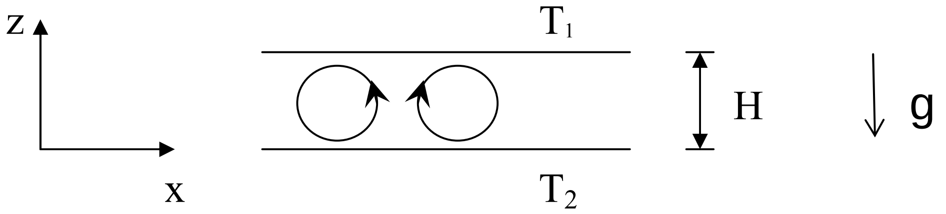
8. KLASSIKALISED NÄITED

Lorenzi atraktor

- elimineeritakse ruumiline muutus

Näide: Lorenz , liikumine vedeliku kihis



Liikumisvõrrandid

$$\frac{\partial}{dt}\nabla^2\psi = -\frac{\partial\left(\psi,\nabla^2\psi\right)}{\partial(x,z)} + \nabla^4\psi + g\alpha\frac{\partial\theta}{\partial x}$$

$$\frac{\partial}{dt}\theta = -\frac{\partial(\psi,\theta)}{\partial(x,z)} + \frac{\Delta T}{H}\,\frac{\partial\psi}{\partial x} + \kappa\nabla^2\theta$$

ΔT - temperatuuri vahe
H - kihi paksus
Ψ - voolu funktsioon (stream function)

$$v = \frac{\partial\psi}{\partial x} \quad , \quad u = \frac{\partial\psi}{\partial z} \qquad \text{kiirused}$$

g - raskuskiirendus
α - soojuspaisumise koefitsient
ν - kinemaatiline viskoossus
κ - soojusjuhtivus

$$\nabla^2 = \frac{\partial^2}{\partial x^2}\left(+\frac{\partial^2}{\partial y^2}\right) + \frac{\partial^2}{\partial z^2}$$

Oletame lahendi kujul:

$$\frac{a}{\left(1+a^2\right)\kappa}\psi = X\sqrt{2}\sin\left(\frac{\pi a}{H}x\right)\sin\left(\frac{\pi}{H}z\right)$$

$$\frac{\pi R_a}{R_c \Delta T}\theta = Y\sqrt{2}\cos\left(\frac{\pi a}{H}x\right)\sin\left(\frac{\pi}{H}z\right) - Z\sin\left(\frac{2\pi}{H}z\right)$$

$R_a = g\alpha H^3 \Delta T / (\nu\kappa)$ Rayleigh number

$R_c = \dfrac{\pi\left(1+a^2\right)^3}{a^2}$ R_n kriitiline väärtus

$\min R_c = \dfrac{27}{4}\pi^4$ kui $a^2 = \dfrac{1}{2}$

X – konvektiivse liikumise intensiivsus

Y – tõusvate ja langevate voogude temperatuuri vahe

sama märk tähendab, et soe vedelik tõuseb, külm langeb

Z – vertikaalse temperatuuri jaotuse erinevus lineaarsest

positiivne väärtus näitab, et suurimad gradiendid on kihi piiridel

Lõpptulemus

$$\begin{aligned}
\dot{X} &= -\sigma X + \sigma Y \\
\dot{Y} &= -XZ + rX - Y \\
\dot{Z} &= XY - bZ
\end{aligned}$$

$$\dot{(\ )} = \frac{d}{d\tau} \quad , \quad \tau = \frac{\pi^2}{H^2}\left(1 + a^2\right)\kappa t$$

