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Physical modelling of the piano string scale

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Abstract

Several stages of physics-based mathematical modelling are described for the design of the piano string scale. Strings are assumed to be perfectly flexible, and piano hammers are described by a nonlinear hysteretic model. It is also assumed that the parameters of the hammers for the whole hammer set are determined experimentally beforehand. Simulation procedures are used to systematically adjust the structure of the piano scale to its optimal value. The efficiency of the piano scale is improved by the analysis of the numerically simulated string motion and spectra of the string vibrations excited by the impact of the hammer. The set of variables to be optimized includes the linear mass density and tension of the piano strings and the position of the striking point. In addition, the problem of choosing appropriate tensions for neighbouring strings terminated on separate bass and treble bridges is considered.
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1. Introduction

The grand piano is a musical string instrument with more than two hundred years of history. As a result, the design of a modern grand piano is a masterpiece of art. However, its musical features have been reached empirically, as a result of hundreds of experiments and long-term practical experience.

The musical performance of each instrument is mainly determined by the piano string scale. This scale is a summary table of the full collection of the string lengths, string diameters, diameters of wrapping wires for the bass strings, and the distance along the string from the hammer striking point to the nearer string termination. The scaling of the piano is in general a complicated theoretical problem. The rules of piano scale design are based on simple physical principles, on more or less necessary practical requirements and on purely empirical original data. Considerable amounts of piano string scale data has been collected and described, for example, in [1–5].

The author's first practical experience in this field was gained through participation in designing and constructing a new model of the mini-sized *Estonia-Minion* piano. Using well-known simple principles, the novel piano string scale was calculated. Briefly, this procedure was presented in [6]. The results of these studies were rather good. The first samples of the new piano were completed by Tallinn Piano Factory by the end of 1995, and these instruments are characterized by a remarkable sound projection.

At the end of nineties, the construction of the medium-sized *Parlour* piano, which has been manufactured by Tallinn Piano Factory for many years, was modernized, and the new string scale of this piano was developed. Some aspects of the scale design of this instrument were analyzed in [7].

Until now the rules for piano scale design have been derived without taking into account the properties or parameters of piano hammers. It is evident that the process of exciting the string by striking with a hammer is a very important component of the sound formation. The sound

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of the piano depends mostly on the detailed motion of strings excited by the impact of the hammers and, therefore, the spectrum of the string vibrations is strongly determined by its elastic features.

The purpose of this paper is to provide a guideline for deployment of a physics-based method for piano scale design. This can be divided into two stages. Firstly, mathematical modelling of the hammer–string interaction allows prediction of the spectrum of the piano string motion. Secondly, this knowledge is used for appropriate altering or "improving" of initial scales to increase the efficiency and quality of the sound production of the instrument.

The first performance criterion which is proposed here for the string scale optimization, or improvement of its efficiency, is the simple and reasonable physical assumption of the similarity of spectra of neighbouring notes over the compass of the piano. The analysis of the numerically simulated force vs. time dependencies of the hammer–string interaction is the second factor of the scale to be examined. This analysis provides the possibility of choosing and optimizing the mass and tension of bass strings in order to avoid negative phenomena such as multiple contacts between the hammer and string.

The numerical simulation of the hammer–string interaction is based on the physical models of a piano hammer described in [8,9]. These models are based on the assumption that the woollen hammer felt is a microstructural material possessing history-dependent properties.

The elastic and hereditary parameters of piano hammers were obtained experimentally using a special piano hammer testing device that was developed and built in the Institute of Cybernetics at Tallinn University of Technology [10,11].

In this paper a number of simplifying assumptions regarding the string and string supports are introduced. Thus, the piano string is assumed to be an ideal flexible string, but the coupling of strings at the end supports is neglected, and the bridge motion is also ignored. Nevertheless, it is hoped that the application of the proposed procedure in piano scale design can improve the tone and the acoustical performance of grand pianos. In what follows, the first stage of procedures describes how the basic parameters are calculated. The second stage, divided into four subsections, describes how these parameters can be enhanced in order to get a better design of piano scale.

