GRAND PIANO MANUFACTURING IN ESTONIA: THE PROBLEM OF PIANO SCALING

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Abstract. The present-day state of piano manufacturing in Estonia is described based on the cooperation between the Tallinn Piano Factory and the Institute of Cybernetics at Tallinn Technical University. Scaling of the medium-sized piano *Estonia* is considered in detail.

1. INTRODUCTION

A brief survey of the development of Estonian piano companies from the late l8th century to 1995 is given in [1]. Some problems, related to design of a mini-sized piano developed in 1995 by the Tallinn Piano Factory in collaboration with the Institute of Cybernetics, are also described there. The first samples of the new piano *Baby Grand* were completed by the end of 1995.

Today, Tallinn Piano Factory manufactures approximately 275 grand pianos per year. These include *Baby Grand*, medium-sized grand pianos *Parlour*, and grand pianos

Concert. Grand pianos *Estonia* are exported worldwide, mainly to the US, Iceland, Belgium, Finland, Norway, Russia, etc, and they have drawn wide attention.

The pianos *Parlour* and *Concert* have been manufactured by Tallinn Piano Factory for many years. Nowadays the construction of the pianos is modernized.

The present-day market demands are very high, especially concerning the piano construction and design, the final quality of the piano frame and the details made of wood, and the stability of all the piano parameters during the period of exploitation.

The most significant work was made to improve the iron frame of the pianos *Parlour* and *Concert* in order to achieve the frame stiffness and dynamic stability. The calculation of the iron frame was carried out by means of the computer program COSMOS/M using the 3D model of the frame. Owing to that, the stability of the piano sound improved.

The studies of the piano soundboard continue as well. This problem is rather complicated and up to the present time only experimental analysis of the prototypes of the soundboard has been performed.

Scaling of the piano is in general a theoretical problem. Considerable amount of data has been collected about it. The method of interactive approach to the evaluation of the mensure of the piano *Baby Grand* is described in [1]. Nevertheless, some problems of the piano mensuration have not been considered earlier. Next we present an analysis of the mensure of the medium-sized piano *Parlour*.

2. PIANO SCALE

Mensure (or piano scale) is the summary table of the full collection of the string lengths, string diameters, diameters of the winding wires of the bass strings, and the positions of the striking point (the distance of the hammer from the nearest string end), the string tensions, etc. The piano scale is based on practical requirements and pure empirical data. Main requirements to the determination of the piano mensure are discussed in [2, 3, 4]. They are the following.

1. The length L of the shortest string is equal to 52 mm, and its diameter d_1 is equal to 0.775 mm. These values have been chosen on the basis of practical experience to provide the maximum permissible stress of the steel wire, and by taking into account some indeterminate considerations about a ”good voicing” in treble notes.

2. For approximately sixty upper notes of the grand piano the position of the striking point l becomes little by little displaced from 1/8 to 1/24 of the whole string length in the direction to the higher notes.

3. The distribution of the string tension must be more or less smooth function of the key number to provide a uniform loading of the iron frame.

The string tension T is calculated as

$$T = (2fL)^2\mu = (2f)^2LM = \pi\rho_s(Lfd_1)^2, \qquad (1)$$

where T is the string tension, f is note frequency, μ is linear mass density of the string, M is the entire string mass, and ρ_s is the density of the steel core (7860 kg/m^3).

For a good instrument, the value Lfd_1 should be a constant. However, it is very difficult to achieve such a distribution of the tension. Therefore, a linear or parabolic law of the tension distribution is mostly used.

Since the stiffness of the string increases sharply with its diameter (proportionally to d_1^4), inharmonicity is especially noticeable in the case of the bass strings. Because wrapped strings are more flexible than solid strings of the same diameter, the inharmonicity of the bass strings is reduced substantially by using wrapped rather than solid strings of the same weight.

There are no reliable recommendations available about how to choose the wrapping wires for the bass notes. A simple way is to choose the thinnest core wire to avoid the string inharmonicity.

The next step in the calculation of the piano scale is the matching of the linear mass density of the string in order to obtain the frequency of the given note. The set of frequencies for each note of the grand piano is defined by

$$f_{n+1} = f_n \sqrt[12]{2} = 1.05946 \, f_n \, . \tag{2}$$

Thus, the region of the note frequencies is exactly determined from f=27.5 Hz for the first note $A2$ to f=4186 Hz for the last note $c5$.

In this paper we shall consider the problems of the distribution of the string tension and the determination of the diameters of the winding wires on the example of the medium-sized piano *Parlour*. Since we are interested only in these two problems, we may assume that the string lengths and the striking point ratios for all the strings are given. The number of strings in choir for each note N is also known. The upper view of the soundboard with the piano strings is presented in Fig. 1.

