Pitch Glide Effect Induced by a Nonlinear String–Barrier Interaction

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Abstract. Interactions of a vibrating string with its supports and other spatially distributed barriers play a significant role in the physics of many stringed musical instruments. It is well known that the tone of the string vibrations is determined by the string supports, and that the boundary conditions of the string termination may cause a short-lasting initial fundamental frequency shifting. Generally, this phenomenon is associated with the nonlinear modulation of the stiff string tension. The aim of this paper is to study the initial frequency glide phenomenon that is induced only by the string–barrier interaction, apart from other possible physical causes, and without the interfering effects of dissipation and dispersion. From a numerical simulation perspective, this highly nonlinear problem may present various difficulties, not the least of which is the risk of numerical instability. We propose a numerically stable and a purely kinematic model of the string–barrier interaction, which is based on the travelling wave solution of the ideal string vibration. The model is capable of reproducing the motion of the vibrating string exhibiting the initial fundamental frequency glide, which is caused solely by the complex nonlinear interaction of the string with its termination. The results presented in this paper can expand our knowledge and understanding of the timbre evolution and the physical principles of sound generation of numerous stringed instruments, such as lutes called the tambura, sitar and biwa.

INTRODUCTION

It is well known that the fundamental frequency of a vibrating string is determined by the type of the string termination. Pitch glide is a short-lasting phenomenon where at the beginning of the string vibration the fundamental frequency is slightly larger then the nominal (desired) value. This effect can be clearly audible in some instruments, such as in the Finnish kantele [1]. Generally, this phenomenon is associated with and discussed in context with the nonlinear modulation of stiff string tension [2, 3]. This paper investigates the pitch glide that is solely generated by the nonlinear string–barrier interaction, apart from other possible physical causes, and without the interfering effects of dissipation and dispersion.

The problem at hand is twofold. First, one needs to solve the highly nonlinear string–barrier collision problem. Second, one needs to accurately estimate the short-lasting and relatively weak effect of the pitch glide. Much effort has been devoted to modelling the collision dynamics of a vibrating string with a distributed unilateral constraint during the past decades. Over the years many authors have solved this problem using different approaches. The problem was considered by Schatzman [4], Burridge *et al.* [5], and Cabannes [6], who used the method of characteristics and assumed that the string does not lose energy when it hits an obstacle. Krishnaswamy and Smith [7], Han and Grosenbaugh [8], Bilbao and Torin [9, 10], Chatziioannou and Walstijn [11], and Taguti [12] used a finite difference method to study the string interaction with the barrier. Vyasarayani, *et al.* [13] described the movement of the sitar string with a set of partial differential equations. Rank and Kubin [14], Evangelista and Eckerholm [15], and Siddiq [16] used a waveguide modelling approach to study the plucked string vibration with nonlinear limitation effects.

However, not much attention has been dedicated to the study of initial pitch glides that are induced by the same barriers studied in the above papers. Although, few reports of the numerical model's ability to reproduce the effect have been previously published [9, 17]. The aim of the current work is to briefly analyse these results.

The organization of the paper is as follows. In Sec. ”String–barrier collision model”, problem description and numerical model are presented. In Sec. ”Estimation of fundamental frequency”, a novel method for estimating the pitch glide is presented and briefly explained. Sections ”Results” and ”Conclusions” present the main results and conclusions.

STRING–BARRIER COLLISION MODEL

The string–barrier interaction problem is solved using a method based on the d’Alembert’s travelling wave solution, making it numerically stable and accurate. The numerical model is best explained in [17, 18]. Problem is studied in a single transverse polarization. Figure 1 a shows the schematic drawing of the problem at hand.

