PIANO HAMMER: THEORY AND EXPERIMENT

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Abstract

The main introduction to the theory of piano hammer is provided. It is based on the hysteretic (hereditary) model of piano hammer and upon the huge number of experimental data. The experimental studies have been made using a piano hammer testing device. The several sets of piano hammers produced by various firms were tested. It is experimentally proved that the piano hammer possess history-dependent properties, or just as well is made of a material with memory. The good agreement of the experiment with the theory is verified. The hereditary and elastic parameters of various hammers were obtained by numerical simulation of experimental data. It is established that the hammer parameters strongly depend on the diameter of the striking string. The quantitative estimation of the air humidity influence on the stability of the hammer parameters is given. The nice distinction between the different piano hammers produced by various firms is found out. The simulated flexible string vibration spectra excited by different piano hammers are presented. The comparison of hammers in a frequency-domain is carried out also.

INTRODUCTION

The theoretical ground of the piano hammer study is the hereditary model of the hammer derived in [1] in the form

$$F(u(t)) = F_0\left[u^p(t) - \frac{\varepsilon}{\tau}\int_0^t u^p(\xi)\exp\left(\frac{\xi - t}{\tau}\right)d\xi\right]. \tag{1}$$

According to this model, the real piano hammer possesses history-dependent properties, or in other words, is made of a material with memory. Here $F(u)$ is the force exerted by hammer and u is the hammer compression. The hammer stiffness F_0 and compliance nonlinearity exponent p are the elastic parameters of hammer, and constants ε and τ are the hereditary parameters. This analytical model of the nonlinear hereditary hammer takes into account all the important features of the hammer-string interaction.

It has been shown [2] that physical assumptions about the history-dependent properties of the hammer are confirmed by the experiment. The several complete sets of piano hammers produced by Schneider, Renner, Abel etc. were tested. Thus it is experimentally proved that the hereditary model of hammer in the form (1) clarifies the dynamical features of piano hammer rather well.

The experimental studies have been made using a piano hammer testing device. This device makes it possible to obtain the dynamical force-compression characteristics of the hammer, and it description is given in [2].

The hereditary and elastic parameters of various hammers were obtained by numerical simulation of experimental data. This procedure is described in [1,2], and the method of calculation of string vibration spectra excited by a piano hammer is presented in [3].

The main problems considered here are:

- determination of the hammer parameters,
- comparison of the different piano hammers produced by various firms,
- influence of the air humidity on the stability of the hammer parameters.

PIANO HAMMER TESTING DEVICE

The experimental arrangement shown in Fig. 1 gives a possibility to obtain the dynamical force-compression characteristics of piano hammer. In the measurements the hammer struck a piece of the string, fixed on the force sensor. To avoid the influence of the shank deformation, it is made of a rigid titanium tube. A piezoelectric wide-band ceramic plate is used as a force sensor. To measure the hammer compression, an infrared optical system has been developed. The analogue force-time and compression-time signals from these two sensors are converted into two sets of data by a digital signal processor. The device is controlled by a personal computer via a RS232 cable.

The calibration of the optical sensor was provided by registration of the output signal when the shank falls freely down. The diameter of the working aperture or the maximum value of the hammer compression was found by using the micrometer. It is equal to 2.0±0.02 mm. While falling down for such a small distance, the force of gravity changes the shank velocity just a little. Therefore, we may assume that during the measurement the shank velocity is a constant value. In that way the displacement of the flag through the optical aperture is proportional to time, and we can find the relationship between the output signal and the flag displacement, and thus the hammer compression. The real accuracy of the hammer compression measurement is less than 0.02 mm.