$\sigma = \dfrac{\nu}{\kappa}$ Prandtl'i number

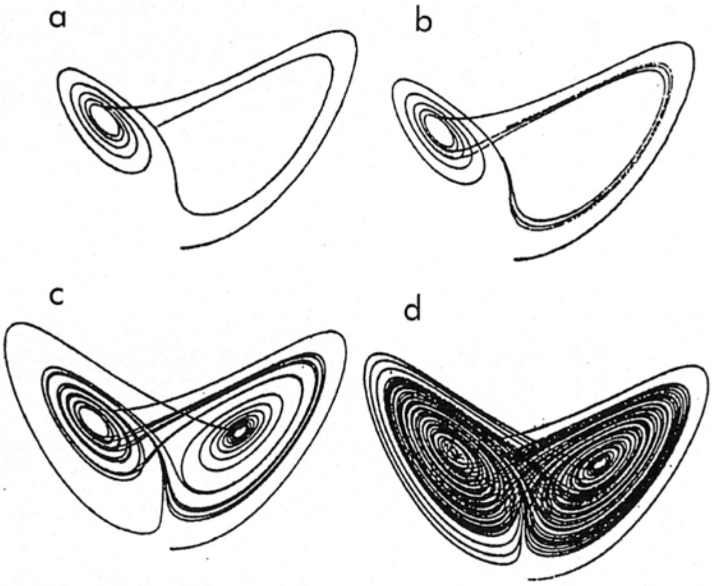
$r = R_a / R_c$

$b = 4 / \left(1 + a^2\right)$

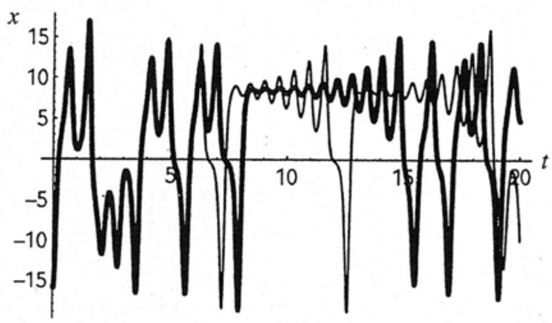
Vt. E.N. Lorenz. Deterministic Nonperiodic Flow

J. Atm. Sci., 1963, 20, 130-141.

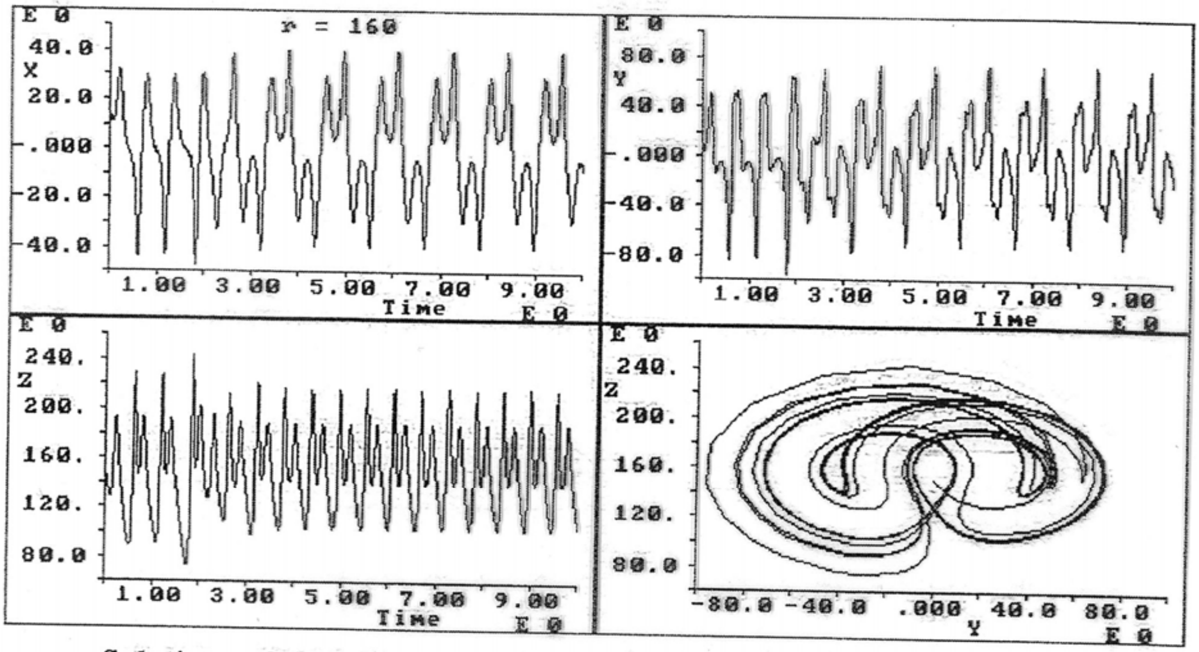
erinevate algtingimuste korral. Jäme joon vastab algtingimustele $x(0) = -15,80$, $y(0) = -17,48$, $z(0) = 35,64$. Peene joone korral on $y(0)$ ja $z(0)$ samad, kuid $x(0) = -15,79$.



