

## EXPERIMENTAL AND THEORETICAL STUDIES OF PIANO HAMMER

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### ABSTRACT

Based upon the large number of experimental data obtained using a special piano hammer testing device, it has been shown, that all the present-day piano hammers have as a quality the hysteretic type of the force-compression characteristics. This not a chance because such a hysteretic character has been developed step-by-step following the history of evolution of piano hammers since the instrument was created.

The dynamical behaviour of the modern piano hammer can be described by different mathematical hysteretic models. In addition to the four-parameter nonlinear hysteretic model of piano hammer, another new three-parameter hysteretic model was developed. It is very similar to the nonlinear Voigt model and permits a description of the dynamical hammer felt compression that is consistent with experiments. The both models are based on an assumption that the hammer felt made of wool is a microstructured material possessing history-dependent properties. The equivalence of these models is proved for all the realistic values of hammer velocity.

The continuous dependencies of the hammer parameters on the key number are obtained, which is the first known case of such an analysis. The application of hysteretic models to numerical simulation of the grand piano hammer-string interaction is demonstrated. The flexible string vibration spectra excited by different piano hammers are analyzed. All that together, leads to a new method for piano stringing-scale design.

### 1. INTRODUCTION

During for two last years the piano hammer testing device [1, 2, 3] was intensively used for piano hammer studies. The several whole sets of piano hammer were compared, and the dozens of piano hammers produced by various firms (*Schneider, Renner, Abel, Immadigawa*) were tested. The effect of the mechanical treatment (hammer voicing) on the piano hammer parameters, and the air humidity influence on the stability of these parameters were also investigated.

The method of the piano hammer parameters determination was based on the hysteretic (hereditary) model of the hammer felt [5]. It has been experimentally shown [2, 3, 4] that physical assumptions related to the history-dependent properties of the hammer are confirmed by the experiment. It was experimentally proved that the hereditary constitutive model of felt clarifies the dynamical features of piano hammers rather well.

The theoretical ground of the studies was the piano hammer model derived in [5] in the form

$$F(u(t)) = F_0 \left[ u^p(t) - \frac{\varepsilon}{\tau_0} \int_0^t u^p(\xi) \exp\left(-\frac{\xi-t}{\tau_0}\right) d\xi \right]. \quad (1)$$

Here  $F(u)$  is the force exerted by a hammer and  $u$  is the hammer compression. The instantaneous hammer stiffness  $F_0$  and compliance nonlinearity exponent  $p$  are the elastic parameters of a hammer, and constants  $\varepsilon$  and  $\tau_0$  are the hereditary parameters. This analytical four-parameter model describes the dynamical behaviour of the microstructural material, and for some certain set of the hammer parameters represent the unique force-compression curve. But, in spite of this evident fact, the numerical simulation of this model (1) demonstrates very similar force-compression curves, obtained for the different sets of hammer parameters. A close and subtle analysis of this phenomenon results the new more simple hysteretic model of piano hammer.

### 2. THREE-PARAMETER HAMMER MODEL

The impact of the hammer can be described by the equation of motion

$$m \frac{d^2 u}{dt^2} + F(u) = 0, \quad (2)$$

with the initial conditions

$$u(0) = 0, \quad \frac{du}{dt}(0) = V. \quad (3)$$

Here  $m$  and  $V$  are the hammer mass and the velocity respectively, and  $F(u)$  is defined by Eq. (1).

The equation (2) with the function (1) may be written also in the form

$$m \frac{d^2 u}{dt^2} + m\tau_0 \frac{d^3 u}{dt^3} + F_0 \left[ (1-\varepsilon)u^p + \tau_0 \frac{d(u^p)}{dt} \right] = 0. \quad (4)$$

The analysis of this equation shows that the second term is much smaller than the first one, and also the other terms. This fact corresponds to the non equality  $F(t) \gg \tau_0 dF/dt$ , which is valid for all the known values of  $\tau_0$  ( $< 10 \mu\text{sec}$ ), and for any reasonable value of the piano hammer velocity – up to 10 m/s. Therefore, the second term may be ignored, and introducing the new parameters

$$Q_0 = F_0(1-\varepsilon), \quad \alpha = \tau_0/(1-\varepsilon), \quad (5)$$

we have

$$m \frac{d^2 u}{dt^2} + Q_0 \left[ u^p + \alpha \frac{d(u^p)}{dt} \right] = 0. \quad (6)$$