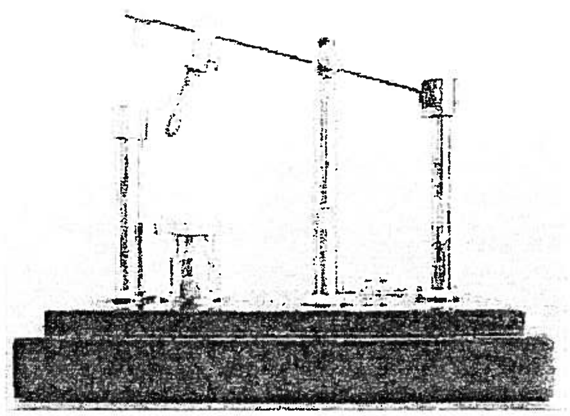

Fig 1. Piano hammer testing device.

The force sensor was calibrated by striking of its flat surface by a rigid hammer specially made of wood. The integration over the time of the registered electric signal gives the value of the hammer pulse. The mass and the velocity of the hammer are known values, therefore the value of the acting pulse is also known. The equality of these pulses gives the value of the force calibration coefficient, which is approximately equal to K_0 = 11 N/V with accuracy about 10%. However the further measurements indicate that the value of the force calibration coefficient is not a constant and may vary under the measurement conditions. This problem will let discussed below.

PIANO HAMMER PARAMETERS

What values are the hammer parameters? This question is not so simple and evident. It is obvious that such parameters as the compliance nonlinearity exponent p, the hereditary amplitude ε and the relaxation time τ may be presented as the hammer parameters. But the hammer stiffness F_0 that dimensionality is N/mm^p cannot be a parameter, because its value depends on another parameter p.

The hereditary model of the hammer in the form (1) is much more natural than other models of hammer. It includes all the values needed to obtain the dimensionless description of the model. Instead of the dimensional values it is better to use the non-dimensional time and dimensionless values of the hammer compression and hammer stiffness f_0

$$t \Rightarrow \frac{t}{\tau}, \quad u \Rightarrow \frac{u}{\tau V}, \quad f_0 = \frac{\tau(\tau V)^p}{mV} F_0 , \tag{2}$$

where m is the mass and V is the hammer velocity. In dimensionless form the mathematical model of the hereditary hammer-string interaction can be described by the equation

$$\frac{d^2u}{dt^2} + f_0 \left[u^p(t) - \varepsilon \int_0^t u^p(\xi) \exp\left(\xi - t\right) d\xi \right] = 0 , \tag{3}$$

with the initial conditions

$$u(0) = 0, \quad \frac{du}{dt}(0) = 1. \tag{4}$$

Here in Eqs. (3) and (4) we have dimensionless values of the hammer compression u and time t, and the "excellent" initial conditions. Therefore, f_0 is the real hammer parameter as well as the compliance nonlinearity exponent p and hereditary amplitude ε. The another parameters V and τ are the latent parameters.

The main merit of the hereditary hammer model is not only the possibility to describe the hysteretic behaviour of the dynamical force-compression characteristics of the hammer, but the dependence of its slope on the hammer velocity also. In Fig. 2 are shown the force-compression characteristics of Renner's hammer for the various hammer velocity. This is clear from Fig. 2, with a diminishing of the hammer speed, the slope of the loading part of the curves is decreased. The values of the hammer parameters obtained by numerical simulation of these curves are displayed in Table 1.

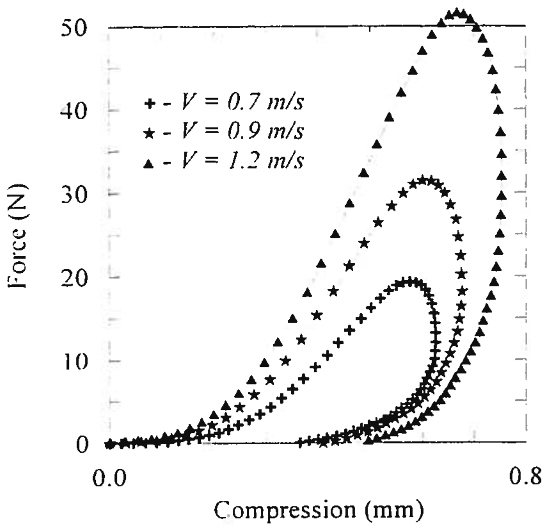

Fig 2. Force-compression characteristics for the various initial hammer velocities.