2. First stage

2.1. Piano string scale and basic formulae

Usually the total number of notes of a grand piano is equal to 88. The number of strings is much larger, because the number of strings per note changes from one and two strings for the bass notes to three strings for the treble notes.

The relationships connecting the velocity c of a transverse nondispersive wave travelling along the string, the string vibration frequency f, the string length L, the string tension T, and the linear mass density of string μ are the following:

$$c = 2fL, \quad T = \mu c^2. \tag{1}$$

The fundamental frequencies of piano notes are exactly predetermined by equal temperament according to the relationship

$$f_{n+1} = f_n \sqrt[12]{2} = 1.05946 f_n, \quad n = 1 \dots 88. \tag{2}$$

Thus the standard frequency for note A_4 ($n = 49$) is equal to $f_{49} = 440$ Hz, and the note frequencies over the compass of the piano are exactly stated from $f_1 = 27.5$ Hz for the first note A_0 to $f_{88} = 4186$ Hz for the last note C_8.

The distribution of the string tension must be a more or less smooth function across the compass of the piano to provide a uniform loading of the iron frame. The string tension is calculated as

$$T = (2fL)^2\mu = (2f)^2LM = \pi\rho_s(Lfd)^2, \tag{3}$$

where M is the entire string mass, d is the diameter of the steel string core, and ρ_s is the density of the steel core (7860 kg/m^3). The last equality in (3) is valid and used only for plain strings.

Generally, it is considered that for a good instrument the value of Lfd should be a constant. However, it is very difficult to achieve such a distribution of the tension. Frequently, for treble notes the strings are adjusted to have the same tension, while for the bass strings the linear or parabolic law of tension distribution is usual.

Since the stiffness of a string increases sharply with its diameter (proportionally to d^4), the inharmonicity is especially noticeable in the case of the bass strings. As wrapped strings are more flexible than plain strings of the same diameter, the inharmonicity of bass strings is reduced substantially by using wrapped, rather than plain, strings of the same weight.

There are no reliable recommendations available about how to choose the wrapping wires for the bass notes. A simple way is to choose the thinnest core wire to avoid string inharmonicity. This dilemma regarding the determination of wrapping wires was considered in [7]. In the current paper we only assume that, according to relationships (3), the knowledge of such string parameters as length and mass is quite enough to accomplish the design task.

2.2. String and hammer models

In this paper it is assumed that the piano string is an ideal (flexible) string. The displacement $y(x,t)$ of such a string obeys the simple wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}. \tag{4}$$

As in [12], the following system of equations describing the hammer–string interaction is employed

$$\frac{dz}{dt} = -\frac{2T}{cm}g(t) + V, \tag{5}$$

$$\frac{dg}{dt} = \frac{c}{2T}F(t), \tag{6}$$

where $g(t)$ is the outgoing wave created by the hammer strike, $F(t)$ is the acting force; m, $z(t)$, and V are the hammer mass, the hammer displacement, and the hammer velocity, respectively. The hammer felt compression is determined by $u(t) = z(t) - y(0,t)$. Function $y(0,t)$ describes the string deflection at the contact point $x = 0$, and is given by [13]

$$y(0,t) = g(t) + 2\sum_{i=1}^{\infty} g\left(t - \frac{2iL}{c}\right) - \sum_{i=0}^{\infty} g\left[t - \frac{2(i+a)L}{c}\right] - \sum_{i=0}^{\infty} g\left[t - \frac{2(i+b)L}{c}\right]. \tag{7}$$

It is assumed that the string of length L extends from $x = -aL$ on the left to $x = bL = (1-a)L$. Parameter $a = l/L$ is the fractional length of the string to striking point. It determines the actual distance l of the striking point from nearest string end. The initial conditions at the moment when the hammer first contacts the string, are taken as $g(0) = z(0) = 0$, and $dz(0)/dt = V$.

The piano hammers are not only the basic sound-generating elements of an instrument, their properties also are among the most important factors in determining its tone quality. The experimental testing of piano hammers that consist of a wood core covered with several layers of compressed wool felt, demonstrates that all hammers have a hysteretic type of force–compression characteristics. A main feature of hammers is that the slope of the force–compression characteristics is strongly dependent on the rate of loading. This feature directly affects the loudness, the brightness, and the tone quality of the instrument.