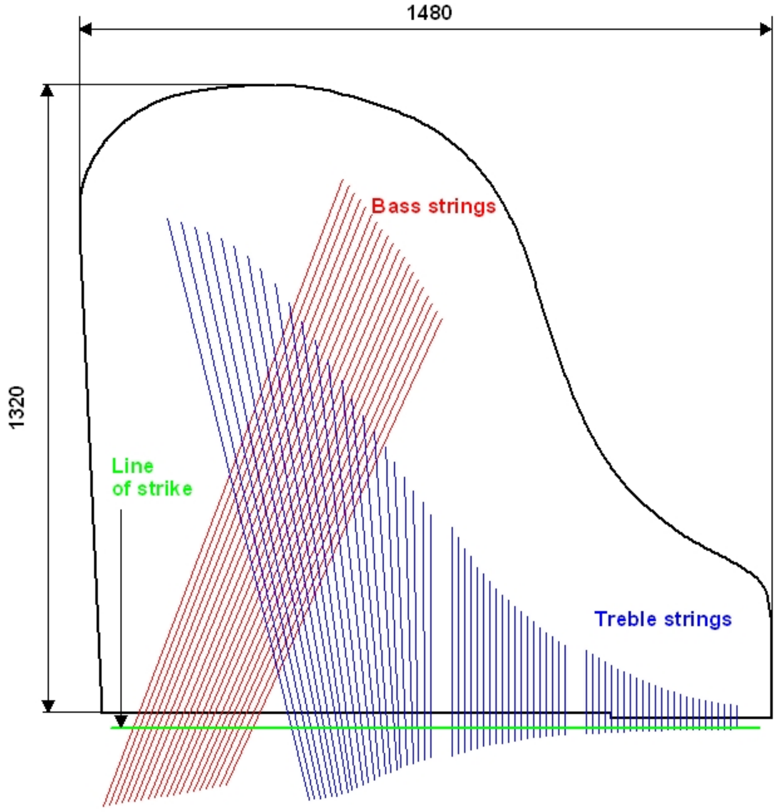

Fig. 1. Position of strings over the soundboard of the medium–sized grand piano *Parlour*.

3. STRING TENSION

The total number of notes in the piano is 88. The number of strings is much larger, because the number of strings in a choir, N, changes from one string for the lower bass notes to three strings in treble. As it was mentioned above, the distribution of the tension of strings must be a more or less smooth function of the key number to provide a uniform loading of the frame. For the first version of the piano *Baby-Grand*, a smooth function of tension distribution per choir was chosen. Therefore, since the number of strings per note changes, the difference of tension per string for these notes is big. So, if the number of strings in choir changes from one to two, and from two to three, then the discontinuity of the relative tension is equal to 70 and 40%, respectively.

Evidently, the main piano manufacturers are trying to reduce the discontinuity of tension when the number of strings in choir varies. Let us consider minimization of the jump of the tension.

According to the construction of the medium-sized piano , the first ten notes ($A2-F\sharp 1$) have only one string per note. The notes from eleven to twenty five ($G1-A0$) have two strings per note, and the other notes consist of three strings. Let us denote the tension of strings of the tenth and the eleven note by T_{10} and T_{11}, respectively. To obtain the minimum jump of the tension between these notes, r_{12}, the relative tension per string must equal to the relative tension per note

$$r_{12}=\frac{2(T_{10}-T_{11})}{(T_{10}+T_{11})}=\frac{2(2T_{11}-T_{10})}{(T_{10}+2T_{11})}\,. \tag{3}$$

From this equation we have

$$T_{10}=\sqrt{2}T_{11}\,. \tag{4}$$

Exactly in the same way, for the notes $A0$ and $A\sharp 0$ we may write

$$r_{23}=\frac{2(T_{25}-T_{26})}{(T_{25}+T_{26})}=\frac{2(3T_{26}-2T_{25})}{(3T_{26}+2T_{25})}\,, \tag{5}$$

and

$$T_{25} = \sqrt{\frac{3}{2}} T_{26} \, . \tag{6}$$

The relations (3) and (5) give the minimum jump of relative tension in cases when the number of strings in choir varies from one to two and from two to three:

$$r_{12} = 0.3431 \, , \;\; r_{23} = 0.2020 \, . \tag{7}$$

However, practically it is very difficult to achieve exactly such distribution of the tension, because the step of the wire diameter is equal to 0.025 mm for the steel core and 0.05 mm for the winding wire.

Using the results obtained so far, we may suggest the most suitable (ideal) distribution of the string tension for all notes of a piano. It has been shown [1] that in treble the tension of the string does not depend on the key number and is approximately equal to 620 N. Therefore, we may choose T_{26}=620 N. According to (6), we have T_{25}=760 N.

