10. KOKKUVÕTE KAOOTILISTEST PROTSESSIDEST

Kaose tekkimise stsenaariumid

1) perioodi kahendumine ehk Feigenbaumi stsenaarium

    logistiline kujutis

    Lorenzi atraktor

    Hénoni kujutis

    katsed vedela heeliumiga

2) juhumuutus ehk Manneville´i – Pomeau stsenaarium

    omaväärtuste muutumine kontrollparameetri muutumised

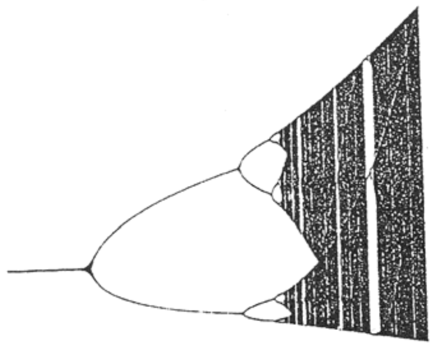
    katsed vedelikus

3) Hopfi bifurkatsioonide jada ehk

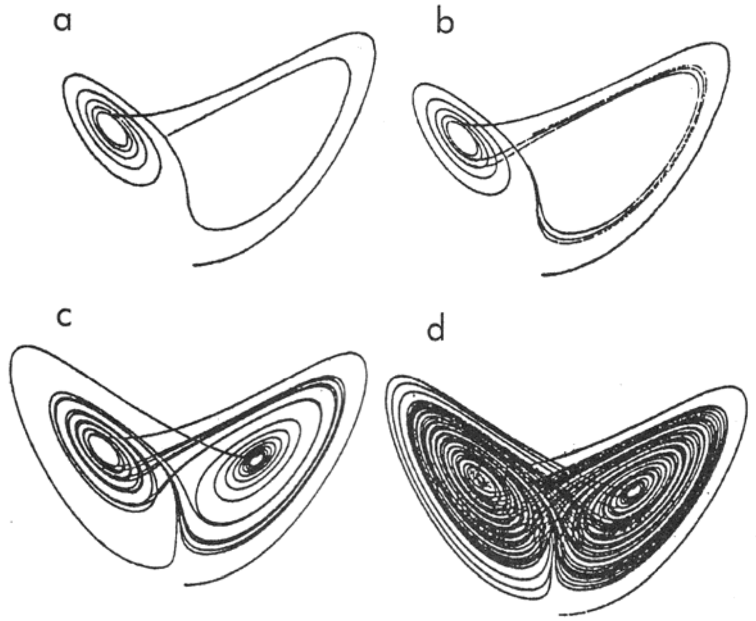
   Ruelle´i – Takensi – Newhouse´i; stsenaarium

    Raylegh´ – Bénardi konvektsioon

    Taylori keerised

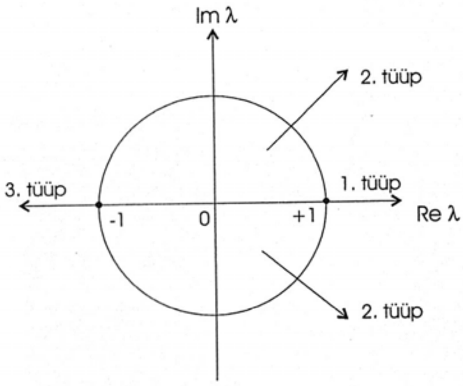


Joonis 6.2. Feigenbaumi diagramm logistilisele kujutisele.

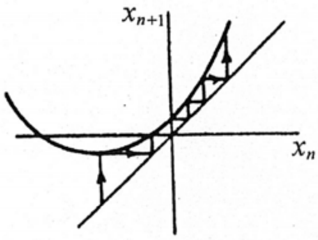


Joonis 6.5. Lorenzi atraktor (6.3.1): (a) iteratsioonide arv 4870; (b) iteratsioonide arv 11 000; (c) iteratsioonide arv 25 000; (d) iteratsioonide arv 150 000.