It has been shown, that nonlinear hysteretic models can be used to describe the dynamic behavior of the hammer felt [8,9,11]. These models are based on the assumption that the woollen hammer felt is a microstructural material possessing history-dependent properties. Such a physical substance is called either a hereditary material or a material with memory.

According to a four-parameter hereditary model of the hammer presented in [8,11], the nonlinear force $F(t)$ exerted by the hammer is related to the felt compression $u(t)$ by the following expression:

$$F(u(t)) = F_0\left[u^p(t) - \frac{\varepsilon}{\tau}\int_0^t u^p(\xi)\exp\left(\frac{\xi - t}{\tau}\right)d\xi\right]. \tag{8}$$

Here the instantaneous hammer stiffness F_0 and compliance nonlinearity exponent p are the elastic parameters of the felt, and ε and τ are the hereditary parameters.

Another three-parameter hereditary model of the hammer is presented in [9] in the form

$$F(u(t)) = Q_0\left[u^p + \alpha\frac{d(u^p)}{dt}\right]. \tag{9}$$

In this case the parameter Q_0 is the static hammer stiffness; compliance nonlinearity exponent p is also an elastic parameter, and α is the retarded time parameter.

The parameters of the hammers in these models were obtained experimentally by measuring a whole hammer set of recently produced unvoiced Abel hammers. The results of these experiments, and continuous variations in the hammer parameters vs. key number, are presented in [9,11,14]. A best match to the whole set of hammers was approximated using

$$\begin{aligned} F_0 &= 15500\exp(0.059n),\\ Q_0 &= 183\exp(0.045n),\\ p &= 3.7 + 0.015n,\\ \varepsilon &= 0.9894 + 8.8\times10^{-5}n,\\ \tau &= 2.72 - 0.02n + 9\times10^{-5}n^2,\\ \alpha &= 259.5 - 0.58n + 6.6\times10^{-2}n^2 - 1.25\times10^{-3}n^3\\ &\quad + 1.172\times10^{-5}n^4, \end{aligned} \tag{10}$$

for hammer number $1 \leqslant n \leqslant 88$. Here the dimension of parameter τ is (μs), and for α is (ms). The dimension of F_0 and Q_0 is (N/mm^p).

It was shown in [9] that the two models of piano hammer describing by Eqs. (8) and (9) are effectively equivalent for practical application. A three-parameter model is used here for simulation of the hammer–string interaction. This model is chosen due to its simplicity, but also because it is significantly more suitable for the numerical calculations that follow from the large value difference of the time-dimensional parameters τ and α. For this reason we can provide the numerical simulation of the hammer loading described by the three-parameter model with a much larger time sampling rate than would be the case using the four-parameter model.

The hammer masses of this set were approximated by

$$m = 11.074 - 0.074n + 10^{-4}n^2, \quad 1 \leqslant n \leqslant 88. \tag{11}$$

The mass of hammer 1 (A_0) is 11.0 g, and the mass of hammer 88 (C_8) 5.3 g. The spectrum of the string motion excited by the hammer is calculated directly from the force history $F(t)$ [12]. The general expression for the string mode energy level is

$$E_i = 10\log\left[\frac{2M\omega_i^2}{mV^2}(A_i^2 + B_i^2)\right], \tag{12}$$

where

$$A_i = \frac{sin(ai\pi)}{i\pi c\mu}\int_0^{t_0} F(s)cos(\omega_i s)ds,$$

$$B_i = -\frac{sin(ai\pi)}{i\pi c\mu}\int_0^{t_0} F(s)sin(\omega_i s)ds.$$

Here $\omega_i = \pi i c L^{-1} = i\omega_0$ is the string mode angular frequency; t_0 is the contact time.

The presented models of piano string and hammer are used here as a tool for simulation of the hammer–string interaction.