On the other hand, experienced piano makers recommend the value of tension for the first string T_1 = 1320 N. The tension of other bass strings must decrease from this value to T_{25} = 760 N, following the linear law. Here we may choose the slope of the tension distribution between the first and the ten note arbitrarily. For example, if we choose T_{11} = 840 N, then from (4) we have T_{10} = 1188 N. These values of tension are acceptable because we may use rather small diameter of the core wires and the permissible stress of the strings will not be exceeded (see below). Thus, using the linear law of the tension distribution between notes 1 and 10 ($A2 - F\sharp 1$), and between notes 11 and 25 ($G1 - A0$), we may hope that the tension distribution as a whole is almost ideal. Such tension dis-

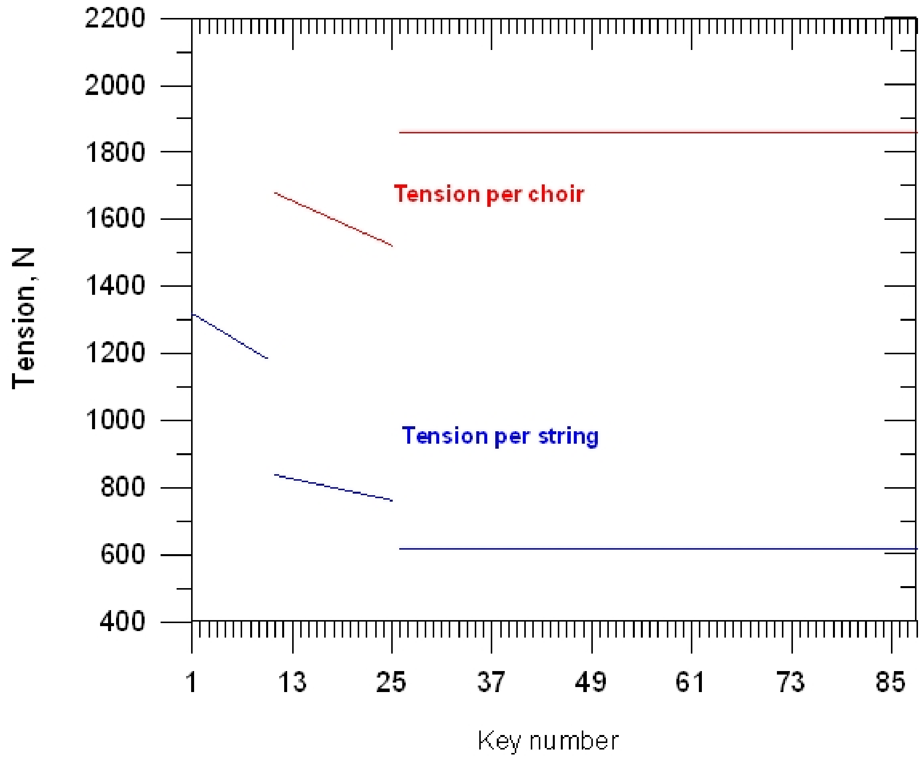

Fig. 2. Ideal tension distribution.

tributions per string and per choir are shown in Fig. 2, and we shall try to obtain these distributions also in practice.

4. DETERMINATION OF THE CORE DIAMETERS

The next step in piano scaling is the determination of string diameters. Now, we have the ideal (preliminary) string tensions T_{0n}, the string lengths L_n, and the frequencies f_n for all the notes. Thus, we may calculate the preliminary linear mass densities μ_{0n} for all the 88 notes

$$\mu_{0n} = (2 f_n L_n)^{-2} T_{0n}, \;\; n = 1 \ldots 88\,, \tag{8}$$

and may calculate the diameters of the string core and of the copper winding wires.

The diameters of the strings without winding were calculated as

$$d_{1n} = \sqrt{\frac{4\mu_{0n}}{\pi\rho_s}},\ n = 1...88, \tag{9}$$

and rounded off to 0.025 mm. The new linear mass density and tension of these strings were recalculated by formule

$$\mu_n = \frac{\pi}{4}\rho_s {d_{1n}}^2,\ T_n = (2 f_n L_n)^2 \mu_n,\ n = 1...88. \tag{10}$$

The diameter of the core in the wrapped strings must be as small as possible, but the stress of each string should be at least twice lower than the tensile strength for the steel wire. Thus, we must take into account the tensile strength [σ] of the steel wire. For simplicity, with the accuracy of 1.5 % we may approximate the dependence of the tensile strength of the wire on the core diameter d_1 by formula

$$[\sigma] = 321.235(1 - 0.3982 d_1 + 0.1033 d_1^2), \tag{11}$$

where units are for [σ] - kg/mm^2, and for d_1 - mm. Taking into account that suitable relative string tension

$$\sigma_n = \frac{T_n}{\pi d_{1n}^2 [\sigma]} \tag{12}$$

can not exceed 0.5, we may choose the diameter of the core wires also for the wrapped strings.

