PIANO STRING-BRIDGE INTERACTION AND ROLE OF DUPLEX SCALE

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Abstract: Estonia Piano Factory is among the few companies in the world that build their pianos with movable double duplex scale. It enables the technicians to change the tuning according to their wishes and necessary acoustics. The special sample of Parlour Grand Piano Estonia L190 was fabricated for experimental studies. Using B&K impact hammer and accelerometers attached to the bridge as a tool, the frequency response functions and two dimensional motion of piano treble bridge were examined in cooperation with IRCAM. The influence of the bridge impedance and duplex scale tuning on the string vibrations was considered. It was shown that inharmonicity of vibrating string appears not due to string stiffness, but due to the bridge impedance and the presence of a duplex scale. The mathematical models of the string-bridge interaction were also discussed. These results should prove extremely useful in developing an accurate model of the real string vibration excited by piano hammer.

1. Introduction

Only a few grand piano companies in the world produce pianos with movable double duplex scale. These companies are Steinway, Yamaha, Fazioli, Pfeiffer, and Estonia. The traditional duplex scale was created by Steinway in 1872. The illustration of a grand piano duplex scale is shown in Figure 1.

The duplex scale is the part of a string between the front tip of the bridge and the brass bridge before the string pin. Their length can be adjusted and "tuned" in harmony with the main strings by sliding of the brass bridge on an inox steel surface, fixed to the iron frame.

Figure 1. Yamaha grand piano duplex scale.

In general, the duplex scale lengths must remain in a fifth or octave relationship with the length of the related string and can be tuned by moving the brass bridges. It does serve to enhance the tonal quality of the note being played by adding that extra sound one octave higher than the actual note, adding a pleasant resonance.

This system of doubling the octave of the notes is only effective in the middle and upper range of the piano. Therefore the duplex scale is present in that part of the piano only. Duplex scaling is not effective on all pianos due to the differences and size and overall design and has in fact been installed on some pianos without proper engineering only to result in tone that is too harsh or even worse, or in other words, in a piano that is very difficult to tune properly. One should not assume that a piano with a duplex scale is better than one without. The tone quality of the given piano should be the final criterion by which we judge its effectiveness.

There are also other opinions that the duplex scale does not improve the sound quality, and the "art of tuning the duplex scale" is a myth although most piano tuners have been taught to believe it by the manufacturers, because it makes for a good sales pitch.

Nonetheless, a proper understanding of the role of the duplex scale is certainly preferable. Evidently, the tuning the duplex scale can optimize the mechanical impedance of the bridge. The duplex scale is needed to allow the bridge to move more freely, and not for producing sound. However, for impedance matching, the tuning needs only be approximate, which is what is done in practice.

The experimental investigation of the duplex scale and the mathematical modeling of this problem allow clarify the influence of the duplex scale tuning on the sound quality of the piano.

2. Model of string-bridge interaction

The displacement $y(x,t)$ of the ideal (flexible) string obeys the second-order wave equation

$$\frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} \qquad (1)$$

The relationships connecting the velocity c of the transverse wave of the string vibration, angular frequency ω, string length L, string tension T, and the linear mass density of string μ are

$$\pi c = \omega L, \qquad T = \mu c^2 \qquad (2)$$

Discussing the model of the string-bridge interaction we may consider a string of finite length, stretched between one rigid support (agraffe) at $x=0$, and the bridge having the transverse mechanical impedance $Z(\omega)$ at $x=x_0$. Therefore, for the simple-harmonic oscillations of frequency $\omega/2\pi$, the boundary conditions for the string will be $y=0$, at $x=0$, and

$$y = \frac{T}{i\omega Z}\frac{\partial y}{\partial x} \qquad (3)$$

at $x=x_0$.