Table 1. Numerically determined parameters of the Renner's hammer (n=5).

Parameter	Hammer velocity (m/s)		
	V=0.7	V=0.9	V=1.2
F_0 (N/mm^p)	15600	19600	21000
p	3.7	3.7	3.7
ε	0.996	0.996	0.996
τ (μs)	2.55	2.10	1.73
f_0	2.0E-10	2.0E-10	2.0E-10

The analysis of parameters presented in Table 1 show that for the different hammer velocities the hammer stiffness F_0 is not a constant value, but nevertheless the non-dimensional stiffness f_0 is the real constant parameter of the hammer indeed.

STRING DIAMETER INFLUENCE

The next set of measurements was performed to clarify whether the hammer parameters depend on the diameter of the striking string. For this purpose the another Renner's hammer (n=4) struck different strings and the flat surface of the force sensor with a constant velocity V=1.24 m/s.

In case when the hammer strikes not a flat surface, but the string, it is obvious that the conditions of the strike are quite not the same. The thin string penetrates into the hammer more easily, and the form of the curve of the force-compression characteristics must depend on the diameter of the striking string.

During the measurements unexpectedly one serious problem was detected. It was found out that the sensitivity of the force sensor strongly depends on the conditions of the strike. The piece of string placed on the force sensor essentially reduces its sensitivity, and therefore the value of the force calibration coefficient K_0 is not a constant.

This problem was solved using the hereditary model of the hammer by numerical simulation of the experiment. It turned out, that it is possible to obtain not only the force-compression characteristics of the hammer which are presented in Fig. 3, but all the hammer parameters and the values of the force calibration coefficient K_0 also. These data are displayed in Table 2.

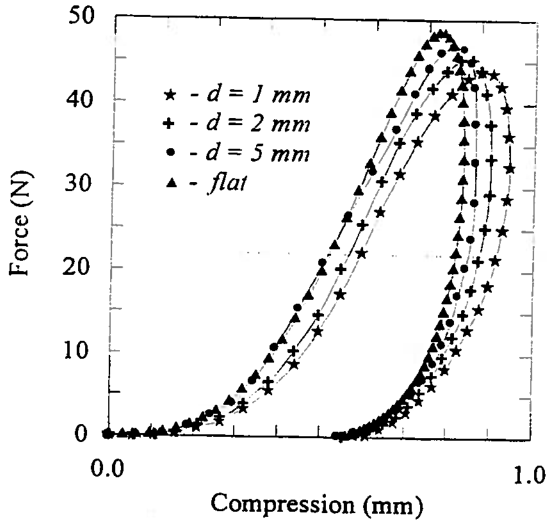

Fig 3. Force-compression characteristics for the various string diameters.

Table 2. Numerically determined parameters of the Renner's hammer (n=4).

Parameter	String diameter (mm)			
	flat	d=5	d=2	d=1
F_0 (N/mm^p)	13250	11200	9700	8200
p	3.69	3.69	3.69	3.69
ε	0.995	0.995	0.995	0.995
τ (μs)	1.70	1.70	1.70	1.70
f_0	1.2E-10	1.0E-11	9.0E-11	7.6E-11
K_0 (N/V)	11.11	15.67	13.58	11.38

Thus we can assert that the value of the hammer stiffness significantly depends on the diameter of the striking string.

PIANO HAMMER FEATURES

For the various hammers comparison were chosen the hammers (n=21) produced by Abel (Ab), Immadigawa (Im), and two Renner's hammers: old type (Or) and new type (Nr).

