Proc. Estonian Acad. Sci. Engin., 1995, **1**, 2, 172-182

A SIMPLE GRAND PIANO HAMMER FELT MODEL

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Presented by J. Engelbrecht

Received 8 December 1995, accepted 31 January 1996

Abstract. Using a reasonable physical assumption, a simple model of the hammer felt was developed. The model is in a good agreement with an experimental force versus deformation relationships obtained both for various numbers of hammers and felt stiffness. Such a model allowed us to calculate the hammer-string interaction for all notes of the grand piano by evaluating only one parameter - the Young's modulus of the felt material. As a result, we can choose the hammers for a piano by matching the masses of the hammer and the felt stiffness. Consequently, this will improve the quality of the instrument.

Key words: grand piano, hammer model, Hertz's law, hammer-string interaction.

1. INTRODUCTION

In the grand piano, the hammers are significant in the sound formation. Therefore, choosing a hammer for a certain piano model is very important. A good set of hammers with matching parameters gives the quality of the instrument. This choice is based on the mathematical simulation of the hammer-string interaction. For this calculation, however, the exact values of the physical parameters of each hammer must be known. The grand piano has eighty eight hammers, and all of them are different – they have various stiffnesses, radii of the curvature, masses, etc.

Some parameters can be obtained by simple measurements. So, we may suppose that such parameters as the radius of the head curvature and the mass of the hammer are known for all the hammers of the grand piano. But it is very difficult to say anything about the stiffness or the Young's modulus of the felt material because the construction of the hammer head is rather complicated. For this reason, there are no universal mathematical models suitable for all hammers.

2. PROPERTIES OF FELT STIFFNESS

The first model of the piano hammer was proposed by Ghosh [1] who considered felt as a material obeying the Hooke's law in the Hertz form

$$F(u) = Eu^p, \quad (1)$$

where

F – the force due to the felt compressed by the collision upon the string;

u – felt compression;

E – a constant;

p – the compliance nonlinearity exponent.

The real samples of hammers had values of p between 1.5 and 3.5 with no definite trend of p from bass to treble as discussed in [2]. The value of p = 1 gives a simple linear system, but it has the unmusical property because loud notes are equivalent to amplified soft notes. Values greater than p =1 are not entirely due to the peculiarities of the felt; the geometry of the rounded contact would already give p =1.5 (known as the Hertz's law) even for locally reacting Hookean material. The value $p > 3$ probably causes too much contrast between a very harsh tone color when playing *fortissimo* and a very blend when playing *pianissimo*. Hall [3] uses this nonlinear model of the felt in the Hertz form to model the piano string - hammer interaction and obtains a better agreement with the earlier data than using his previous calculations, based on a completely linear model.

Suzuki and Nakamura [4] describe the properties of the hammer more explicitly. They present the results of measurements of dynamic relationships between the hammer felt deformation and the applied force. Three types of hammers – soft, medium and hard, acting on the three various strings, were discussed. Our tests of the hammer felt model were based on the experimental data presented by Suzuki and Nakamura.

All the previous piano hammer models are static(in addition to [5,6]) and deal with one hammer, the parameters of which must be obtained experimentally by static or dynamic loading. These data are not suitable for others hammers, and at least eighty eight experiments are needed to prove the data for all hammers.

The model of the hammer felt presented here is based on the known geometrical parameters of the hammers and describes the force-compression characteristics of all the grand piano hammers by evaluating only one parameter – the Young's modulus of the felt. In this case, the value of the Young's modulus is limited and is changed approximately from 160 MPa for soft and medium hammers to 260 MPa for hard hammers.

Based on the presented model, we can calculate the interaction between the hammer and the string for all the numbers of the grand piano keys and choose the masses of the hammers and felt stiffnesses in order to improve a quality of the instrument.

3. HAMMER FELT MODEL

When we derive a static hammer felt model, we must describe the deformation of the felt. Under the acting force, it depends only on the stiffness parameters of the felt material, identical for all the hammers of the grand piano, and on the geometrical domain of interaction between the hammer and the string, as shown in Fig. 1. Also, we assume that the thickness of the felt is rather large as compared to the felt compression. Thus, the existence of the hammer wood kernel may be neglected. In Fig. 1, the following notation is used: u is the dent of the felt; $d = 2r$ is the string diameter; R is the radius of the curvature of the hammer head.