5. DETERMINATION OF THE DIAMETERS OF THE WINDING WIRES

If we know the entire mass of the wrapped string $M = \mu_0 L$ and diameter of core d_1, then the diameters of the winding wires can be found easily. In Fig. 3(a) the cross-section of a doubly-wound string is shown. It is the ideal cross-section, but we may suppose that it is very close to reality. Because almost the entire length of the core is wrapped, the number of coils of the winding is equal to $m_2 = L/d_2$ for the first winding, and $m_3 = L/d_3$ for the second one. Here and below the index n is omitted. The diameters of the coils are $(d_1 + d_2)$ and $(d_1 + 2d_2 + d_3)$, respectively. Thus, the lengths of the winding wires are

$$L_2 = \pi m_2 (d_1 + d_2) = \frac{L}{d_2} \pi (d_1 + d_2)\,,$$

$$L_3 = \pi m_3 (d_1 + 2d_2 + d_3) = \frac{L}{d_3} \pi (d_1 + 2d_2 + d_3)\,, \tag{13}$$

and the mass of the string is

$$M = \mu_0 L = \frac{\pi L}{4} \left[\rho_s {d_1}^2 + \pi \rho_c d_2 (d_1 + d_2) + \pi \rho_c d_3 (d_1 + 2d_2 + d_3) \right]$$

$$= \frac{\pi L}{4} \left[\rho_s {d_1}^2 + \pi \rho_c (d_2 + d_3)(d_1 + d_2 + d_3) \right]\,, \tag{14}$$

where ρ_c is the density of the copper winding wire (8950 kg/m^3).

Introducing the notation $d_{23} = d_2 + d_3$, Eq. (14) yields

$$\mu_0 = \frac{\pi}{4} \left[\rho_s {d_1}^2 + \pi \rho_c d_{23} (d_1 + d_{23}) \right]\,. \tag{15}$$

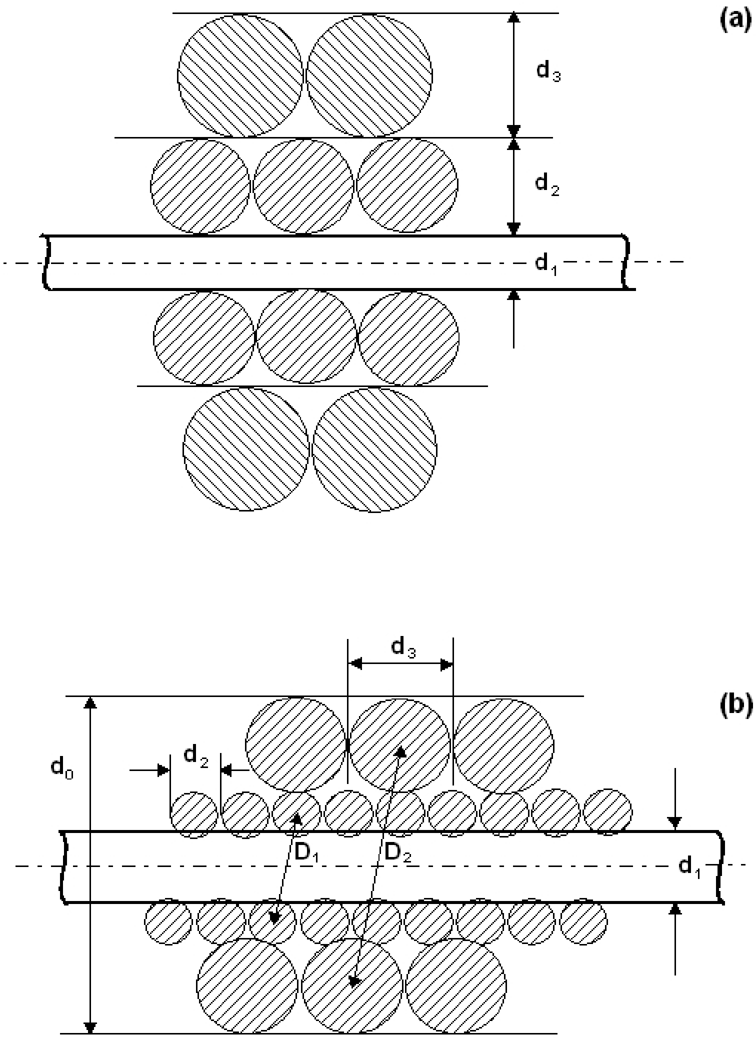

Fig. 3. Ideal (a) and real (b) cross–section of a doubly–wound piano string.