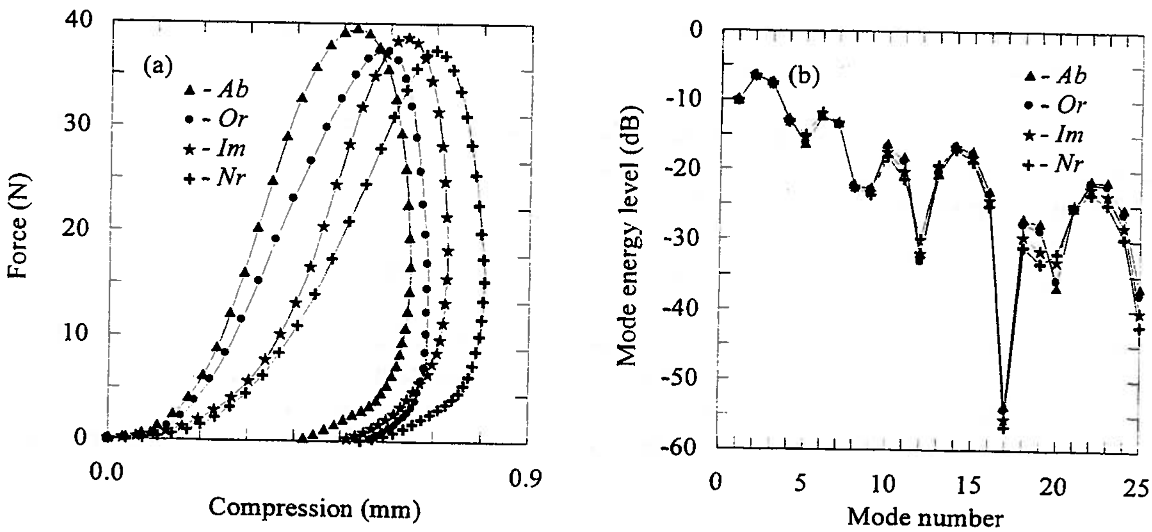

Fig 4. Various hammers comparison.
(a) Force-compression characteristics and (b) spectra envelopes.

The experimental data of the dynamical hammers compression is presented in Fig. 4(a). The initial hammer velocity is equal to V=1 m/s. The values of the numerically determined parameters for all hammers are displayed in Table 3. With the exception of the hammer stiffness F_0, in spite of the quite different form of the force-compression characteristics, the hammer parameters are much the same. To compare the hammers in frequency-domain the flexible string vibration spectra excited by all the hammers were calculated. The results for the note F_2 (f=87 Hz) are presented in Fig. 4(b).

Table 3. Numerically determined parameters of the various hammers (n=21).

	Hammer type			
	Ab	*Or*	*Nr*	*Im*
F_0 (N/mm^p)	44000	31400	10900	17400
p	3.80	3.70	3.73	3.80
ε	0.998	0.998	0.994	0.996
τ (μs)	2.3	2.0	2.73	2.67
f_0	4.8E-10	4.2E-10	5.6E-10	5.3E-10

The difference between the hammers in frequency-domain is not essential up to 17[th] harmonic. The largest difference of the mode energy level at n=11 is equal to 3 dB. The main reason why the spectra of all the hammers considered look similar is the almost equal values of the non-dimensional stiffness f_0 also. This is the additional corroboration that in the hereditary hammer model the non-dimensional stiffness f_0 is the working parameter.

AIR HUMIDITY INFLUENCE

The air humidity influence was investigated by testing of the new type of Renner's hammer (n=20). In