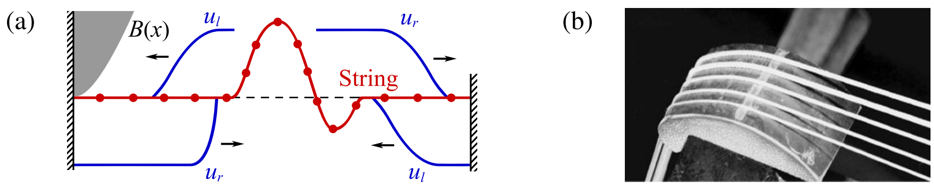

FIGURE 1. (a) The scheme of the problem. The travelling waves u_r and u_l, shown for two different time moments (unmarked solid lines). The form of the string (solid lines marked with circles). The position of the barrier is shown by the grey formation. The profile of the barrier is described by the function $B(x)$. (b) *Sawari* located at the nut of the *Chikuzen biwa* neck.

In order to explore only the effect of the influence of the barrier on the string motion, and thus on the produced pitch glide, we eliminate the possible contribution that the lossy and dispersive wave propagation may introduce to this problem. The wave equation for the lossless *ideal* string is in the form

$$\partial_t^2 u = c^2 \partial_x^2 u, \tag{1}$$

where $u(x,t)$ is the string displacement, $c = \sqrt{T/\mu}$ is the speed of the travelling waves, T is the tension, and μ is the linear mass density of the string.

We normalise Eq. (1) by choosing $c = 1$ and by introducing the following dimensionless variables:

$$t \Rightarrow t/P_0, \quad x \Rightarrow x/\lambda, \quad u \Rightarrow u/\lambda, \tag{2}$$

where P_0 is the fundamental period and λ is the corresponding wavelength. The speaking length L of the string is chosen to be a half of the wavelength, i.e., $L = 0.5$. This ensures that the fundamental frequency $f = 1$, since for Eq. (1) it holds that $f = c/(2L) = c/\lambda$.

The barrier is considered to be absolutely rigid. Ensuring the conservation of energy of the system. Additionally, it is selected to have a parabolic cross-section profile. Parabola is described by the function $B(x) = (2R)^{-1}x^2$, where R is the radius of the barrier’s curvature at $x = 0$. The selection of a parabolic barrier is inspired by the nut of the *Chikuzen biwa* called *sawari*, shown in Fig. 1 b. Position of the barrier relative to the string is shown in Fig. 1 a.

The string plucking condition is introduced as follows. We assume that at the point $x = l = 0.8L$ the emerging travelling wave $\hat{u}$ is of the form

$$\hat{u}(t) = \begin{cases} a\,(t/t_*)^2 \exp 2\,(1 - t/t_*)\,, & \text{for } 0 \leqslant t \leqslant t_*, \\ a, & \text{for } t_* < t < \infty, \end{cases} \tag{3}$$

where a is the amplitude parameter, t_* is the duration of the string excitation. In our case $t_* = 2$, i.e., the excitation prolongs for two periods P. It can be shown that plucking condition (3) corresponds to a bell-shaped initial force *cf.* [18]. The string displacement time series $u(l,t)$ that is used below is ”recorded” at the plucking coordinate $x = l$.

ESTIMATION OF FUNDAMENTAL FREQUENCY

The pitch glide is numerically estimated using a method introduced by Peterson *et al.* in [19], where it is also shown that this approach is more accurate and faster compared to other methods that are based on the autocorrelation function (ACF) or the fast Fourier transform algorithm (FFT). Following is a short overview of the method.

The fundamental period P_0 is assumed to correspond to the shift that leads to the smallest difference, i.e., largest similarity, of the examined time series and its shifted copy. We define a functional that measures similarity of a given function and its shifted counterpart. This functional is used as an objective function to a minimization problem which solution corresponds to the fundamental period of the given function [19].

A similarity measure between the function $u(t) = u(l, t)$ and its shifted counterpart $u(t + \tau)$ is selected as follows:

$$U[u](\tau) = \int [u(t+\tau) - u(t)]^2\, dt = \int u^2(t+\tau)dt + \int u^2(t)dt - 2 \cdot \mathrm{ACF}[u](\tau), \tag{4}$$