3. Second stage

3.1. Choosing strings' tension in case of the discontinuity of the length of strings

The process of piano string scale optimization is presented here through an example of an improvement of a scale of *Estonia-Minion* piano designed by Tallinn Piano Factory together with the Department of Mechanics and Applied Mathematics at the Institute of Cybernetics at Tallinn University of Technology in 1995.

The *Estonia-Minion* piano is a small instrument. Its total length is only 163 cm (5′4). The maximum lengths of the bass strings and location of the bass bridge are strongly determined by technological conditions. The form and location of the treble bridge is firmly determined by its position near the main diagonal of the soundboard. The upper view of the soundboard of this piano with the piano strings is presented in Fig. 1.

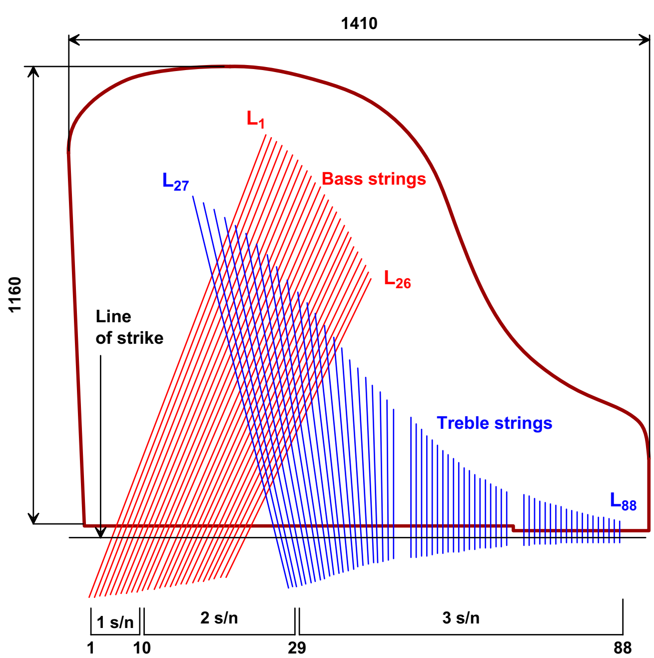

Fig. 1. Position of strings over the soundboard of the mini-sized Estonia-Minion piano.

According to the construction of this instrument, the first 10 notes ($A_0 - F_1^\sharp$) have only one string per note. The notes from 11 to 29 ($G_1 - C_3^\sharp$) have two strings per note, and the other notes consist of three strings. Those strings that structure the first twenty six notes ($A_0 - A_2^\sharp$) terminate on the bass bridge, and the other strings terminate on the treble bridge.

As the value of frequency for each note is known, and the lengths of the strings are also prescribed, in order to complete the piano scaling the values of appropriate linear mass densities and tensions of the strings must be calculated.

For example, in Table 1 a part of the new improved piano scale is presented for strings whose parameters are then used in the numerical modelling of the hammer–string interaction.

Here d_1 is the diameter of the winding wire, and s/n is the number of strings per note. The lengths of the strings were taken from [6]. The first string is a double-wrapped string, and for this string d_1 is the sum of the diameters of both copper wires.

It is well-known that, due to the technical requirements and the recommendations of experienced piano makers, the tension must be approximately constant across the treble strings. In our piano the value of string tension for notes with $n \geqslant 30$ was chosen to be approximately 675 N per string. However, in practice, it is very difficult to achieve such a distribution of the tension exactly, because the step of the wire diameter is equal to 0.025 mm for the steel core, and 0.05 mm for the winding wire. Nevertheless, by choosing the wire diameters carefully, an almost constant string tension for treble notes with $n \geqslant 30$ was obtained in [6]. For this reason, and due to the fact that with increasing key (note) number n the diameters of these strings decrease smoothly and continuously, the oscillation spectra of neighbouring strings are alike and very similar. This spectral similarity characterizes also the high sound quality of the piano.