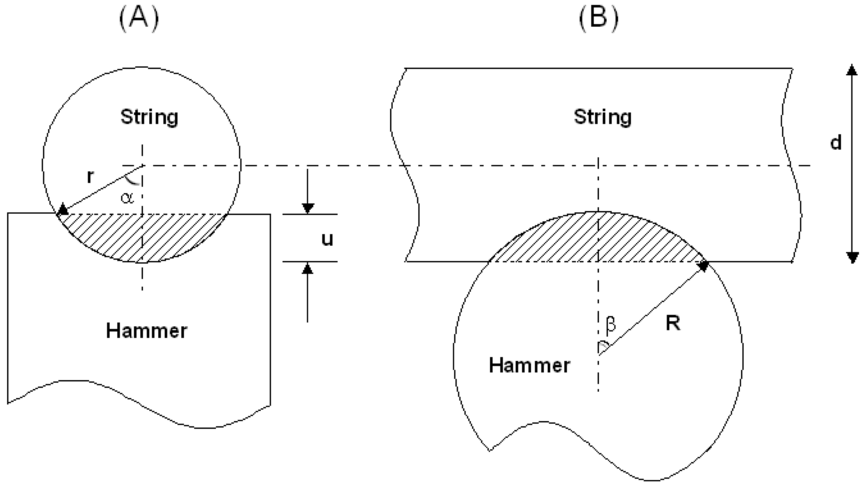


Fig. 1. Geometry of the hammer-string interaction: A – cross-section across the string, B – cross-section along the string.

When the hammer strikes, the string deforms the felt and the kinetic energy of the hammer is transformed into the deformation energy of some volume of the felt. We assume that this energy is concentrated mainly on the region of interaction of these two bodies – the cylindrical hammer and the cylindrical string (shaded in Fig. 1), and that the density of this energy is constant in the volume of interaction.

Now we can find the value of the force causing deformation

$$F(u) = \frac{\partial U(u)}{\partial u}. \tag{2}$$

Here, $U(u)$ is the energy of deformation proportional to the energy density of some volume $V(u)$ of the felt.

Suppose that the total deformation energy is proportional to the dent of the felt and to the volume of deformation

$$U(u) = U_0 \, u \, V(u), \qquad U_0 = \text{const}. \tag{3}$$

Because rather rough assumptions are used here, the volume of deformation is approximately equal to

$$V(u) = 2RS\sin^2\beta\,, \tag{4}$$

where S is the area of the region of interaction shaded on the cross-section (A) in Fig. 1

$$S = \frac{1}{2}r^2 q(u)\,, \tag{5}$$

and

$$q(u) = \begin{cases} 2\alpha - \sin 2\alpha\,, & \text{if } u \le r \\ \pi + 4[(u/r) - 1]\,, & \text{if } u > r\,. \end{cases} \tag{6}$$

The angles α and β are shown in Fig. 1.

Taking into account that the felt compression is small, i.e. $u \ll R$, we obtain

$$\cos\alpha = 1 - \frac{u}{r}\,, \tag{7}$$

and

$$\sin^2\beta = \frac{2u}{R}\,. \tag{8}$$

The volume of deformation is now calculated as

$$V(u) = 2ur^2 q(u)\,. \tag{9}$$

Substituting Eqs. (3), (4) and (9) into Eq. (2), we find the force acting on the string due to the felt deformation

$$F(u) = 2U_0 r^2 u(2 + \frac{u}{q}\frac{\partial q}{\partial u})q(u)\,. \tag{10}$$

It easy to show that the second term in the parentheses is restricted for any u

$$\frac{3}{4} \le \frac{u}{q}\frac{\partial q}{\partial u} \le \frac{4}{\pi}\,, \tag{11}$$

so that the two terms in parentheses in Eq. (10) may be substituted by one constant.