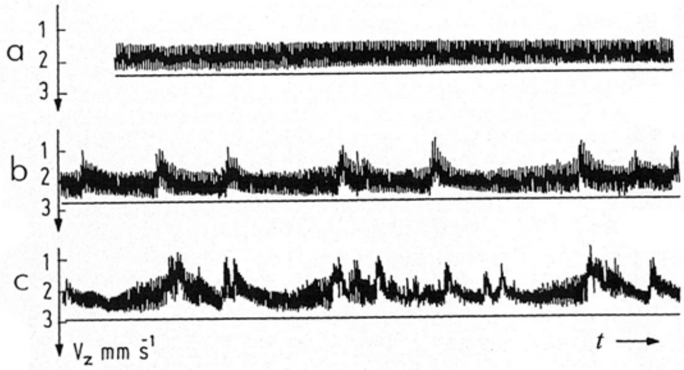
Juhumuutus



Joonis 11.1. Omaväärtuste muutumine ühikringi suhtes.



Joonis 11.2. Iteratsioon esimest tüüpi juhumuutuse puhul.



Joonis 11.3. Esimest tüüpi juhumuutuva kaose tekkimine Rayleigh'-Benard'i eksperimendi andmetel.

Kaose identifitseerimine

Jääb üle loetleda tunnused, mis ilmnevad kaootilistel protsessidel:

1) tundlikkus algtingimuste suhtes;

2) laiaribaline Fourier' spekter, kus ülekaalus on madalad sagedused;

3) positiivne Ljapunovi eksponent;

4) atraktori fraktaalne struktuur faasiruumis;

5) korrapäraste liikumiste bifurkatsioonid, mis ilmnevad ühe (või mitme) kontrollparameetri muutmisel;

6) mitteregulaarsete intervallide esinemine korrapärasel liikumisel, mis viib juhumuutuvale kaosele.

Nii või teisiti, kui antud protsessi analüüsimisel tekib kahtlus kaose võimalikus ilmingus, on vaja kontrollida järgmist:

1) kas protsessis on mittelineaarsus oluline;

2) kas algtingimused protsessi parameetrid on determineeritud või juhuslikud suurused (funktsioonid);

3) milline on protsessi ajaline kulg (muutujate esitus aegridadena);

4) milline on faasiportree;

5) milline on Poincaré kujutis;

6) milline on Fourier' spekter;

7) mis juhtub süsteemi parameetrite muutumisel.

TABLE 2-1 Classes of Motion in Nonlinear Deterministic Systems

Regular Motion—Predictable: Periodic oscillations, quasiperiodic motion; not sensitive to changes in parameters or initial conditions

Regular Motion—Unpredictable: Multiple regular attractors (e.g., more than one periodic motion possible); long-time motion sensitive to initial conditions

Transient Chaos: Motions that look chaotic and appear to have characteristics of a strange attractor (as evidenced by Poincaré maps) but that eventually settle into a regular motion

Intermittent Chaos: Periods of regular motion with transient bursts of chaotic motion; duration of regular motion interval unpredictable