where $\mathrm{ACF}[u](\tau) = \int u(t)u(t+\tau)dt$ is ACF of $u(t)$, and $u^2(\xi) \equiv [u(\xi)]^2$. In (4) the integration interval depends on the type of functions: for periodic functions the integration is carried out over an interval with the length of a period and for functions on a finite interval over a subinterval where $u(t + \tau)$ and $u(t)$ are defined simultaneously. The functional U is an unbiased measure both for periodic and for functions on finite interval. Clearly, the measure U has a local minimum at the fundamental period and we can use it as objective function for finding the fundamental period estimate of a quasiperiodic function $u = u(t)$: $P_{0,\mathrm{est}}[u] = \mathrm{argmin}_{\tau>0} U[u](\tau)$, where $P_0 = \mathrm{argmin}_{\tau>0} U(\tau)$ means that P_0 is a positive minimum point of a function $U(\tau)$ [19]. In current work, the time series $u(t)$, to be evaluated, is broken up into short windows (frames) of finite size, which overlap each other. The fundamental period P_0 is estimated for each of them. Fundamental frequency f is found as $f = 1/P_0$.

In general, the computational complexity of finding the fundamental period of a signal is $O[(n - \lfloor P_0 \rfloor)\lfloor P_0 \rfloor]$, where n and P_0 are the length and the fundamental period of the signal, respectively. Compare this to the computational complexity $O(n \log n)$ of the FFT [19]. A C library `libfperiod` and Python package `iocbio.fperiod` based on the aforementioned similarity measure for estimating the fundamental period are made publicly available[1].

RESULTS

Figures 2 a and 2 b show the results. The results are calculated for window size $t = 4P$ and for overlap value of $3P$. The estimation of fundamental frequency starts after the force stops acting on a string, i.e., for $t > t_* = 2P$.

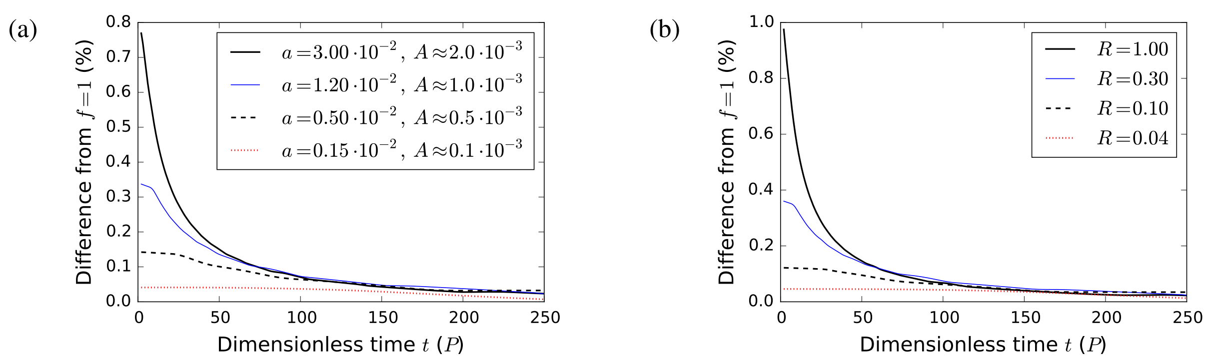

FIGURE 2. (a) Pitch as a function of dimensionless time, shown for four amplitude parameter a values, and for barrier radius $R = 0.7$. (b) Pitch as a function of dimensionless time, shown for four values of barrier radius R, and for amplitude parameter $a = 0.03$ ($A \approx 2 \cdot 10^{-3}$).

Figure 2 a shows the barrier induced pitch glide for four values of the amplitude parameters a and for the resulting initial string displacement amplitudes A. A typical pitch gliding behaviour is recognizable, where with the passage of time the pitch value asymptotically approaches the nominal frequency $f = 1$. One can see that for greater amplitude the pitch difference from the nominal value is greater, especially at the beginning of the string vibration. More intensive pitch glide happens during the first 120 periods, approximately. After 120 periods have elapsed, the pitch differs from the nominal value by less than 0.1%.