It is obvious that for the cross-section of the string (Fig. 3(a)), the value of d_{23} may be obtained from the quadratic equation (15). In this case the values d_2 and d_3 should be chosen arbitrarily from the technological conditions, observing the condition $d_2 < d_3$. Scale of the piano *Parlour* is shown in Table 1.

Table 1. Scale of the medium-sized piano *Parlour*

n	L [mm]	l[mm]	L/l	N	T[N]	σ	μ[g/m]	d_1[mm]	d_2[mm]	d_3[mm]
1	2	3	4	5	6	7	8	9	10	11
1	1415.0	175.8	8.05	1	1306.8	0.45	215.75	1.350	1.000	1.700
2	1399.0	173.8	8.05	1	1292.9	0.47	194.49	1.300	0.900	1.650
3	1383.0	171.6	8.06	1	1261.2	0.46	172.98	1.300	0.700	1.650
4	1366.0	169.5	8.06	1	1268.2	0.49	158.90	1.250	0.650	1.600
5	1350.0	167.5	8.06	1	1246.0	0.50	142.36	1.225	0.600	1.600
6	1333.0	165.4	8.06	1	1228.3	0.50	128.23	1.225	0.500	1.450
7	1316.0	163.1	8.07	1	1202.0	0.49	114.72	1.225	0.400	1.400
8	1298.0	160.8	8.07	1	1197.2	0.50	104.66	1.200	1.700	
9	1281.0	158.7	8.07	1	1188.7	0.52	95.05	1.175	1.600	
10	1263.0	156.5	8.07	1	1188.9	0.52	87.11	1.175	1.500	
11	1251.0	154.8	8.08	2	840.7	0.46	55.93	1.025	1.150	
12	1233.0	152.6	8.08	2	801.2	0.46	48.89	1.000	1.050	
13	1215.0	150.4	8.08	2	821.0	0.47	45.96	1.000	1.000	
14	1196.0	148.0	8.08	2	821.5	0.49	42.28	0.975	0.950	
15	1178.0	145.6	8.09	2	781.3	0.47	36.93	0.975	0.850	
16	1159.0	143.3	8.09	2	790.0	0.47	34.37	0.975	0.800	
17	1140.0	140.9	8.09	2	796.1	0.48	31.89	0.975	0.750	
18	1121.0	138.6	8.09	2	781.1	0.49	28.83	0.950	0.700	
19	1102.0	136.0	8.10	2	780.5	0.49	26.56	0.950	0.650	
20	1083.0	133.7	8.10	2	776.8	0.49	24.38	0.950	0.600	
21	1273.0	157.2	8.10	2	763.5	0.46	15.45	0.975	0.350	
22	1261.0	155.7	8.10	2	750.7	0.45	13.79	0.975	0.300	
23	1248.0	154.1	8.10	2	731.8	0.44	12.23	0.975	0.250	
24	1233.0	152.0	8.11	2	705.7	0.42	10.76	0.975	0.200	
25	1218.0	150.2	8.11	2	743.8	0.47	10.36	0.950	0.200	
26	1201.0	148.1	8.11	3	612.2	0.29	7.81	1.125		
27	1180.0	145.5	8.11	3	605.7	0.31	7.13	1.075		
28	1153.0	142.0	8.12	3	649.1	0.33	7.13	1.075		
29	1120.0	137.9	8.12	3	625.1	0.34	6.49	1.025		
30	1080.0	133.0	8.12	3	652.4	0.36	6.49	1.025		
31	1034.0	127.3	8.12	3	638.8	0.37	6.17	1.000		
32	985.0	121.3	8.12	3	650.7	0.37	6.17	1.000		
33	939.0	115.5	8.13	3	631.0	0.38	5.87	0.975		
34	890.0	109.5	8.13	3	636.3	0.38	5.87	0.975		
35	840.0	103.3	8.13	3	636.2	0.38	5.87	0.975		
36	793.0	97.5	8.13	3	636.4	0.38	5.87	0.975		
37	752.0	92.5	8.13	3	642.4	0.39	5.87	0.975		
38	713.0	87.6	8.14	3	648.2	0.39	5.87	0.975		
39	675.0	82.9	8.14	3	652.1	0.39	5.87	0.975		
40	638.0	78.4	8.14	3	654.0	0.39	5.87	0.975		
41	604.0	74.2	8.14	3	624.6	0.39	5.57	0.950		
42	570.0	69.9	8.15	3	624.4	0.39	5.57	0.950		
43	541.0	66.4	8.15	3	631.4	0.40	5.57	0.950		
44	511.0	62.7	8.15	3	632.2	0.40	5.57	0.950		

Table 1 (continued)