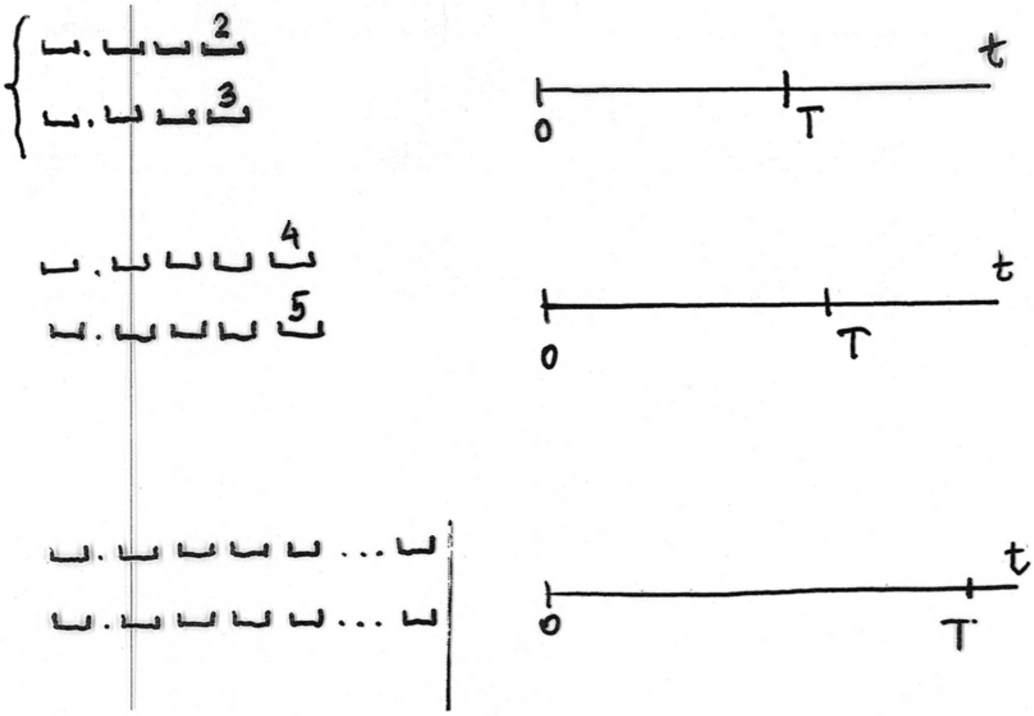
Limited or Narrow-Band Chaos: Chaotic motions whose phase space orbits remain close to some periodic or regular motion orbit; spectra often show narrow or limited broadening of certain frequency spikes

Large-Scale or Broad-Band Chaos—Weak: Dynamics can be described by orbits in a low-dimensional phase space $3 \leq n < 7$ (1–3 modes in mechanical systems) and usually one can measure fractal dimensions < 7; chaotic orbits traverse a broad region of phase space; spectra show broad range of frequencies especially below the driving frequency (if one is present)

Large-Scale Chaos—Strong: Dynamics must be described in a high-dimensional phase space; large number of essential degrees of freedom present; difficult to measure reliable fractal dimension; dynamical theories currently unavailable

Predictability horizon J. Lighthill, 1986

Time after which solutions with initial conditions that are "nearest neighbours" to the accuracy of specification being used, become remote from one another differ beyond recognition



physical limit

Ennustatavuse horisont:

aeg millest alates naaberalgandmetest arvutatud protsessi parameetrid erinevad teineteisest kaootiliselt

KAOSE SÜMBOLID

Feigenbaumi diagramm
Mandelbroti fraktal
Lorenzi liblikas
Barnsley sõnajalg

ja veel:

Sierpinski vaip (kolmnurk)
Julia hulgad
Cantori hulk
Kochi lumehelbeke
Smale'i hobuseraud
. . . .