Joonis 6.5. Lorenzi atraktor (6.3.1): (a) iteratsioonide arv 4870; (b) iteratsioonide arv 11 000; (c) iteratsioonide arv 25 000; (d) iteratsioonide arv 150 000.



Joonis 6.6. Lorenzi atraktori x-komponent erinevatel algtingimustel. *Kaplan, Glass, 1995.*



Solutions of the Lorenz equations with $r = 160$. After an initial transient that lasts until about $t = 3$, the solutions are periodic (but not sinusoidal).

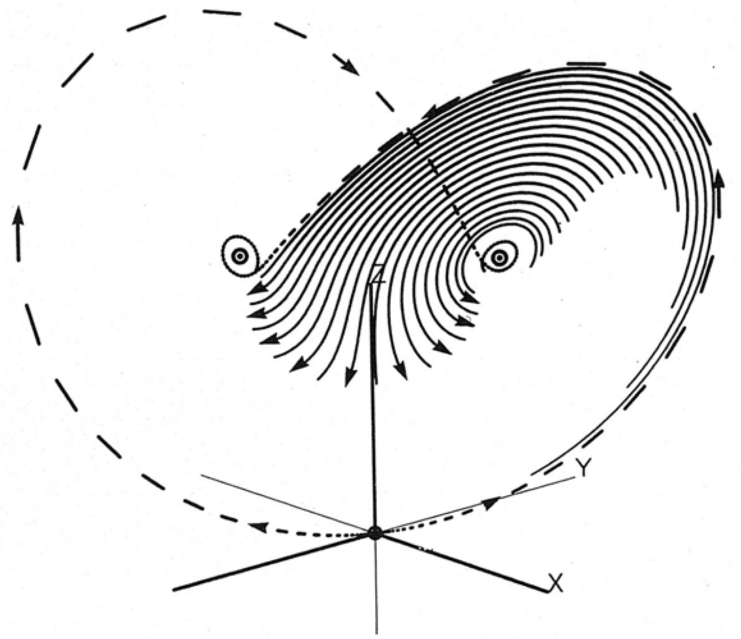


Figure 11.8 The outset of the origin (broken curves) reinserts near opposite spiral fixed points

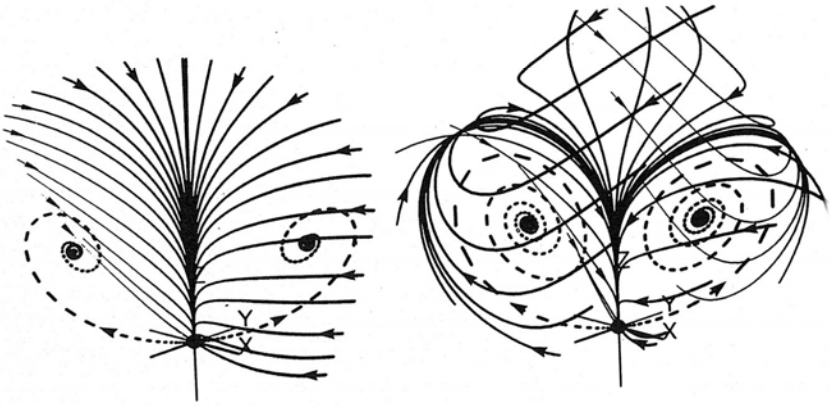
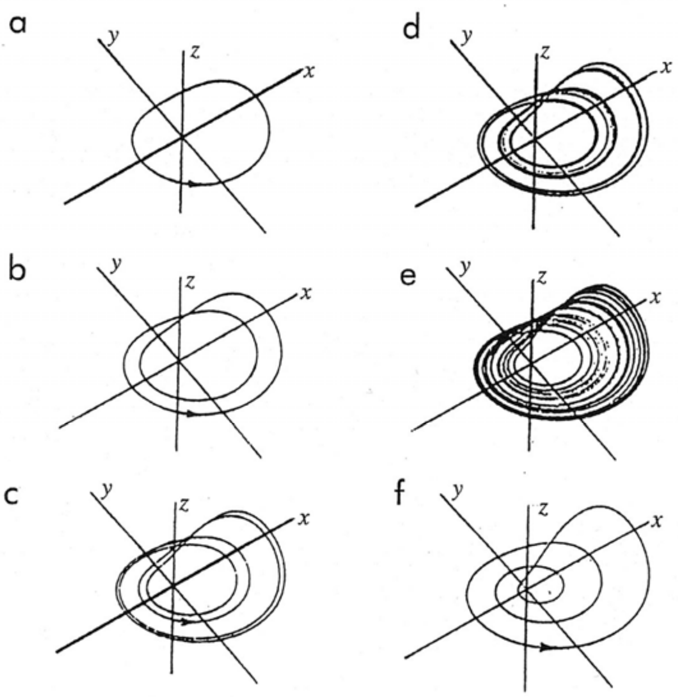