We can suppose that the n-th trial function is

$$\psi_n(x,\omega) = \sin\left[\frac{\omega_n}{c}(x - a_n)\right] \qquad (4)$$

where ω_n and a_n must be obtained from boundary conditions. Using (3) and assuming $(T/\omega x_0 Z) << 1$, we can derive

$$\omega_n \approx \frac{n\pi c}{x_0}\left[1 + \frac{T}{i\omega x_0 Z(\omega)}\right], \qquad a_n = 0 \qquad (5)$$

As a result, the fundamental function at $x=x_0$ is

$$\psi_n(x,\omega) \approx (-1)^{n-1}\frac{n\pi c}{\omega x_0}(\sigma_n + i\chi_n) \qquad (6)$$

where

$$\sigma = -\frac{\mu c X}{R^2 + X^2}; \quad \chi = \frac{\mu c R}{R^2 + X^2} \qquad (7)$$

and $Z(\omega)=Z_n(\omega_n)=R_n(\omega_n)-iX_n(\omega_n)$.

Thus, the real part of the mechanical impedance of the bridge as well as the imaginary causes the damping of the free vibrations. The damping constant depends also on the string length x_0. For this reason the oscillation of a short string is dying faster, then oscillation of a long string.

It is also important that the string partials are not exact integral multiplies of the fundamental string frequency due to the influence of the bridge impedance, in according to formula (5).

In fact, in real grand pianos the strings are not terminated on the bridge, but passing the bridge they terminate on the hitch-pin rail of the frame. Therefore we can suppose also another mathematical model of the string-bridge interaction. Probably, it is possible to consider the bridge as an extra load attached to a string of finite length L, stretched between agraffe and the brass bridge before the string pin.

This load changes the frequencies of free vibration of the string and also changes the shape of the standing waves of free vibration. If the load is a force concentrated at the point x_0 of the string, and it is a simple-harmonic, of frequency $\omega/2\pi$, so that $f(x,t)=F(x)\exp(-i\omega t)$, we can represent the steady–state shape of the string with the same factor, $y(x,t)=Y(x)\exp(-i\omega t)$. For a concentrated force of magnitude equal to the string tension in the string, $f(x,t)=T\ \delta(x-x_0)\ \exp(-i\omega t)$, the Green's function $g(x|x_0)$ is a solution of equation

$$\frac{d^2 g}{dx^2} + \left(\frac{\omega}{c}\right)^2 g = -\delta(x - x_o) \qquad (8)$$

The Green's function must also satisfy the boundary conditions at the end of the string. If the string supports are rigid, $g(x|x_0)$ must be zero at $x=0$ and $x=L$. Therefore the Green's function for $0< x < x_0 < L$ is

$$g(x\,|\,x_0) = A\sin\frac{\omega(L - x_0)}{c}\sin\frac{\omega x}{c} \qquad (9)$$

and for $0 < x_0 < x < L$ is

$$g(x\,|\,x_0) = A\sin\frac{\omega(L - x)}{c}\sin\frac{\omega x_0}{c} \qquad (10)$$

where

$$A = \frac{c/\omega}{\sin(\omega L/c)} \qquad (11)$$

In case the load is an impedance $Z(\omega)$, attached at the point x_0, the reaction force is $f(x,t)= i\omega Z(\omega)Y(x)\ \delta(x-x_0)\ \exp(-i\omega t)$, and the shape of the string will be

$$Y(x) = \frac{iZ(\omega)}{\mu c}\frac{Y(x_0)}{\sin(\omega L/c)}G(x\,|\,x_0) \qquad (12)$$

where the Green's function for $0 < x < x_0 < L$ is

$$G(x \mid x_0) = \sin\frac{\omega(L-x_0)}{c}\sin\frac{\omega x}{c} \qquad (13)$$

and for $0 < x_0 < x < L$ the Green's function is

$$G(x \mid x_0) = \sin\frac{\omega(L-x)}{c}\sin\frac{\omega x_0}{c} \qquad (14)$$

Setting $x=x_0$ provides the equation for the resonant frequencies

$$\sin\frac{\omega L}{c} = \frac{iZ(\omega)}{\mu c}\sin\frac{\omega(L-x_0)}{c}\sin\frac{\omega x_0}{c} \qquad (15)$$

or

$$\tan\frac{\omega L}{c} = \frac{-(iZ/\mu c)\sin^2(\omega x_0/c)}{1-(iZ/2\mu c)\sin(2\omega x_0/c)} \qquad (16)$$

The formulae above are well known, and were derived seventy years ago by Philip McCord Morse [1] (first edition of this book was in 1936).