Mappings and fixed points

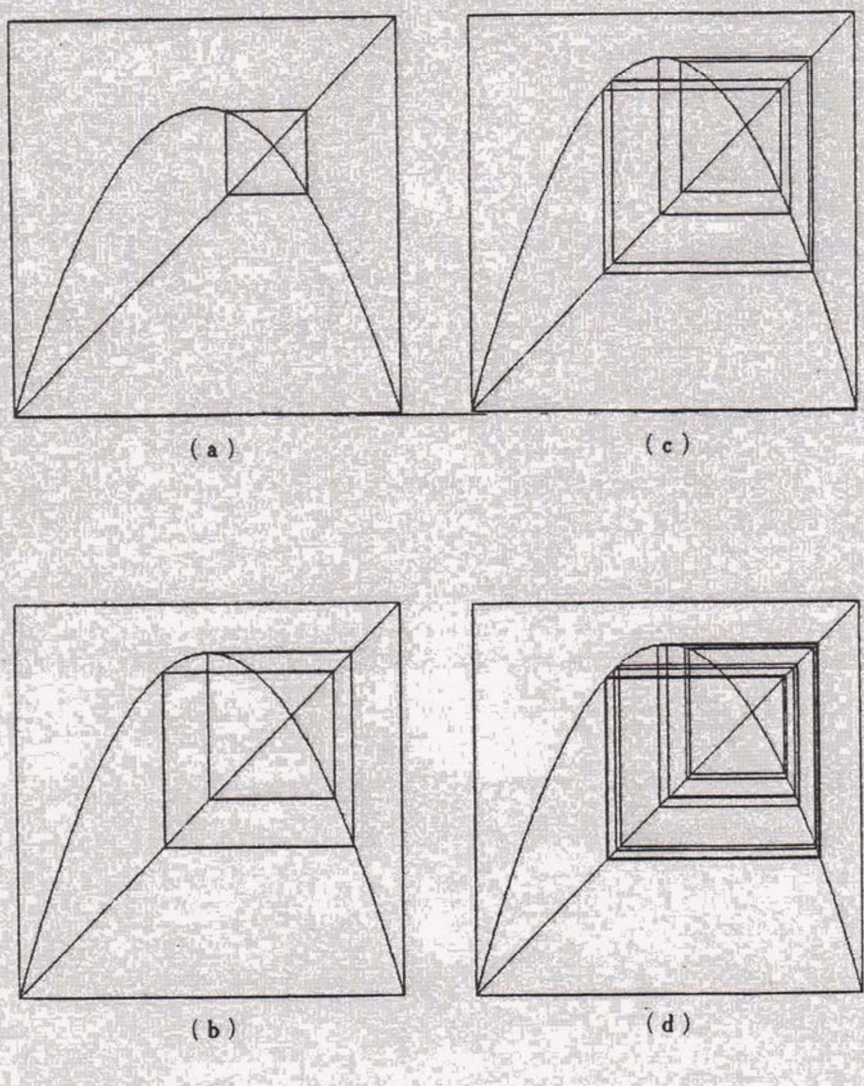


Figure 3.9. Periodic points of the logistic map. (a) The period 2 points ($R = 3.1$). (b) The period 4 points ($R = 3.5$). (c) The period 8 points ($R = 3.56$). (d) The period 16 points ($R = 3.5685$).

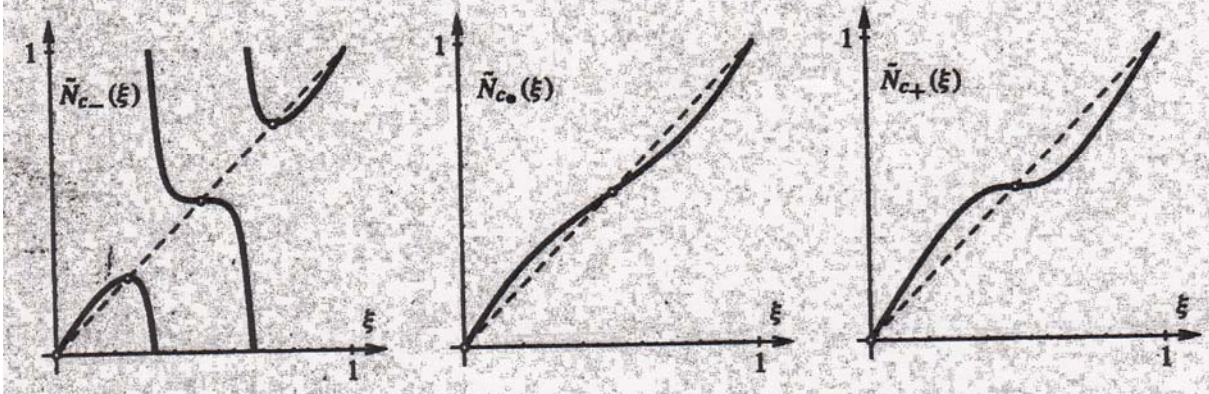


Figure 2.18. The maps induced by the cubic polynomials c_-, $c_\bullet$ and c_+, respectively