Figure 11.10 Outstructures of the saddle fixed point at the origin, for $R = 5$ (left) and $R = 10$ (right)

Lorenzi struktuuri tekkimise tee võib olla üsna sarnane p. 6.2 vaadeldud perioodi kahendumisega. Selle demonstreerimiseks vaatame veelgi lihtsamat näidet, nn. *Rössleri süsteemi*

$$\begin{aligned} \dot{x} &= -y - z, \\ \dot{y} &= x + ay, \\ \dot{z} &= b + zx - cz, \end{aligned} \tag{6.3.2}$$

kus a, b, c on konstandid ja ainus mittelineaarne liige on zx kolmandas võrrandis. Vaatleme selle süsteemi käitumist, valides $b = 2$ ja $c = 4$ ning lugedes teguri a kontrollparameetriks.



Joonis 6.7. Rössleri atraktor (6.3.2), kui $b = 2$, $c = 4$ ja kontrollparameetrit a varieeritakse: (a) $a = 0,3$; (b) $a = 0,35$; (c) $a = 0,375$; (d) $a = 0,386$; (e) $a = 0,398$; (f) $a = 0,411$. *Thomson, Stewart, 1986.*

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IUTAM Symposium on 50 Years of Chaos: Applied and Theoretical

How a Broken Egg Attractor Has Influenced the Dynamics of My Life[1]

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$$\frac{d^2u}{dt^2} - \mu(1 - \gamma u^2)\frac{du}{dt} + u^3 = B\cos\nu t, \quad 0 < \mu \ll 1$$

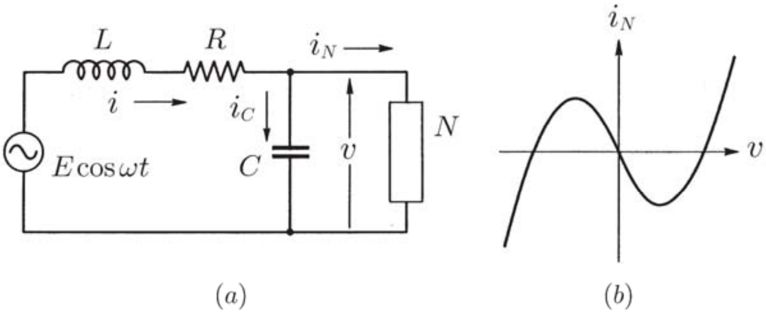


Fig. 5. (a) Periodically forced self-oscillatory circuit (b) Characteristics of negative-resistance element

$$\frac{d^2x}{d\tau^2} - \mu(1 - x^2)\frac{dx}{d\tau} + \alpha x + x^3 = B\cos\nu\tau$$