Thus, according to Eq. 2 we can determine the new piano hammer model in the form

$$Q(u(t)) = Q_0 \left[ u^p + \alpha \frac{d(u^p)}{dt} \right], \quad (7)$$

where  $Q(u)$  is the force exerted by a hammer,  $Q_0$  is the static hammer stiffness, and  $\alpha$  is the retarded time. This hysteretic model is very similar to nonlinear Voigt model and permits a description of the hammer felt compression that is consistent also with experiments.

For very slow deformation, the loading and unloading of the hammer felt occur near the limit curve that is the same curve for both hysteretic models (Eqs. (1) and (7)). Due to the equalities (5), this curve is (see also [5]) determined by

$$F(u) = F_0(1 - \varepsilon)u^p = Q(u) = Q_0u^p. \quad (8)$$

However, for very fast loading these two models are quite different. The limit curve for the first model is described [5] by equation

$$F(u) = F_0u^p. \quad (9)$$

With the increasing rate of loading the position of this curve is not changed, but only the amplitude is increased. Instead of this, for the fast loading the limit curve in a frame of the second model does not exist at all, because the force  $Q(u)$  exerted by hammer is proportional to the hammer velocity  $V$  and its value is unlimited

$$Q(u) = p\alpha V Q_0 u^{p-1}. \quad (10)$$

For this and some other reasons, which are not discussed here, the first model (Eq. 1) is more physical and reasonable by nature, than the second (Eq. 7) one. Nevertheless, the three-parameter model, which is more simple, describes the dynamical behaviour of piano hammer also rather well for the hammer velocity up to 10 m/s.

### 3. PIANO HAMMER QUALITY

All the piano hammers tested in experiments were presented by Tallinn Piano Factory. The several series of measurements were carried out in order to compare the various types of piano hammers. Here the result of one of the tests is presented.

The four different hammers produced by *Abel* (Ab), *Immadi-gawa* (Im), and two *Renner's* hammers - old type (Or) and new type (Nr) were tested. The mass of each hammer was in the range 9.2 - 9.7 g. Such hammers correspond to  $E_2 - A_2$  notes. The hammer velocity just prior to impact was approximately 1 m/s (different for each hammer to obtain the equal upper level of the acting force). The results of experiments are presented comparatively in Figure 1. The numerical simulation of experiment was provided by using both the hammer models. In Table 1 are displayed the values of the numerically determined hammer parameters for all hammers and the exact values of hammer velocities.

| Parameter                   | Hammer type |       |       |       |
|-----------------------------|-------------|-------|-------|-------|
|                             | Ab          | Or    | Nr    | Im    |
| $F_0$ (kN/mm <sup>p</sup> ) | 44.0        | 31.4  | 16.3  | 23.2  |
| $\tau_0$ ( $\mu$ s)         | 2.30        | 2.0   | 1.82  | 2.0   |
| $p$                         | 3.80        | 3.70  | 3.73  | 3.80  |
| $\varepsilon$               | 0.998       | 0.998 | 0.996 | 0.997 |
| $Q_0$ (N/mm <sup>p</sup> )  | 88.0        | 62.8  | 65.4  | 69.6  |
| $\alpha$ ( $\mu$ s)         | 1150        | 1000  | 455   | 667   |
| $p$                         | 3.80        | 3.70  | 3.73  | 3.80  |
| $V$ (m/s)                   | 1.05        | 1.12  | 1.15  | 1.13  |

Table 1: Piano hammer parameters.

Because the hammer parameters within frame of two models are related to each other by equalities (5), in Figure 1 there are no noticeable differences between the curves obtained using by each model.