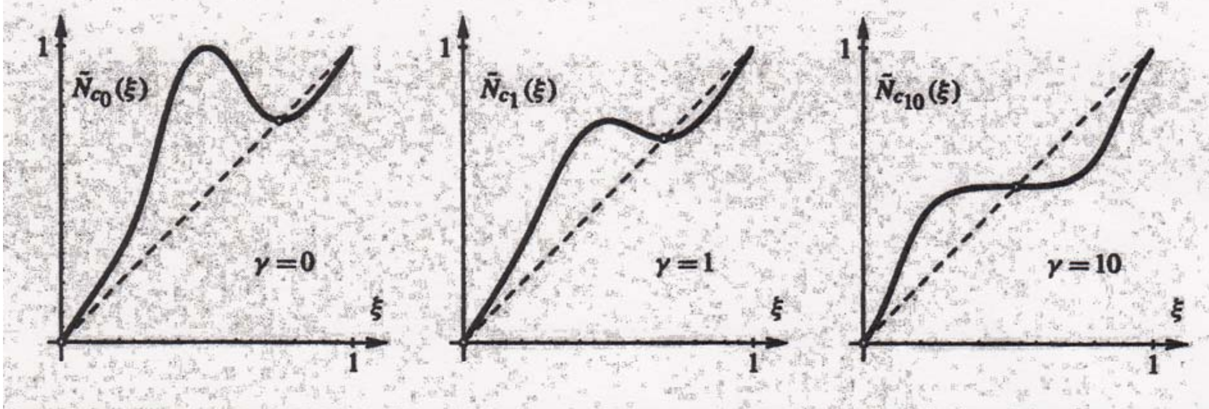
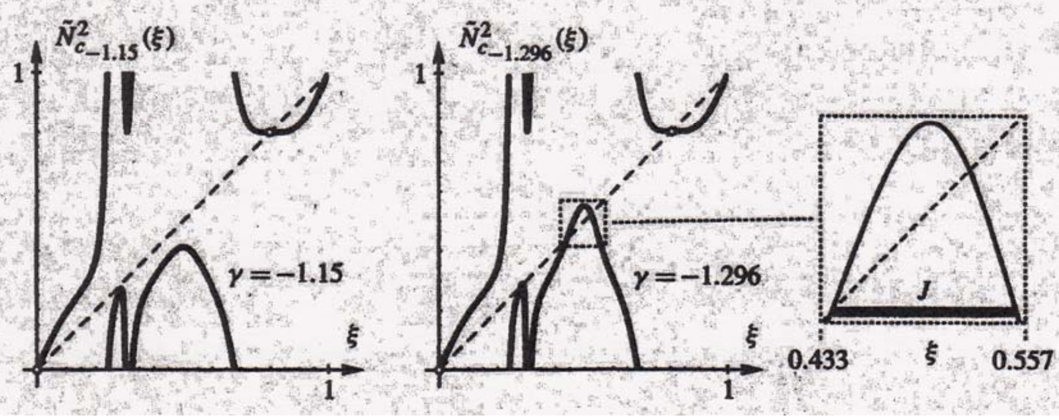


Figure 2.19. Depending on the parameter γ the maps $\tilde{N}_{c_\gamma}$ induced by the cubic polynomials c_γ may exhibit regular (above) and chaotic behaviour.



KAOSE VÄLTIMINE

kaose kontroll, kaose „taltsutamine“

eesmärk: kaootilise režiimi vältimine juhtparameetrite muutmise teel

tehniliselt võimalik ka lisada stabiliseerivaid abisüsteeme (lisavõnkumised).