However, there is a specific point of the piano scale, where the smooth continuity of all functions is broken. This is due to the transition of strings from the bass bridge to treble bridge. In this case, the two strings of the note $A_2^\sharp$

Table 1
Parameters of strings

Note	n	f (Hz)	L (mm)	l (mm)	T (N)	μ (g/m)	d (mm)	d_1 (mm)	s/n
A_0	1	27.5	1239.2	150.2	1350	290.6	1.500	3.70	1
$A_2^\sharp$	26	116.5	831.2	99.2	625	16.7	0.950	0.40	2
B_2	27	123.5	1031.0	123.0	793	12.2	0.975	0.25	2
C_3	28	130.8	1007.4	120.1	747	10.8	0.975	0.20	2
$C_3^\sharp$	29	138.6	985.3	117.4	773	10.4	0.950	0.20	2
D_3	30	146.8	964.1	114.8	626	7.8	1.125		3
A_4	49	440.0	399.3	47.0	687	5.6	0.950		3
A_5	61	880.0	208.3	21.9	672	5.0	0.900		3

($n = 26$), which terminate on the bass bridge are much shorter than the strings of note B_2 ($n = 27$), which terminate on the treble bridge (see Fig. 1) With increasing note number n the continuity of the lengths of strings terminated on the treble bridge then resumes. Therefore, in order to avoid a rough discontinuity in the sound spectra of neighbouring notes where strings are terminated on the different bridges, the mass and the tension of the string $n = 26$ must be chosen very accurately. The procedure for determining the masses and tensions of the strings in this domain is described below.

Again, the aim is to obtain similarity in the spectra of neighbouring notes. However, the data presented in Table 1 demonstrates the appearance of two additional problems. The first problem is the discontinuity in the number of strings per note. This problem was considered in [7], and the procedure of relative string tension jump minimization in cases when the number of strings per note varies from one to two and from two to three was discussed.

The second problem arises from the need to use wrapped strings for notes $n \leqslant 29$. It is connected by the fact that with decreasing of key (note) number n the linear mass density of the strings should be increased in the direction of the bass notes. It is obtained by the increases of 0.025–0.05 mm in string diameter. The maximum practical diameter for plain strings is limited by inharmonicity, and the largest plain string normally used is about 1.125 mm. Further increases in the linear mass density of the strings are provided by using wrapped strings, which, as was mentioned above, are more flexible than plain strings of the same diameter. Thus, beginning from the string $n = 29$, and up to the first string $n = 1$, the strings must be wrapped. The linear mass density of these strings must rise smoothly from a value of $\mu = 7.81$ g/m (the linear mass density of the string $n = 30$) up to $\mu = 290$ g/m (the linear mass density of the first string).

Unfortunately, due to technological demands, the diameter d_1 of the winding wire must be greater than 0.2 mm. As a results the combination of diameters of core and winding wires shown in Table 1 was used. This combination provides the smoothest and most continuous dependence of the linear mass density on the string number n, for notes with $n = 30$, 29, 28, and 27. The linear mass density of these wrapped strings is calculated according to formulae presented in [7], and is also proved by experimental testing. Since the linear mass density of the strings $n = 29$, 28, and 27 is determined, the tension of these strings is defined by formulae (3), and displayed in Table 1. The result of the string parameters determination is presented in Fig. 2a.

The spectral envelopes are simulated using the basic formulae presented above. The three-parameter hammer model (Eq. (9)) is explored, and the values of hammer parameters are computed by formulae (10). The results were obtained by solving the system of Eqs. (5) and (6) for initial hammer velocity $V = 2$ m/s. The string oscillation spectra were calculated in according to formula (12) for one string per note. For this purpose the acting mass of a hammer is defined as being the total hammer mass defined by expression (11) divided by the number of strings per note s/n. Visual inspection of the simulated curves shows that it was possible to achieve a good resemblance between all four spectra envelopes simultaneously.