If we introduce deformation or nondimensional compression y of the felt according to

$$y = \frac{u}{d}\,, \tag{12}$$

and replace the arbitrary constant U_0 by the expression

$$U_0 = E\frac{d^2}{R}\left(1 + \frac{d}{2R}\right)^{-1/2}\,, \tag{13}$$

then we obtain

$$F(y) = E\frac{d^3}{R}\left(1+\frac{d}{2R}\right)^{-1/2} yq(y)\,, \tag{14}$$

with the function

$$q(y) = \begin{cases} 2\arcsin\phi - 2(1-2y)\phi\,, & \text{if } y \le 0.5 \\ 8y+\pi-4\,, & \text{if } y > 0.5\,, \end{cases} \tag{15}$$

and

$$\phi = 2\sqrt{y(1-y)}\,, \tag{16}$$

where now the constant E is a Young's modulus of the felt material (or the constant directly proportional to the Young's modulus). The constant U_0 in the form of Eq. (13) was obtained in the following way.

According to the Hertz's law, the force acting on the two connected elastic spheres with radii R_1 and R_2, respectively, is given by

$$F(u) = \frac{4E}{3(1-\sigma^2)}\left(\frac{R_1 R_2}{R_1+R_2}\right)^{1/2} u^{3/2}\,, \tag{17}$$

where E is the Young's modulus and σ is the Poisson's ratio. As mentioned above, the value p = 1.5 of the compliance nonlinearity exponent is not in agreement with the experimental data obtained for the real felt deformation. By choosing U_0 in the form as in Eq. (13), for the small deformation $y \ll 1$, Eq. (14) gives

$$F(u) = \frac{\sqrt{2}E}{R}\left(\frac{rR}{r+R}\right)^{1/2} u^{5/2}\,, \tag{18}$$

which is similar to Eq. (17) and so U_0 in the form Eq. (13) can be used by the analogy of the Hertz's law to describe of the hammer-string interaction as in Eq. (14). In this case, the value of the compliance nonlinearity exponent p = 2.5 in Eq. (1) and that obtained here are in a good agreement with the experimental results discussed in .

4. MODEL AND EXPERIMENT COMPARISON

There is only one way to judge the success of the present model. It is the comparison with the experimental data. In [4] the relationships of dynamic force versus deformation are presented for hard, medium and soft hammers A_0, A_3 and A_6 (key numbers n = 1, n = 37 and n =73, respectively). These nonlinear relationships show a significant influence of hysteresis characteristics. Because the model presented here is rather simple and ignores this effect, only the increasing parts of the experimental characteristics are discussed. It is also suggested that the unloading and the loading of the felt are similar.

Unfortunately in [4], such parameters as the masses of the hammers and their dimensions are not determined. Thus, the comparison between the experimental and the theoretical results is not always correct. Table 1 shows the primary parameters of the grand piano (the parameters known those easy to measure).

Table 1

Primary parameters of the grand piano strings and hammers

Parameter			Notes A_0	A_3	A_6	C_4
String						
	Note frequency	f, Hz	28	220	1760	262
	Length	L, mm	2016	777	115	620
	Distance of hammer from nearest string end	l, mm	243	91	8.1	74.4
	Diameter	d, mm	4.9	1.075	0.875	1.025
	Tension	T, N	1629	834	774	670
	Linear mass density	μ, g/m	130.7	7.1	4.7	6.3
Hammer						
	Key number	n	1	37	73	40
	Mass	m, g	13	10.6	8.2	8.9
	Radius of the head	R, mm	17	11	5	8