ELSEVIER

Physica D 86 (1995) 198–211

PHYSICA D

Techniques for the control of chaos

William L. Ditto[a], Mark L. Spano[b], John F. Lindner[c]

[a] Applied Chaos Laboratory, Georgia Institute of Technology, Atlanta, GA 30332, USA
[b] Naval Surface Warfare Center, 10901 New Hampshire Ave., Silver Spring, MD 20903, USA
[c] Department of Physics, The College of Wooster, Wooster, OH 44691, USA

Abstract

The concepts of chaos and its control are reviewed. Both are discussed from an experimental as well as a theoretical viewpoint. Examples are then given of the control of chaos in a diverse set of experimental systems. Current and future applications are discussed.

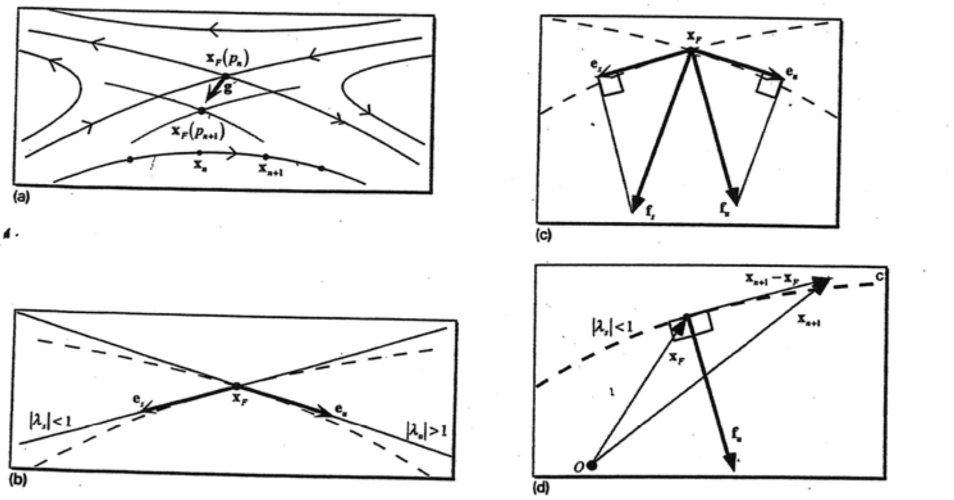


Fig. 4. (a) Movement of unstable fixed point due to parameter change. (b) The eigenvectors around the unstable fixed point which may be nonorthogonal, $e_u^T e_s \neq 0$. The eigenvalues satisfy $|\lambda_s| < 1 < |\lambda_u|$. (c) New "perpendicular" basis vectors $\{f_u, f_s\}$ in terms of old "parallel" basis vectors $\{e_u, e_s\}$ found by demanding $f_s^T e_s = 1 = f_u^T e_u$ and $f_u^T e_s = 0 = f_s^T e_u$. (d) Graphical representation of an iterate of the map near the unstable fixed point. Control is achieved by changing a system parameter such that the projection of $x_F - x_{n+1}$ onto f_u is zero as shown. In words this is simply the requirement that there be no relative motion between the fixed point x_F and the mapped point x_{n+1} perpendicular to the stable direction.

Algoritm

Olgu

$$\boldsymbol{x}_{n+1} = f\left(\boldsymbol{x}_n, p\right)$$

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \qquad p \text{ - juhtparameeter}$$

püsipunkt

$$\boldsymbol{x}_F = f\left(\boldsymbol{x}_F, p\right)$$

juhtparameetri muutusel

$$\boldsymbol{x}_F\left(p_n\right) = f\left(\boldsymbol{x}_F\left(p_n\right), p_n\right)$$

uus püsipunkt on iseloomustatud siirdevektoriga

$$\boldsymbol{x}_F\left(p_{n+1}\right) \sim \boldsymbol{x}_F\left(p_n\right) + \left(p_{n+1} - p_n\right)\boldsymbol{s}$$

$$\boldsymbol{s} \equiv \frac{d}{d\,p}\,\boldsymbol{x}_F\left(p\right)\bigg|_{p=p_n} \sim \frac{\boldsymbol{x}_F\left(p_{n+1}\right) - \boldsymbol{x}_F\left(p_n\right)}{p_{n+1} - p_n}$$