1	2	3	4	5	6	7	8	9	10	11
45	481.0	58.9	8.17	3	628.8	0.39	5.57	0.950		
46	455.0	54.4	8.36	3	631.6	0.40	5.57	0.950		
47	428.0	49.8	8.59	3	627.3	0.39	5.57	0.950		
48	408.0	46.4	8.79	3	639.8	0.40	5.57	0.950		
49	381.0	42.3	9.01	3	626.3	0.39	5.57	0.950		
50	363.0	39.3	9.24	3	638.1	0.40	5.57	0.950		
51	345.0	36.5	9.45	3	647.0	0.41	5.57	0.950		
52	328.0	34.0	9.65	3	656.4	0.41	5.57	0.950		
53	312.0	31.6	9.87	3	632.1	0.42	5.28	0.925		
54	298.0	29.1	10.24	3	647.2	0.43	5.28	0.925		
55	284.0	26.7	10.64	3	659.8	0.43	5.28	0.925		
56	270.0	24.5	11.02	3	669.4	0.44	5.28	0.925		
57	256.0	22.5	11.38	3	639.4	0.44	5.00	0.900		
58	244.0	20.8	11.73	3	652.0	0.45	5.00	0.900		
59	231.0	19.1	12.09	3	656.0	0.45	5.00	0.900		
60	218.0	17.4	12.53	3	655.7	0.45	5.00	0.900		
61	208.0	16.2	12.84	3	633.4	0.46	4.73	0.875		
62	197.0	14.9	13.22	3	637.7	0.46	4.73	0.875		
63	187.0	13.7	13.65	3	645.0	0.47	4.73	0.875		
64	177.0	12.6	14.05	3	648.6	0.47	4.73	0.875		
65	167.0	11.6	14.40	3	648.1	0.47	4.73	0.875		
66	158.0	10.7	14.77	3	651.2	0.47	4.73	0.875		
67	150.0	9.9	15.15	3	658.8	0.48	4.73	0.875		
68	141.0	9.1	15.49	3	653.4	0.47	4.73	0.875		
69	132.0	8.3	15.90	3	642.8	0.46	4.73	0.875		
70	125.0	7.7	16.23	3	647.0	0.47	4.73	0.875		
71	119.0	7.2	16.53	3	658.2	0.48	4.73	0.875		
72	112.0	6.6	16.97	3	654.4	0.47	4.73	0.875		
73	106.0	6.1	17.38	3	658.0	0.48	4.73	0.875		
74	101.0	5.7	17.72	3	670.5	0.49	4.73	0.875		
75	95.0	5.2	18.27	3	665.8	0.48	4.73	0.875		
76	90.0	4.9	18.37	3	670.8	0.49	4.73	0.875		
77	84.0	4.5	18.67	3	655.9	0.47	4.73	0.875		
78	79.0	4.1	19.27	3	651.2	0.47	4.73	0.875		
79	74.0	3.8	19.47	3	641.3	0.46	4.73	0.875		
80	70.0	3.5	20.00	3	644.1	0.47	4.73	0.875		
81	66.0	3.2	20.63	3	606.6	0.46	4.46	0.850		
82	63.0	3.0	21.00	3	620.4	0.47	4.46	0.850		
83	59.0	2.8	21.07	3	575.3	0.46	4.20	0.825		
84	56.0	2.6	21.54	3	581.7	0.47	4.20	0.825		
85	54.0	2.5	21.60	3	571.0	0.48	3.95	0.800		
86	52.0	2.4	21.67	3	594.3	0.50	3.95	0.800		
87	51.0	2.3	22.17	3	602.2	0.54	3.71	0.775		
88	50.0	2.2	22.73	3	649.7	0.58	3.71	0.775		

But reality is more complicated. The analysis and measurements of the singly- and doubly-wound piano strings carried out at Tallinn Piano Factory show that the real cross-section of the piano string is similar to the scheme shown in Fig. 3(b). During the process of winding the surface of copper wires, contiguous to the core, is deformed. The same deformation of the copper wires takes place between the first and the second winding. Thus, the outer diameter d_0 of the wrapped string is less than $d_1+2d_2+2d_3$. The measurements of the outer diameters of the wrapped strings show that the value of this diameter may be obtained from the empirical formula

$$d_0 = d_1 + 2d_2 - 0.041 d_2 \left(1 + \frac{d_2}{d_1}\right) , \tag{16}$$

for singly-wound string, and from

$$d_0 = d_1 + 2d_2 + 2d_3 - 0.041(d_2+d_3) \left(1 + \frac{d_2+d_3}{d_1}\right) , \tag{17}$$

for doubly-wound string. Thus, the diameter of the coil of the singly-wound string D_1 is

$$D_1 = \sqrt{\left[d_1 + d_2 - 0.041 d_2 \left(1 + \frac{d_2}{d_1}\right)\right]^2 + \frac{{d_2}^2}{4}} , \tag{18}$$

and the diameter of the second winding coil of the doubly-wound string D_2 is

$$D_2 = \sqrt{\left[d_1 + 2d_2 + d_3 - 0.041(d_2+d_3) \left(1 + \frac{d_2+d_3}{d_1}\right)\right]^2 + \frac{{d_3}^2}{4}} . \tag{19}$$