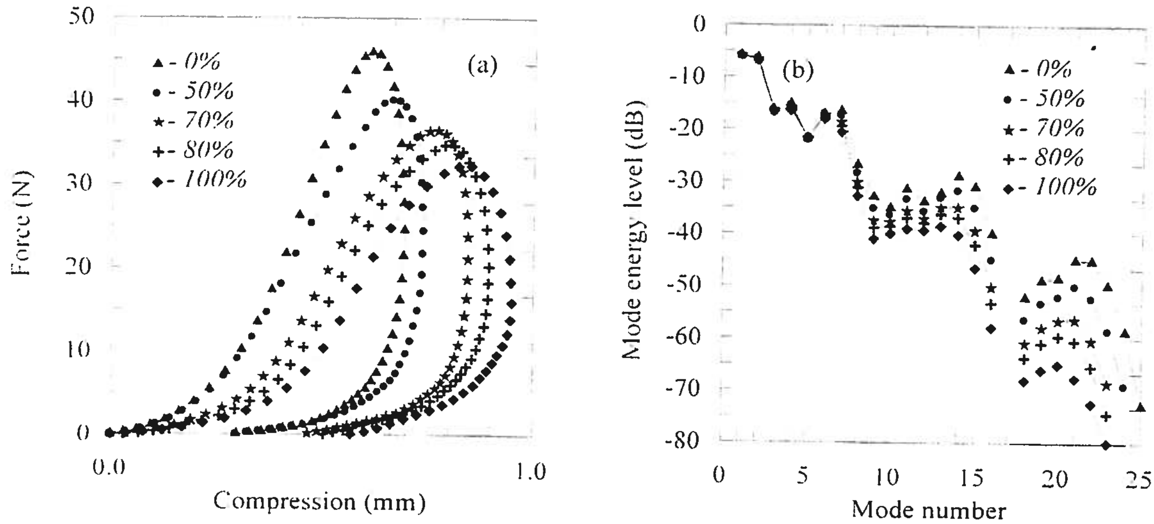

Fig 5. Air humidity influence.
(a) Force-compression characteristics and (b) spectra envelopes.

the absence of the climatic chamber the experiment was provided in the usual room, when the humidity of the air was suitable for the measurements. The simplest type of psychrometer was used as a measuring instrument. The most moisten day was chosen to provide the measurement at the air humidity almost 100%. The very dry (0% humidity) hammer was prepared especially by it heating. The experimentally obtained force-compression characteristics are presented in Fig. 5(a).

Table 4. Hammer parameters for the various air humidity.

Parameter	Humidity				
	0%	50%	70%	80%	100%
F_0 (N/mm^p)	18800	11000	7000	5500	3900
p	3.72	3.72	3.72	3.72	3.72
ε	0.992	0.992	0.992	0.992	0.992
τ (μs)	2.7	2.8	2.9	3.0	3.1
f_0	9.4E-10	8.2E-10	5.2E-10	4.8E-10	4.0E-10

In this experiment the initial hammer velocity is equal to V=1 m/s. The values of the numerically determined parameters of the hammer are displayed in Table 4. Using this Table 4 we may state that, in spite of the significant difference of the force-compression characteristics, the air humidity acts only on the hammer stiffness, and not changes the other hammer parameters.

To consider the air humidity influence in frequency-domain the flexible string vibration spectra were calculated for the note E_2 (f=82 Hz). The results are presented in Fig. 5(b). The level of the 17th harmonic is less than -80 dB. The spectra difference up to the 6th harmonic is very small. The difference of the mode energy level at n=11 is equal to 10 dB. From Fig. 5(a) we may estimate the rate of the higher harmonics attenuation with the increasing of the air humidity.

SUMMARY

To obtain the dynamical force-compression characteristics of the piano hammer, a special device has been developed. This arrangement makes it possible to record the force and compression histories during the hammer-string interaction with a rather high accuracy.

It has been experimentally proved that the piano hammer possess history-dependent properties, or just as well is made of a material with memory. The hereditary model of the piano hammer makes prediction in good agreement with experimental data for various types of hammers and for a broad range of hammer velocities. The dependence of the slope of the force-compression characteristics of the hammer on the rate of loading is obvious (Fig. 2).

The numerical simulation of the hammer-string interaction permits to determine the values of the elastic and hereditary parameters of the hammer. It has been shown that the values of the hammer parameters depend on the diameter of the struck string (Fig. 3).

The comparison of the different piano hammers produced by various firms was carried out. It has been shown that the hammers differ from each other mainly by its stiffness (Table 3).

The quantitative estimation of the air humidity influence on the stability of the hammer parameters was given (Table 4).

The examples of the hammers comparison in frequency-domain were presented also.

Without any doubt the piano hammer testing device in conjunction with the hereditary model of the hammer is a powerful instrument for the piano hammer investigation.

ACKNOWLEDGEMENTS

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