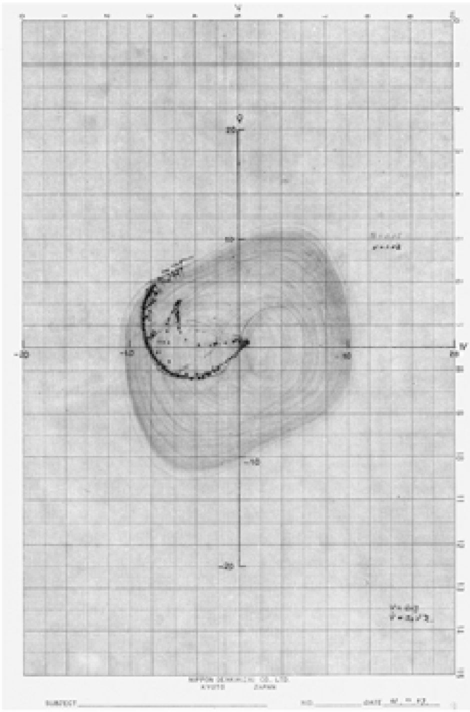


Fig. 1. Broken Egg Attractor first observed in physical system; the parameter values are $\mu = 0.2, \gamma = 8, B = 0.35$, and $\nu = 1.02$ – Brookhaven National Laboratory, Photography Division Negative No. 1-380-90

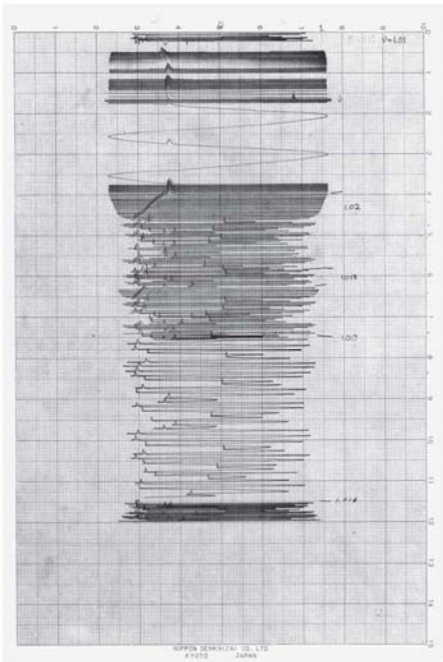


Fig. 2. Waveforms related to Broken Egg Attractor in Fig.1 – Brookhaven National Laboratory, Photography Division Negative No. 9-307-90