Because the bridge can be considered as a very massive load, therefore its mass $M >> \mu L$, then the lowest mode of the working string length is a few lower than that of the fundamental, $\omega_0=\pi c/x_0$, of the unloaded string.

Setting $\omega=\omega_0(1-\varepsilon)$ and using expression (15), we can find the shift ($\varepsilon<<1$) of the fundamental frequency of the string oscillation caused by the bridge impedance

$$\varepsilon = \frac{1}{\pi}\frac{\delta\tan(\pi\beta)}{\tan(\pi\beta)-\delta(1+\beta)} \qquad (17)$$

where

$$\delta = \frac{\mu c}{iZ(\omega)} = \vartheta(\omega) + i\sigma(\omega), \quad |\delta| << 1 \qquad (18)$$

and $\beta=(L-x_0)/x_0$, is the ratio of the lengths of the parts of the strings passing the bridge to the working part of the string x_0. Usually, β changes approximately from 3/5 for the high treble notes to 1/7 in the middle, and the suitable value of this parameter may be preferred by duplex scale tuning.

The dependencies of the frequency shift ε on the possible values of β calculated using formula (17) are presented in Figure 2. Here the values of δ were selected arbitrary. In fact, this value is strongly determined by the bridge impedance by formula (18), and its value causes not only the frequency shift, but also the decay rate of the string oscillations.

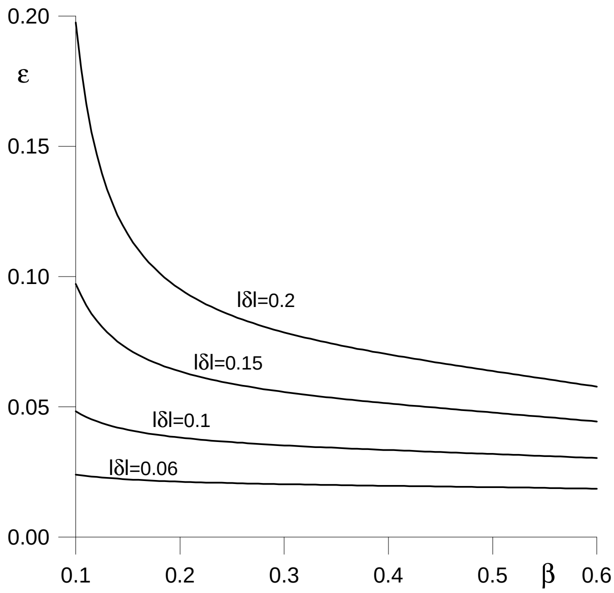

Figure 2. Frequency shift as a function of the duplex scale tuning parameter.

Because the bridge impedance is a very specific and peculiar measure of each instrument, it must be determined in experimental testing of piano.

3. Experimental method

The individual sample of a grand piano shown in Figure 3 was manufactured by Tallinn Piano Factory, and presented to the Institute of Cybernetics as a trial tool. It is built similar to Parlour Grand Piano Estonia, which is produced by Tallinn Piano Factory during quite a few years. This unique instrument is prepared for experimental measurements and tests of the various parts of the piano.

The experimental study of treble bridge vibrations of this piano was carried out by Rene Causse and Philippe Zelmar from IRCAM, Paris during their visit in Tallinn in December 2005 (PARROT Programme).

The Bruel & Kjaer impact hammer of type 8204 was used for the treble bridge excitation in a direction perpendicular to both the soundboard and the string. Two Bruel & Kjaer accelerometers of type 4374 were mounted on the bridge to measure the bridge motion perpendicular and parallel to the soundboard.