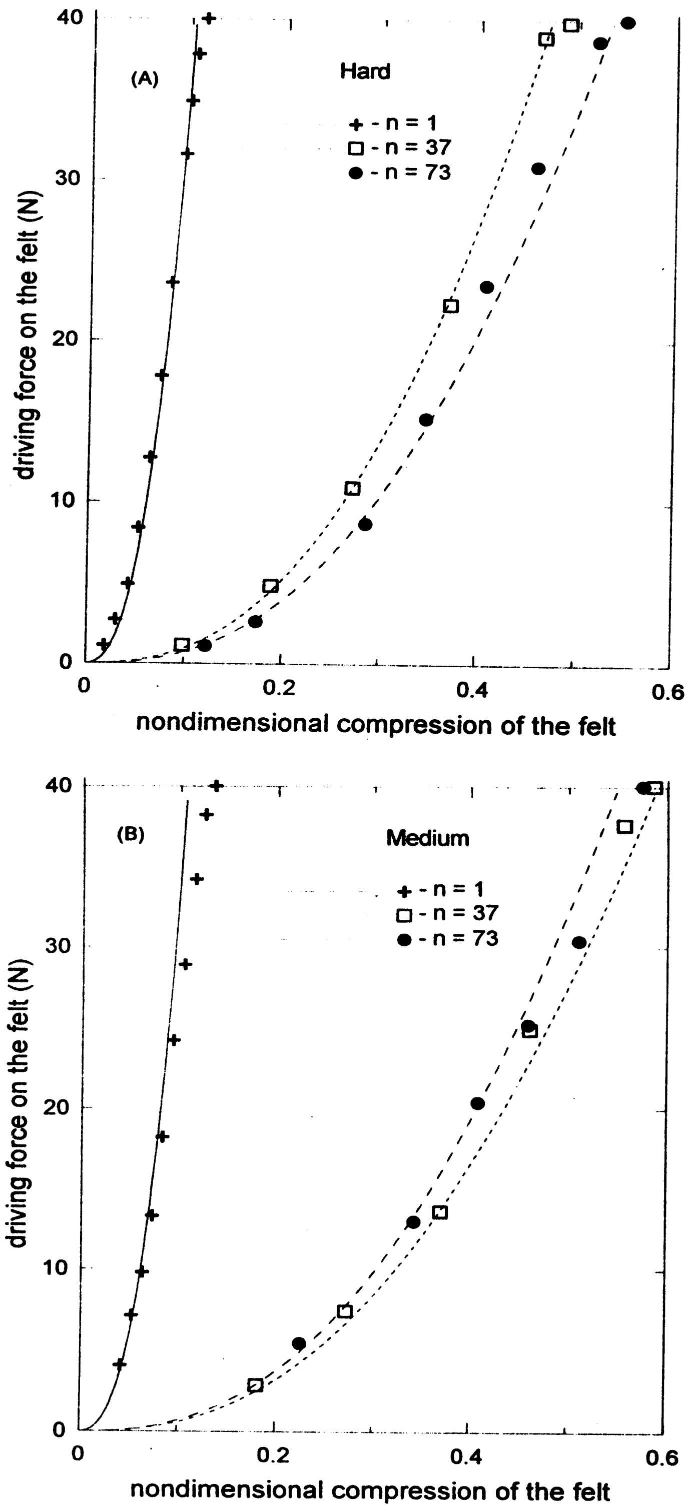
Figure 2 illustrates the comparison of the experimental data and the data calculated using Eq. (14). The values of the Young's modulus used for plotting curves in Fig. 2 for soft, medium and hard hammers are presented in Table 2. It is clear that a sufficiently simple model presented here is in good agreement with the experimental data and can be used to describe of the hammer-string interaction.

It is easy to see that Eq. 14 consists of two parts: the first multiplier depends on the parameters defined only by the key number, and the second one depends only on the dent of the felt. Like in [7], the second multiplier may be approximated in the polynomial form or in the form of the usual power-law dependence [2,3,5]. The first good approximation of the function $f(y) = yq(y)$ in the interval $0 < y < 0.6$ is given by the function

$$f_1(y) = (2.4y^2 + 9y^3)\,, \tag{19}$$

and the second approximation of the function $f(y)$ is

$$f_2(y) = 7.5y^{2.3}\,. \tag{20}$$



(A)
Hard
+ - n = 1
□ - n = 37
● - n = 73
driving force on the felt (N)
nondimensional compression of the felt
0
10
20
30
40
0.2
0.4
0.6
(B)
Medium
+ - n = 1
□ - n = 37
● - n = 73
driving force on the felt (N)
nondimensional compression of the felt
0
10
20
30
40
0.2
0.4
0.6