The next and more complex step is to determine the tension of string $n = 26$ that is terminated on the bass bridge, and which is much shorter than the neighbouring string $n = 27$. A tension must be found which ensures that the spectrum of its vibrations is similar to the spectrum of string $n = 27$. The result of varying the string tension in the numerical simulation of the string $n = 26$ is presented in Fig. 2b. Visual inspection of the spectra envelopes shows that the curve marked by circles ($T = 625$ N) exhibits the closest agreement with the curve marked by diamonds, which corresponds to the spectrum of vibrations of the string $n = 27$. Thus, it seems, the best choice of tension for the string $n = 26$ is approximately $T = 625$ N. Once the tension has been determined, the linear mass density

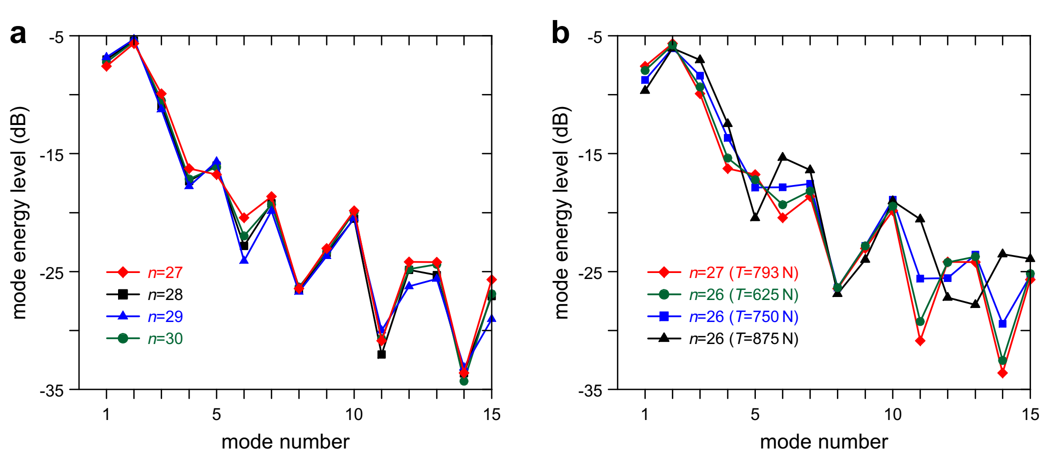

Fig. 2. Comparison of spectral envelopes. (a) All strings are terminated on treble bridge and (b) for string $n = 27$ terminated on treble bridge, and for various tension T of string $n = 26$ terminated on bass bridge.

of the string, and the suitable diameters of core and winding wires can be found. The new optimized or improved parameters of the string $n = 26$ are displayed in Table 1.

3.2. Influence of hammer parameters

In the previous section, the optimal string tension was determined by numerical simulation of the hammer–string interaction. This requires that the elastic properties of the hammer be recognized and included. The effect of the hammer parameters on the sound produced by a piano is evident. In the view of the authors, the design of each piano scale should take into account all parameters of the hammers (which may be different for each musical instrument).

The process of string excitation through striking with a nonlinear nonhysteretic hammer was considered in [12]. The change in the hammer–string interaction as the elastic hammer parameters are varied was demonstrated. The force histories and sample spectra were calculated for various values of the hammer compliance and of the stiffness nonlinearity exponent. Another example of the string excitation by the hammer, which is described by a four-parameter hereditary model, was presented in [13].

In Fig. 3 we demonstrate the influence of the hereditary hammer parameter α on the spectrum of the string vibrations excited by a three-parameter hysteretic hammer.

Using the hammer model in the form (9), force–time curves and sample spectra have been calculated for the string $n = 49$. The parameters of this string are displayed in Table 1. The initial hammer velocity is $V = 1.8$ m/s. Fig. 3 illustrates the option of fixing the other parameters and varying the retarded time parameter α. A value of $\alpha = 310$ µs corresponds to the normal value of retarded time for this hammer. In case of $\alpha = 0$ the hammer loses its hysteretic features. Increasing α means a steeper and faster rise of the force when the process of the hammer compression begins, and a correspondingly faster decay later.

3.3. First bass string case

The first bass string of the grand piano is the longest string of the instrument. In the case of the *Estonia-Minion* piano, its length is 1239.2 mm, and the note frequency is equal to 27.5 Hz. All bass strings terminate on a bass bridge and, as mentioned previously, the string tension distribution must be an almost linear function of key number n to provide a uniform loading of the cast-iron frame. An appropriate tension of 625 N for the last bass string $n = 26$ was determined in Section 3.1. The question of how to choose or determine the tension of the first bass string will now be investigated.