We must note here, that the lateral surfaces of the copper wires are not deformed and, consequently, we may find the number of coils easily. In case of two windings, the length of the first winding is approximately 100 mm less than the string length. Therefore the number of coils of the first winding is

$$m_2 = \frac{L-100}{d_2} . \tag{20}$$

The length of the second winding is 40 mm less than the string length, and the number of coils of the second winding is

$$m_3 = \frac{L-40}{d_3} . \tag{21}$$

The lengths of the copper wires of the first and second windings are equal to

$$L_2 = \pi m_2 D_1 , \qquad L_3 = \pi m_3 D_2 , \tag{22}$$

respectively. Thus, the mass of the singly-wound string is

$$M_1 = \frac{\pi L}{4} \left[\rho_s {d_1}^2 + \pi \rho_c d_2 D_1 \left(1 - \frac{40}{L}\right)\right] , \tag{23}$$

and that of a doubly-wound string

$$M_2 = \frac{\pi L}{4} \left\{\rho_s {d_1}^2 + \pi \rho_c \left[d_2 D_1 \left(1 - \frac{100}{L}\right) + d_3 D_2 \left(1 - \frac{40}{L}\right)\right]\right\} . \tag{24}$$

Table 2. Parameters of the strings

Measured						Theory	
L [mm]	d_1 [mm]	d_2 [mm]	d_3 [mm]	d_0 [mm]	M [g]	d_0 [mm]	M [g]
1415	1.400	1.850		4.95	196	4.92	197
1383	1.400	1.750		4.70	174	4.74	178
1350	1.225	1.500		4.10	127	4.09	129
1165	1.300	0.850	1.950	6.55	283	6.54	282
1145	1.300	0.850	1.500	5.70	213	5.73	211
1130	1.175	0.800	1.350	5.25	174	5.23	173

The parameters of the strings obtained from the measurements and calculated theoretically by formulae (16) - (17) and (23) - (24) are displayed in Table 2. These formulae give the values of the outer diameters of the strings with accuracy better than 1%, and the values of the string masses with accuracy better than 1.5%.

Semiempirical formulae (23) - (24) give us the possibility to find the diameters of the winding wires using the known values of the ideal lineal mass density of the string μ_0 and the diameter d_1 of the core. From (23) and (24) we have for the singly-wound string

$$\mu_0 = \frac{\pi}{4}\left[\rho_s {d_1}^2 + \pi\rho_c d_2 D_1 \left(1 - \frac{40}{L}\right)\right] , \tag{25}$$

and for the doubly-wound string

$$\mu_0 = \frac{\pi L}{4}\left\{\rho_s {d_1}^2 + \pi\rho_c \left[d_2 D_1 \left(1 - \frac{100}{L}\right) + d_3 D_2 \left(1 - \frac{40}{L}\right)\right]\right\} . \tag{26}$$

Now, using (25) we may find immediately the value of the diameter d_2 of the winding wire. Due to technological demands, this diameter must be greater than 0.2 mm and less than 2 mm. If this diameter is greater than 2 mm then we must use the doubly-wound string. In this case we must choose the preliminary diameter d_2 beforehand, and then the diameter d_3 of the second winding wire may be found using (26). The only condition which must be fulfilled is $d_2 < d_3 < 2$. By applying this procedure the values of the copper wire diameters obtained were rounded off to 0.05 mm. These values of d_2 and d_3 are displayed in Table 1.

6. SCALE OF THE MEDIUM–SIZED PIANO

To complete the piano scaling we must calculate new values of the string tension and the relative string stress. Using the diameters d_1, d_2 and d_3 in formulae (25), (26), and (10) we find the string tensions T_n, and by using (12), the relative string stresses σ_n. Now the Table 1 is complete. Distributions of the string tension and relative stress are shown in Figs. 4 and 5. The values of the string tensions $T_1, T_{10}, T_{11}, T_{25}$, and T_{26} obtained are very close to the calculated values. The values of the relative tensions calculated for notes where the number of strings changes from 1 to 2 are: r_{12} = 0.363 per string, and r_{12} = 0.323 per choir. For notes where the number of strings changes from 2 to 3 we have: r_{23} = 0.199 per string, and r_{23} = 0.205 per choir. These values are very close to the ideal values, and we may hope that the needed string tension is achieved.