Kasutame süsteemi omaväärtusi / omavektoreid

λ_s, $\boldsymbol{e}_s$ - stabiilsel suunal

λ_u, $\boldsymbol{e}_u$ - mittestabiilsel suunal

$$\boldsymbol{M}\ \boldsymbol{e}_u = \lambda_u \boldsymbol{e}_u$$
$$\boldsymbol{M}\ \boldsymbol{e}_s = \lambda_s \boldsymbol{e}_s$$

$$\left(\boldsymbol{x}_{\mathrm{n+1}} - \boldsymbol{x}_{\mathbf{F}}\right) \sim \boldsymbol{M}\left(\boldsymbol{x}_n - \boldsymbol{x}_F\right)$$

$$\boldsymbol{x}_{\mathrm{n+1}} \sim \boldsymbol{x}_F + \boldsymbol{M}\left(\boldsymbol{x}_n - \boldsymbol{x}_F\right)$$

$$\left|\lambda_s\right| < 1 < \left|\lambda_u\right|$$

Leiame uued baasvektorid

$\boldsymbol{f}_u$, $\boldsymbol{f}_s$

$$\boldsymbol{M} = \left[\boldsymbol{e}_u\ \boldsymbol{e}_s\right] \begin{bmatrix} \lambda_u & 0 \\ 0 & \lambda_s \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_u^T \\ \boldsymbol{f}_s^T \end{bmatrix}$$

$\boldsymbol{f}_u^T$ on $\boldsymbol{M}$ vasak omavektor, millele vastab omaväärtus $\boldsymbol{e}_{\mathrm{u}}$

Nõuame, et järgmine iteratsioon langeks stabiilsele suunale

$$\boldsymbol{f}_u^T \left(\boldsymbol{x}_{n+1} - \boldsymbol{x}_F\left(p_n\right)\right) = 0$$

st. siirdevektori komponent puudub piki $\boldsymbol{f}_u^T$

Määrame

$$\delta p_n = p_{n+1} - p_n$$

$$\delta \boldsymbol{x}_n = \boldsymbol{x}_n - \boldsymbol{x}_F\left(p_n\right)$$

$$\delta p_n \sim \frac{\lambda_u}{\lambda_u - 1} \quad \frac{f_u^T \, \delta \boldsymbol{x}_u}{f_u^T \, \boldsymbol{s}}$$

δp_n määrab kui palju tuleb muuta süsteemi juhtparameetrit, et kontrollida süsteemi nii, et liikumine toimiks stabiilselt.

Parandus tuleb leida igal perioodil.

Kirjandus:

W.L. Ditto, M.L. Spano, J.F. Lindner. Techniques for the control of chaos. Physica D 86, 1995, 198-211.

H.G.Schuster. Handbook of Chaos Control. Wiley, 2007.

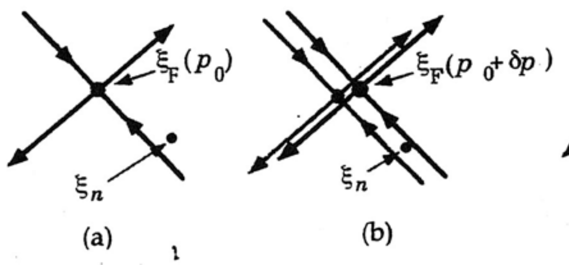


Fig. 5. An outline of the OGY control method: (a) the system state point approaches the unstable fixed point. It will naturally tend to move in along the stable direction and out along the unstable direction; (b) the saddle point (unstable fixed point) is positioned so that the system state point tends to move back toward the original unstable fixed point; (c) the system state point reaches the original unstable fixed point and the perturbation is turned off. In real-world systems with noise, the control would have to applied continuously to balance the state point onto the stable direction and hence onto the unstable fixed point.