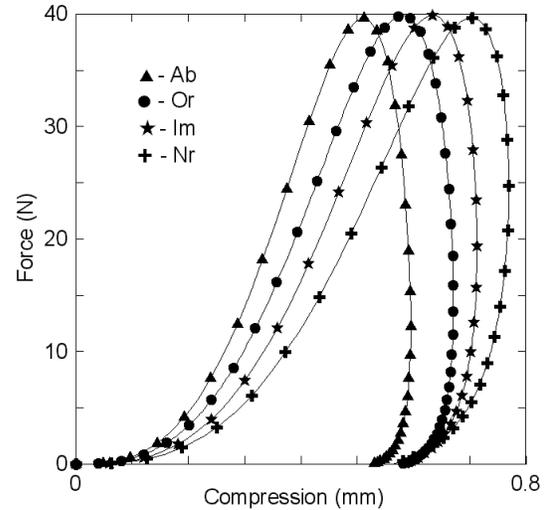


Figure 1: The measured (various symbols) and simulated (solid lines) force-compression characteristics of various piano hammers.

All the hammers considered are of different age and produced by different firms using the particular manufacturing technology. It seems, they have the quite different form of the force-compression characteristics, and the hammer parameters are not the same for each hammer. Nevertheless, the comparison of these hammers in frequency - domain, provided also in [3] demonstrates the similarity of hammers.

In Figure 2 is presented the result of the numerical simulation for note  $F_2$  ( $f=87.3$  Hz). The following set of the flexible string

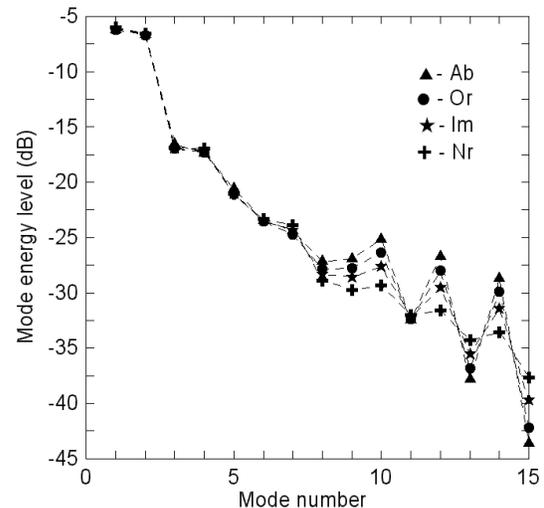


Figure 2: The simulated spectra of the string vibrations excited by various piano hammers.

parameters were used: the string length  $L=1.826$  m, the fractional striking point parameter  $r=0.118$ , the string diameter  $d=1.175$  mm. The linear mass density of this string is equal to  $\mu=8.51$  g/m, and for the simulation note frequency  $f=87.3$  Hz the string tension is equal to  $T=865$  N. The hammer parameters are the same as in Table 1.

The result of the spectra comparison presented in Figure 2 shows, that up to the 8th harmonic, the distinction between the hammers in frequency-domain is not essential. The largest difference of the mode energy level at  $n=15$  is equal to 6 dB. It is interesting, that the newest hammer (Nr) provides the most uniform spectrum of the string vibrations.

In our opinion, the main reason why the spectra of all the different hammers considered look similar is not a fortuity. It is obvious, that the dynamical features of the hammers produced by various firms are very similar indeed, for the various rates of loading. Thus we may state, that in spite of the different technologies that the manufacturers of the piano hammers are using, the mechanical features of their hammers are rather comparable. In particular, the evolution of the hammer manufacturing technology was developed so, to obtain namely the same modern hammer that we have now. And this one is the piano hammer which is made of the material with memory and possesses the hysteretic features.

#### 4. PIANO HAMMER SET

The procedure of the experimental testing of the whole hammer set gives a possibility to obtain the continuous dependencies of the hammer parameters on the key number. The experimentally measured force-compression characteristics of some hammers from the set of *Abel's* hammers were presented in [4], and displayed here in Figure 3. The hammer number  $N$  is displayed in Figure 3 near the corresponding curve.

