ning uhtekõrguse arvutamise küsimustest, madala vee solitonide interaktsiooni ja turbulentsi valitud küsimustest.