Figure 2 b shows the pitch glide for four values of the parabolic barrier's apex radius R. A qualitatively similar result is seen here, with exception of shorter period of the intense pitch gliding, which prolongs for 100 periods,

[1] `https://code.google.com/p/iocbio/`

approximately. The effect is greater for barriers with greater value of radius R. In both presented cases, a closer inspection of the results reveals that the function describing the pitch glide varies in a highly non-trivial manner – indeed, it can not be described by a simple smooth exponentially decaying function. Additionally, the effect of the plucking coordinate x on the pitch was investigated. No significant effect was found.

The simplest physical time domain explanation for the presented results is the following. The pitch is higher at the beginning of the vibration, because of the *effective* shortening of the speaking length of the string due to the spatial extent of the barrier, and the interaction of the string with the barrier. The section of the string that ”binds” or ”clings” to the barrier, as it collides with it, is temporarily forced not to participate in the vibration process. This makes the actively moving part of the string slightly shorter, for a small fraction of the duration of the period, and thus raising the value of the fundamental frequency slightly higher.

CONCLUSIONS

The physical effect of the string–barrier interaction on the initial pitch glide, apart from the interfering effects of dissipation and dispersion, was investigated by means of a numerical method based on the d'Alembert solution to the wave equation [17, 18]. It was found that, indeed, the rigid barrier induces a short-lasting initial fundamental frequency glide especially for bulky barriers and larger string vibration amplitudes. The presented results allow to estimate the exact proportion of the pitch glide phenomenon that is caused solely by the string–barrier interaction and the barrier geometry, apart from other factors, such as stiffness of the string.

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REFERENCES

[1] H. Järveläinen and V. Välimäki, "Audibility of initial pitch glides in string instrument sounds," in *Proc. Int. Computer Music Conf. 2001*, pp. 282-285.
[2] K. A. Legge and N. H. Fletcher, J. Acoust. Soc. Am. 76(1), 5–12 (1985)
[3] T. Tolonen, V. Välimäki and M. Karjalainen, IEEE Trans. on Speech and Tech. 8(3), 300–310 (2000)
[4] M. Schatzman, Journal of Mathematical Analysis and Applications 73(1), 138–191 (1980)
[5] R. Burridge, J. Kappraff and C. Morshedi, SIAM Journal on Applied Mathematics 42(6), 1231–1251 (1982)
[6] H. Cabannes, Applied Mathematics Letters 10(5), 79–84 (1997)
[7] A. Krishnaswamy and J. O. Smith, "Methods for simulating string collisions with rigid spatial obstacles," in *Proc. IEEE Workshop on Applications of Signal Processing to Audio and Acoustics 2003*, pp. 233–236.
[8] S. M. Han and M. A. Grosenbaugh, Journal of Sound and Vibration 237, 337–361 (2004)
[9] S. Bilbao, "Numerical modeling of string/barrier collision," in *Proc. ISMA 2014*, pp. 267–272.
[10] S. Bilbao and A. Torin, Journal of the Audio Engineering Society, 63(5), 336–347 (2015)
[11] V. Chatziioannou and M. V. Walstijn, Journal of Sound and Vibration 339, 262–279 (2015)
[12] T. Taguti, Acoustical Science and Technology 29(3), 203–214 (2008)
[13] C. P. Vyasarayani, S. Birkett and J. McPhee, J. Acoust. Soc. Am. 125(6), 3673–3682 (2009)
[14] E. Rank and G. Kubin, "A waveguide model for slapbass synthesis," in *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing 1997*, pp. 443–446.
[15] G. Evangelista and F. Eckerholm, IEEE Trans. on Audio, Speech, and Language Proc. 18(4), 822–832 (2010)
[16] S. Siddiq, Archives of Acoustics 37(1), 73–79 (2012)
[17] D. Kartofelev, A. Stulov, H.-M. Lehtonen and V. Välimäki, "Modeling a vibrating string terminated against a bridge with arbitrary geometry," in *Proc. Stockholm Music Acoustics Conf. 2013*, pp. 626–632.
[18] A. Stulov and D. Kartofelev, Applied Acoustics 76, 223–229 (2014)
[19] P. Peterson, M. Kalda and M. Vendelin, Am. J. of Physiology – Cell Physiology 304(6), C519–C531 (2013)