The treble bridge impedance was measured in the region of termination of the strings for note F_7 (fundamental frequency f_0=2794 Hz; key number n=81).

Figure 3. Experimental grand piano Estonia.

The frequency dependence of the relative transverse bridge impedance of piano Estonia is shown in Figure 4. In Figure 5 are shown the similar results obtained by P. Zelmar for Petrof piano. The entire collection of results of the piano bridges and duplex scale studies of different pianos is presented in P. Zelmar theses [2].

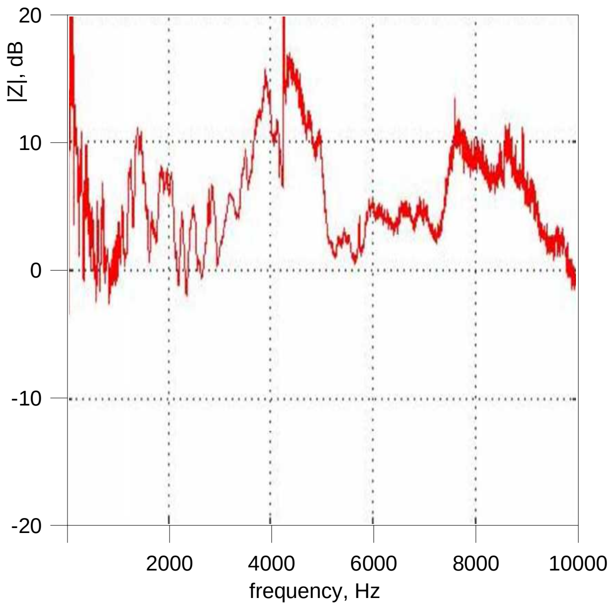

Figure 4. Transverse bridge impedance of Estonia piano in the region of note F_7 (f_0=2794 Hz).

4. Discussion

We have presented theoretical and experimental results of the piano string-bridge interaction. The significant influence of the bridge impedance on the frequency of the string oscillations and its decay rate is shown. It is especially noteworthy that the second and higher modes of piano string

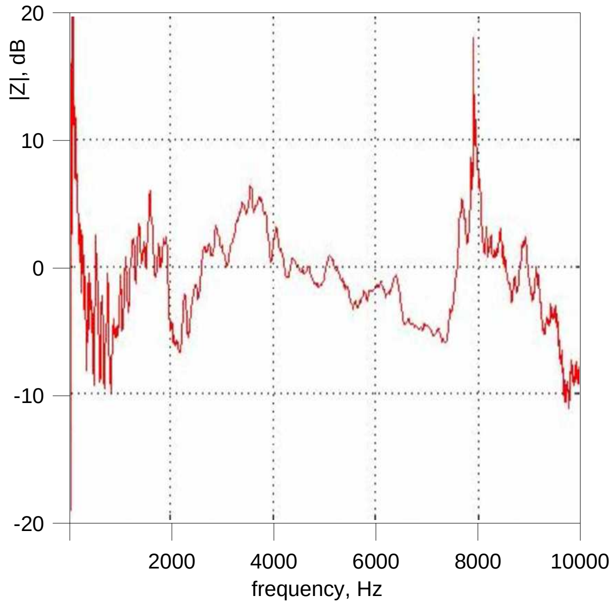

Figure 5. Transverse bridge impedance of Petrof piano in the region of note E_7 (f_0=2637 Hz).

vibrations are not exact integral multiples of the fundamental due to the influence of the bridge impedance, but not due to the string stiffness. The results of the experimental study of the piano bridge motion demonstrate that the bridge impedance is very individual for each piano and it is also the extremely jagged function of frequency. For this reason, in according to formula (18), variation of parameter δ may be also very high. Taking into account that duplex scale tuning of a certain string can also change the bridge impedance for the neighbouring strings, in our opinion the role of the duplex scale tuning is not so efficient, as it might sound.

The results of this study give the clear understanding of the process of the wave transmission through the bridge and the way of the piano soundboard excitation.

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References

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