This problem can also be solved by the numerical simulation of the string excitation by the hammer. In Fig. 4a the force–time dependencies calculated for various string tensions are presented. The initial hammer velocity was chosen to be the rather high value of 5 m/s. It is well-known that for a hard blow the hammer leaves the string just before the first reflection returns from the agraffe (the nearest string termination). Indeed, the reflection pulse may catch up with the hammer and make a renewed contact. This undesirable phenomenon is referred to as multiple contacts. To avoid this unwanted event a value for the string tension must be chosen so as to minimize the influence of the reflected pulse.

The results presented in Fig. 4 demonstrate that with increasing string tension, the second pulse appearance is reduced. Unfortunately, it is very difficult to increase the string tension above about 1350 N, because the relatively short string will be very massive, and thus also too thick. Due to technological demands the maximal diameter of the wrapped string can be 9 mm. Therefore, the string is chosen to be as thick as possible. As a result, for the value of tension $T = 1350$ N selected for this string, the second pulse magnitude is extremely small for all the values of hammer velocity. Having defined a tension for the first and the last strings, the linear law of tension distribution

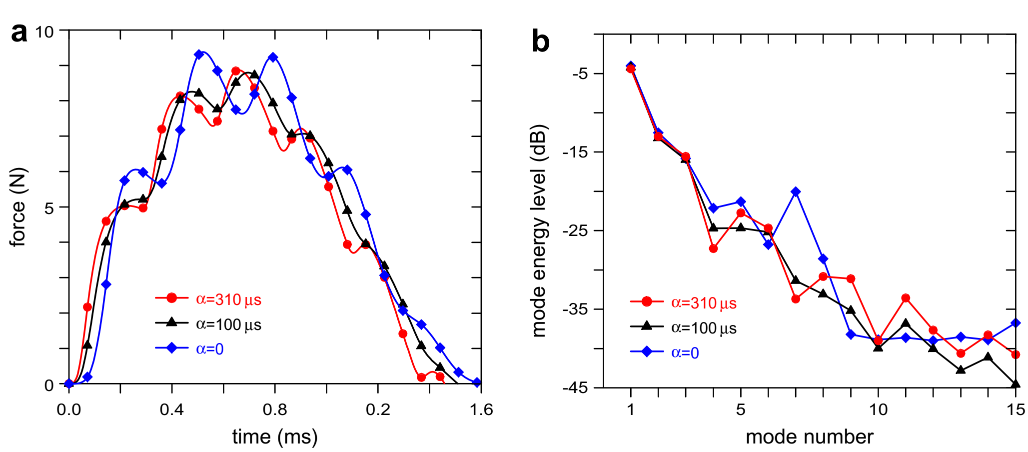

Fig. 3. Force histories (a) and spectral envelopes (b) for string $n = 49$. Varying retarded time parameter α with fixed compliance nonlinearity exponent $p = 4.43$, and static hammer stiffness $Q_0 = 1660$ N/mm^p.

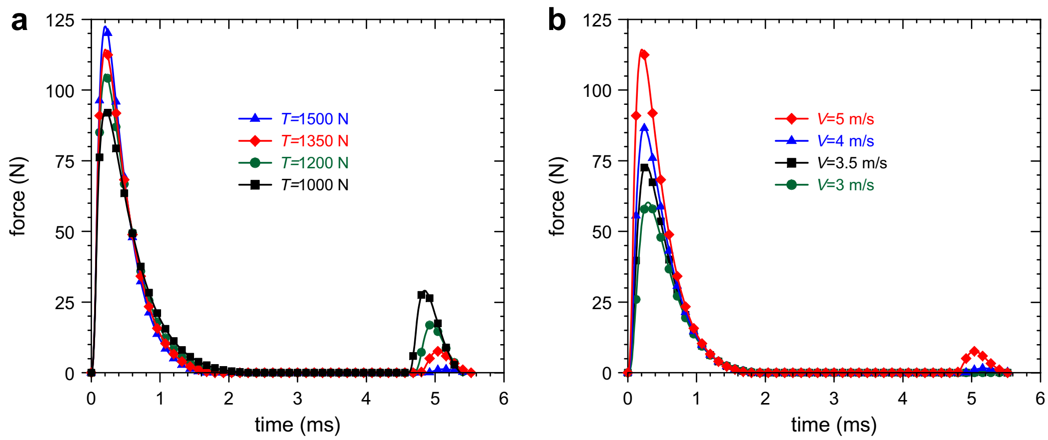

Fig. 4. Force histories computed for first bass string $n = 1$: (a) constant hammer velocity $V = 5$ m/s and (b) constant string tension $T = 1350$ N.