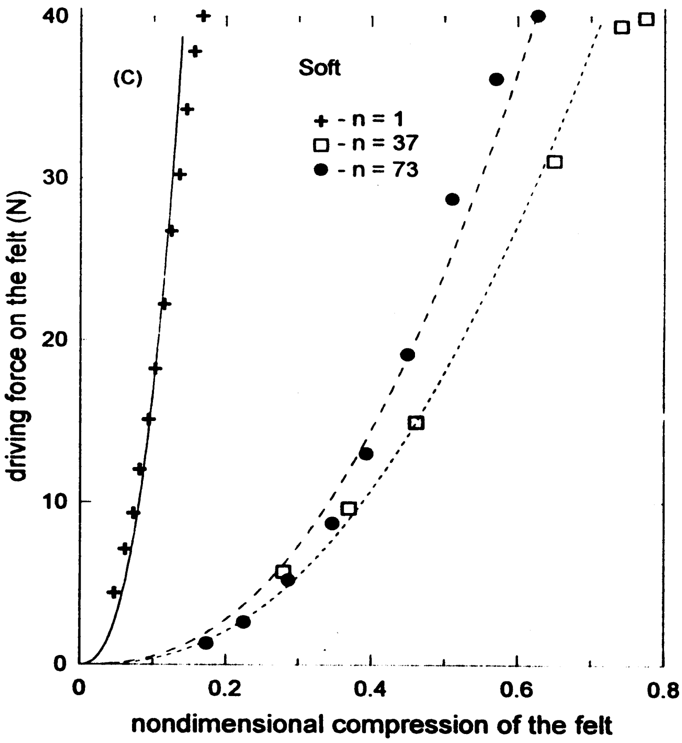


Fig. 2. Force-compression characteristics of the felt for: A – hard hammers, B – medium hammers, and C – soft hammers. Crosses, circles, and squares denote the experimental data points [4]. The solid, dashed, and dotted lines are the calculated curves for each of the key number.

Table 2

Young's moduli of the felt *E*, MPa

Key number	Hammer		
	Hard	Medium	Soft
$n = 1$	170	160	160
$n = 37$	260	160	208
$n = 73$	170	160	240

So the force-compression relationship of the hammer felt may be described as

$$F(y) = F_0(2.4y^2 + 9y^3)\,, \tag{21}$$

or

$$F(y) = F_0\,7.5y^{2.3}\,, \tag{22}$$

where

$$F_0 = E\frac{d^3}{R}\left(1+\frac{d}{2R}\right)^{-1/2}. \tag{23}$$

Eqs. (19) and (20) are also rather good representations of the model developed and may be used to describe the force-compression characteristic of the piano hammers.

The values of the Young's moduli presented in Table 2 demonstrate a sufficiently good agreement with the experimental data both for the hammer number and the felt type. The value of the Young's modulus for the medium hammers is constant and nondependent on the key number. Although Table 2 shows that the hard A_3 hammer is really exceptional, the same is valid for the hard hammers. As referred to above, some parameters of the hammers used in the experiments are not described in [4]. In the manufacture of the piano hammers, various types of the felt materials are used, and so the hammers vary in dimensions. The types of hammers studied in the experiments are not specified. We may suppose that the radius of the curvature of the hard A_3 hammer was not 11 mm but 7 mm, and then the Young's modulus of that hammer is equal to 170 MPa also. Probably for the same reason, the radii of the curvature of the soft A_3 and A_6 hammers were not 11 mm and 5 mm, but less, and the Young's modulus of the soft hammers is constant and approximately equal to 160 MPa.

Table 1 also demonstrates some parameters for the note C_4. These values were used in [8] for the numerical simulation of the piano string excitation. The force-compression characteristics of the hammer were describe in the form

$$F(u) = Ku^p \tag{24}$$

with K = 142.3 N/mm^p, and p = 2.5. The values of these parameters were obtained experimentally. The model of the hammer felt developed here allows us to describe the force-compression characteristics. Figure 3 shows two of the force-compression characteristics for note C_4. The solid line is obtained by using Eq. (14) with E = 122 MPa, and the dashed line is calculated from Eq. (24) using [8] and Table 1. The agreement of these two curves is rather good. It is obvious that a very soft C_4 hammer was used in these experiments.