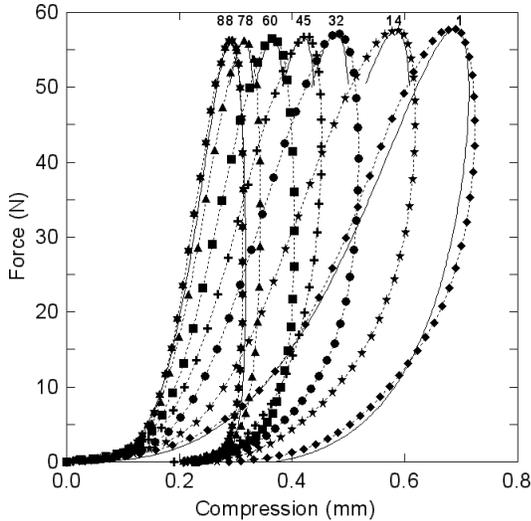


Figure 3: The measured (various symbols) and simulated (solid lines) force-compression characteristics of the hammer set.

During the measurements the initial hammer velocity  $V$  was not a constant value, but it was decreased with the hammer number  $N$ , to obtain approximately the same maximum value of the acting

force for the each hammer tested. Finally, the dependence of the hammer velocity on the hammer number  $N$  is determined as

$$V = 0.849 - 0.004N. \quad (11)$$

The velocity of the 88th hammer is equal to  $V = 0.5$  m/s.

Not all the data observed are used for determining the continuous dependencies. The rejected data corresponded either to possible errors in measurements (strong variation of data), or to hammers with defects. The total number of tests for one set is approximately 80. The percentage of rejected data is nearly 20%. We hope that further experiments should clarify these rejected data, which at the moment still bear certain marks of free choice.

Keeping the dependencies which form and position have qualitatively an obvious trend, we witness clearly regularity dependencies. The number of curves used for the analysis is certainly much more than seven shown in Figure 3, where the curves shown are the typical samples of the observed trend.

The hammer parameters were obtained by numerical simulation using the four-parameter model. In Figure 3 are also presented the simulated curves (solid lines) for the first and last hammers, and only the parts of the simulated curves (near the function maximum) for the others. The continuous dependencies of the hammer parameters on the key number are approximated as

$$p = 3.7 + 0.015N, \quad (12)$$

$$\varepsilon = 0.9894 + 8.8 \cdot 10^{-5}N, \quad (13)$$

$$\tau_0 = 2.72 - 0.02N + 9 \cdot 10^{-5}N^2, \quad (14)$$

for each hammer number  $1 \leq N \leq 88$ .

Regarding the hammer stiffness  $F_0$ , for this set of hammers, it is a linear function in logarithmic scale. The equation of this dependence is the function

$$F_0 = 15.5 \exp(0.059N). \quad (15)$$

Here the unit for relaxation time  $\tau_0$  is ( $\mu$ s), and the unit for  $F_0$  is ( $\text{kN}/\text{mm}^2$ ).

In Figure 4, the hammer parameters calculated for the three-parameter model are presented.

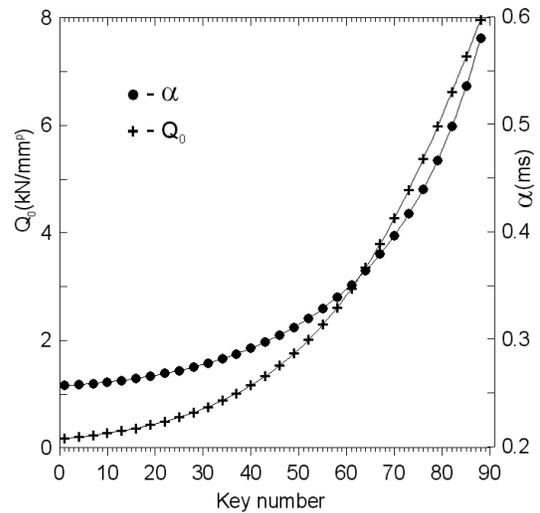


Figure 4: Hammer parameters.

These dependencies of the static hammer stiffness  $Q_0$ , and the retarded time  $\alpha$  were derived from (13) - (15), using (5). For numerical calculations the values of these parameters may be approximated as

$$\alpha = 248 + 1.83N - 5.5 \cdot 10^{-2}N^2 + 8.5 \cdot 10^{-4}N^3, \quad (16)$$

$$Q_0 = 183 \exp(0.045N). \quad (17)$$

Here the unit for retarded time  $\alpha$  is ( $\mu\text{s}$ ), and the unit for  $Q_0$  is ( $\text{N/mm}^p$ ). For the three-parameter model we must choose the same value of the compliance nonlinearity exponent  $p$  (Eq. 12), as for the four-parameter model.