can be applied for all bass strings and, according to relationships (1) and (3), the masses of strings are then also defined. The parameters of this string are displayed also in Table 1.

3.4. Choosing the striking point

As the string vibration spectrum is very sensitive to the position of the fractional striking point a, especially for the treble strings, the numerical simulation of the hammer–string interaction enables the piano string scale to be chosen with a more uniform spectrum.

For approximately 60 upper notes of the grand piano, the position of the striking point determined by parameter a gradually becomes displaced from 1/8 to 1/24 of the whole string length in the direction of the high notes. In Fig. 5, spectra are displayed for note A_5 ($n = 61$) calculated for initial hammer velocity $V = 1.5$ m/s.

The recommended (used in practice) value of the fractional striking point for this note is close to 1/8.8 ($a = 0.114$). Visual inspection of Fig. 5 shows that the odd harmonics in the spectrum corresponding to the recommended striking point are damped significantly. It is

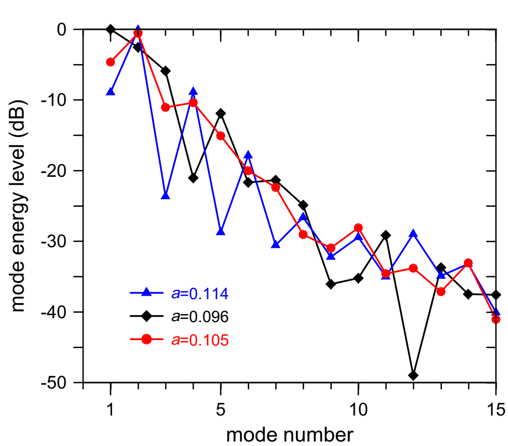

Fig. 5. Spectra for note A_5 ($n = 61$) calculated for different striking point.

likely that such a spectrum was the aim of the designers of a grand piano. However, if a more uniform spectrum is required, another point of hammer impact can be selected. If the criterion of optimization is the homogeneity of the spectrum, then the value of fractional striking point $a = 0.105$ is a better choice.

4. Summary

The influence of the hammer parameters on the sound produced by a piano is obvious. For this reason, the design of the piano scale should take into account all parameters of the hammers that may be used by piano manufacturers. Although, the hammers produced by various firms are significantly different, it is assumed that elastic parameters of hammers for the whole hammer set are experimentally measured beforehand and known. The knowledge of piano hammer characteristics over the compass of the piano enables investigation of the vibrations of all piano strings excited by the impact of the hammer.

Simple and efficient procedures, based on the numerical simulation of the hammer–string interaction have been described and applied for the piano string scale optimization. The presented method is only a first step in the direction of using of mathematical models in the design of new models of grand pianos. A number of simplifying assumptions concerning the string and string supports are introduced. Thus, the piano string is assumed to be an ideal flexible string, but the coupling of strings at the end supports is neglected, and the bridge motion is also ignored. The mobility of strings support is not considered due to of absence of practical data about the piano bridge impedance. Besides, the bridge stiffness varies significantly for different types of instruments and, moreover, it is very difficult to provide experimental testing of the string support.

Further experimental investigations of piano bridges of the real instruments should give a possibility for creation of more realistic models of the string vibrations in order to get results that come closer to measurements in real pianos. Nevertheless, the simplifying assumptions made in this paper are not regarded as a serious weakness of the procedure presented. The new optimized piano scale has been realized at Tallinn Piano Factory, and by experts estimation the sound quality of new pianos is rather high.

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