Thus, the model of the hammer felt presented here in Eq. (14) enables us to calculate the hammer-string interaction for all hammers of the grand piano from the note A_0 to the note C_8 and to match the hammers or to calculate the spectra of the strings vibrations.

5. CONCLUSION

A new version of the grand piano hammer felt model is proposed. This model is in good agreement with the experimental data presented by different authors. It enables us to describe all the numbers of the hammers with the various

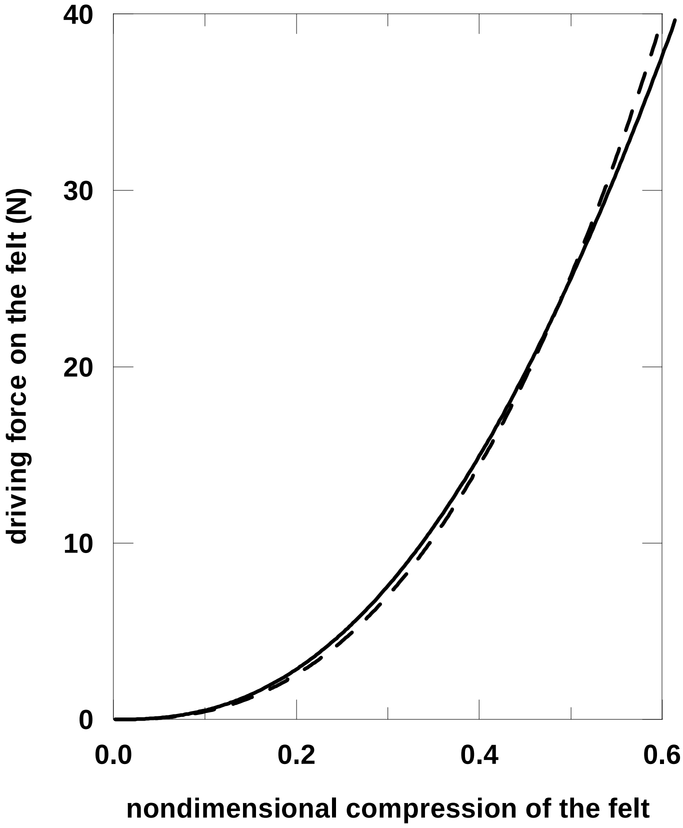


Fig. 3. Comparison of the two force-compression characteristics of the felt for the C_4 note. The solid line is the calculated Eq. 14 curve, and the dashed line is the experimentally obtained [8] curve.

felts (soft, medium and hard) of the grand piano. According to the model, these felts differ in one parameter – the Young's modulus of the felt material. For a set of the grand piano hammers, the Young's modulus has a constant value.

This result implies a way to improve the quality of the grand pianos by matching the parameters of the hammers by numerical simulation of the hammer-string interaction.

ACKNOWLEDGMENT

This study was supported by the Tallinn Piano Factory, Estonia.

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KLAVERI HAAMRI VILDI LIHTNE MUDEL

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Lähtudes põhjendatud füüsikalistest oletustest on välja töötatud haamri vildi lihtne mudel. Mudel on heas vastavuses nii erinevate numbritega haamrite kui ka erineva jäikusega vildi puhul saadud katselise jõu ja deformatsiooni vahelise seosega ning võimaldab arvutada ainult ühe parameetri – Youngi mooduli alusel kontsertklaveri igas sõlmes haamri ja keele vahelise vastastikmõju. Selle alusel saab valida konkreetse klaveri haamreid sobitades haamrite massi ja vildi jäikust ning parandada seega instrumendi kvaliteeti.

ПРОСТАЯ МОДЕЛЬ РОЯЛЬНОГО МОЛОТКА

Анатолий СТУЛОВ

С использованием разумных физических предположений разработана простая модель рояльного молотка. Эта модель хорошо описывает зависимость силы, действующей на молоток, от величины деформации фильца как для различных номеров молотка, так и для различных жесткостей фильца. Модель позволяет рассчитать взаимодействие струны и молотка для всех нот, варьируя только один параметр – модуль упругости фильца. Такая процедура дает возможность подбирать молотки для любого рояля, улучшая качество звучания инструмента.