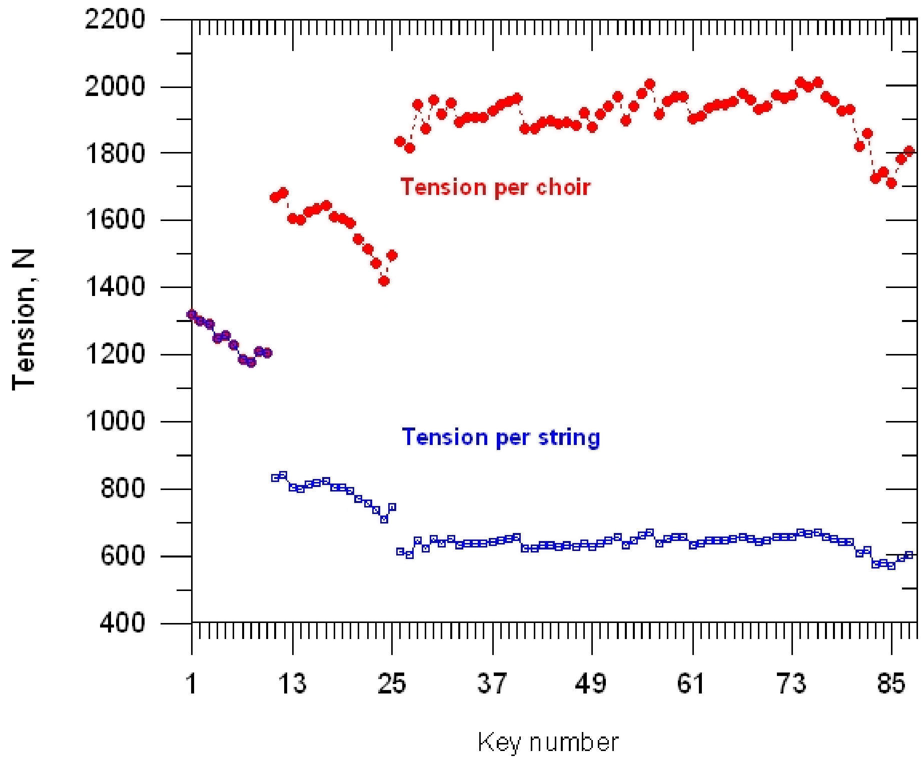

Fig. 4. Tension tension distribution.

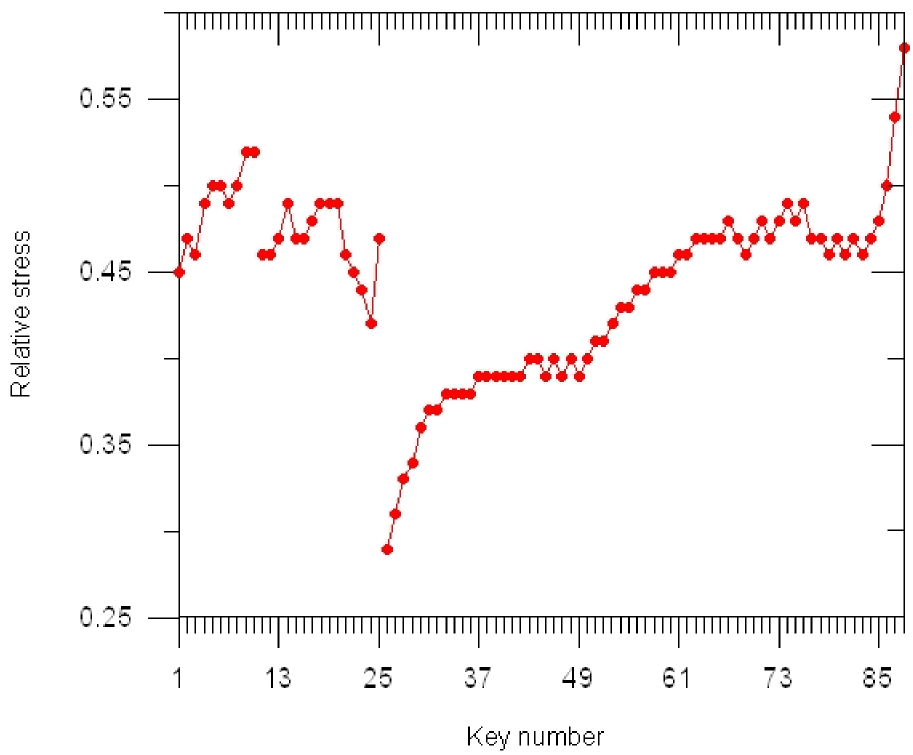

Fig. 5. Relative stress distribution.

The method of evaluation of the mensure of the grand piano presented here, is elementary. Many problems have not been considered. The most complex problem is how to choose the position of the striking point. This problem may be solved correctly only by numerical simulation of the hammer-string interaction that is discussed in [5]. The hammer parameters will be determined by using a hereditary hammer model presented in [6], after experimental testing of the hammer. This problem will be considered in future publications.

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