Most likely, for a good high-quality set of piano hammers the dependencies of the hammer parameters on the key number must be continuous and regular as in Figure 4. On the contrary, if the measured values of the hammer parameters do not show such regular dependencies we may suppose that the hammer set is not of a good quality.

## 5. HAMMER-STRING INTERACTION

The numerical simulation of the hammer-string interaction considered in Section 3, is based on the mathematical model described in [6]. The application of this method for the case of hysteretic hammer was presented also in [7]. It was shown, that for calculation of the string vibration spectra the type of the hammer model is very important. In particular, the influence of the fractional striking point parameter on the sound spectra for the four-parameter model was discussed.

Here we want to demonstrate the importance of the hammer-string interaction study for the piano strings and scale design. No doubt that there are many approved, tested in practice and well tried methods, which are used for creation of a good configuration of tonally related structural parts of the grand piano. But for all that the dynamical features of the piano hammers were not taking into account due to the absence of good models.

Now we have the suitable piano hammer models, and the hammer parameters are also the known values. Thus, we may simulate the hammer-string interaction for the purpose of matching the hammers to some piano scale. Quite the contrary, we may use our knowledge to design of a string scale in according to the piano hammer used. For example, in Figure 5 are presented the simulated spectra for notes  $A_2$  (curves 1 and 2) and  $A_2^\sharp$  (curve 3). These are the neighbor notes, but the first note  $A_2$  consists of two strings, and the second one of three strings per note. Thus, we have a problem how to choose the string tension for each note.

Initially, the following set of the string parameters for note  $A_2$  were used: the string length  $L=1.218$  m, the fractional striking point parameter  $r=0.125$ . This is the wrapped string  $d=0.950$  mm with the winding wire  $d=0.200$  mm. The linear mass density of this string is equal to  $\mu=10.36$  g/m. For note  $A_2^\sharp$  the string parameters are: the string length  $L=1.201$  m, the fractional striking point parameter  $r=0.125$ , the string diameter  $d=1.125$  mm. The linear mass density of this string is equal to  $\mu=7.81$  g/m. For these strings the spectra difference (curves 1 and 3) is approximately equal to 5 dB at 11th and 13th harmonics. Nevertheless, if we choose for note  $A_2$  the another wrapped string:  $d=0.975$  mm with the winding wire  $d=0.250$  mm (curve 3), the spectra difference of the string vibrations may be neglected at all.

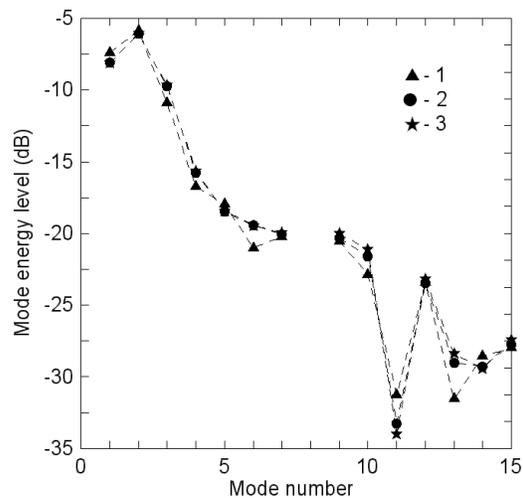


Figure 5: The simulated spectra of the various string vibrations excited by new type of Renner hammer.

## 6. CONCLUSIONS

The new three-parameter hysteretic model of piano hammer developed may be successfully used for description of the hammer-string interaction for a broad range of hammer velocities. The comparison of the different piano hammers produced by various firms shows that in spite of the different manufacturing technologies, the dynamical features of dissimilar hammers are rather comparable. The regular dependencies of the piano hammer parameters on the key number were obtained. The knowledge of the hammer parameters gives the possibility to predict the string vibration spectrum. Apparently, the results obtained will serve as a basis for a new method for piano stringing-scale design.

## 7. REFERENCES

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