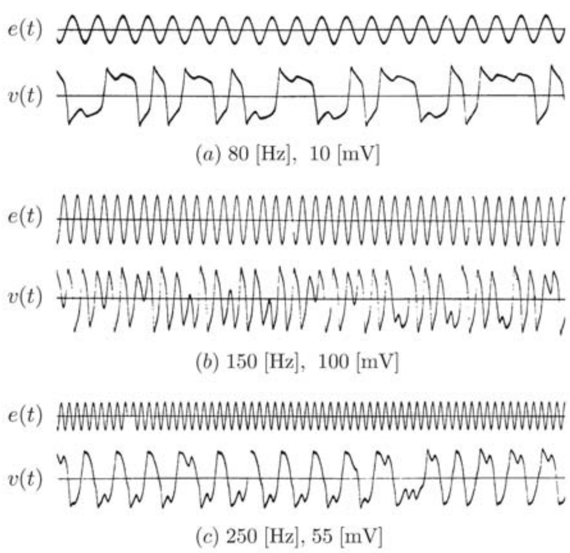


Fig. 8. Various waveforms of asynchronous best oscillations in the circuit II

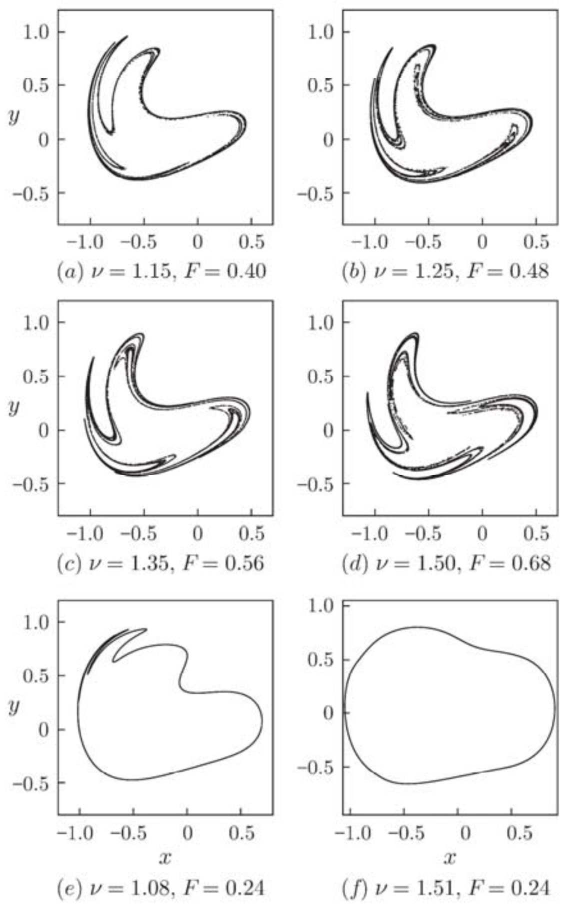


Fig. 11. Attractors representing asynchronous oscillations in Eq.23

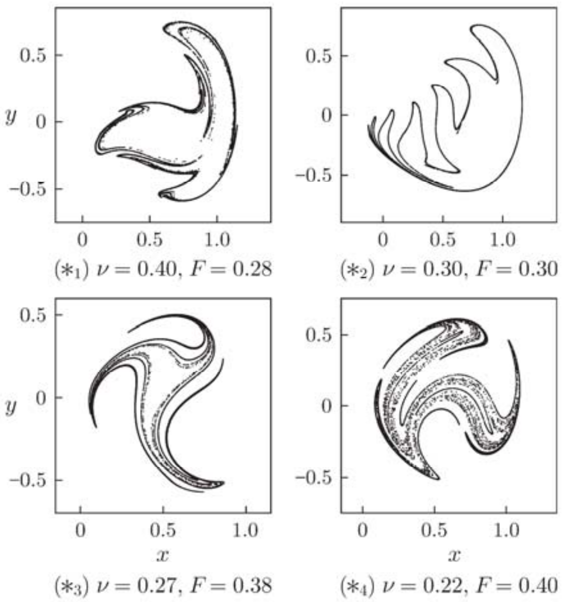
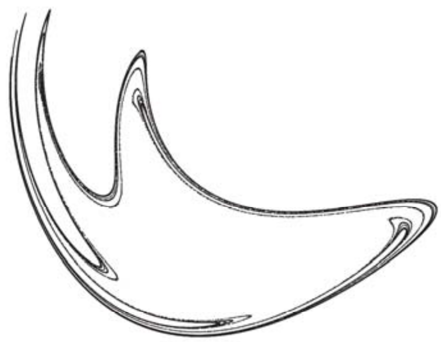


Fig. 12. Chaotic attractors of Eq.23 with $\mu = 0.1$, $a = 1.04$; Group (*)



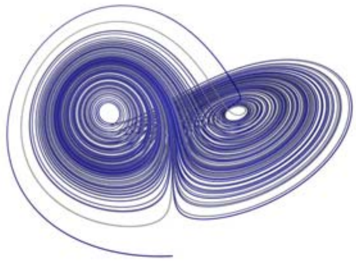
$$\left.\begin{aligned} \frac{dx}{dt} &= y \\ \frac{dy}{dt} &= 0.2(1 - 8x^2)y - x^3 + 0.35\cos t \end{aligned}\right\}$$

Lorenzi atraktori modifikatsioonid

Lorenz (1963)

$$\dot{x} = a(y - x)$$
$$\dot{y} = cx - xz - y$$
$$\dot{z} = xy - bz$$

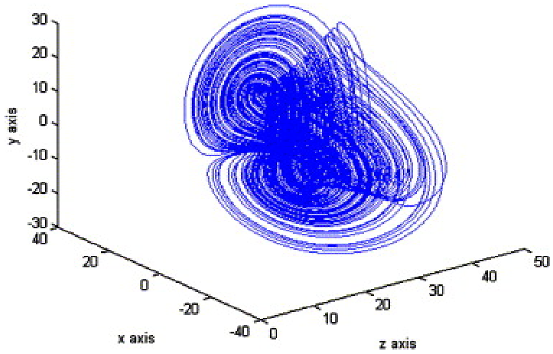
a, c, b, - konstandid



Chen (1999)

$$\dot{x} = a(y - x)$$
$$\dot{y} = (c - a)x - xz - +cy(+m)$$
$$\dot{z} = xy - bz$$

a, b, c, m - konstandid



Lü (2002)

$$\dot{x} = a(y - x)$$
$$\dot{y} = -xz - +cy$$
$$\dot{z} = xy - bz$$