WAVE PROPAGATION IN FUNCTIONALLY GRADED MATERIALS

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Summary The propagation of two-dimensional stress waves in functionally graded materials (FGMs) is studied numerically. Two distinct models of FGMs are considered: i) a multilayered metal-ceramic composite with averaged properties within layers; ii) randomly embedded ceramic particles in a metal matrix with prescribed volume fraction. The numerical simulation demonstrates the applicability of the composite wave-propagation algorithm to the modelling of FGMs without any averaging procedure. The analysis based on simulation shows significant differences in the stress wave characteristics for the distinct models that can be used for optimizing the response of such structures to impact loading.

BACKGROUND

Studies of the evolution of stresses and displacements in FGMs subjected to quasistatic loading [1] show that the utilization of structures and geometry of a graded interface between two dissimilar layers can reduce stresses significantly. Such an effect is also important in case of dynamical loading where energy-absorbing applications are of special interest.

A two-dimensional problem of the impulsive loading of a plate of thickness h and length L >> h is considered. The load is applied transversally at the central region of the plate upper surface of length a (a < L). The material of the plate is assumed to be compositionally graded along the thickness direction. The gradation is described in terms of the volume fraction of a ceramic reinforcing phase within a metal matrix. Both metal and ceramics are assumed to behave as linear isotropic elastic media.

We focus our attention on the case of dynamic loading of a plate where the wavelength of stress pulse is comparable with the plate thickness. This means that the wavelength is much larger than the size of inclusions and the distance of wave travel is relatively small. In addition, the rise time of applied stress pulse (0.75 µs) is much larger than the ratio of the reinforcement dimension to the fastest wave speed in the reinforcing material.

For the comparison of the two distinct models of FGMs, it is sufficient to employ the linear rule of mixtures for the Young's modulus and Poisson's ratio of the graded layer in the multilayered model of metal-ceramic composite with averaged properties within layers. The same individual material properties without any averaging are used in the model of the composite with randomly embedded ceramic particles in a metal matrix.

The idea of direct numerical simulation of FGMs by means of regular [2] or random [3] particle distribution is not a new one: it allows us to analyze the influence of shape and aspect ratio of particles on the FGM behavior [4]. Our goal is simply to show the differences in dynamical behavior of FGMs by using distinct models. The main aim of the paper is to compare the time evolution of the field quantities as a result of wave propagation and interaction with interfaces and gradients in these two models and clarify the significance of the choice of gradation on impact applications.

NUMERICAL SIMULATION

We applied the composite wave-propagation algorithm [5-6] to compare the models of discrete layers with averaging the material properties and of randomly embedded ceramic particles in a metal matrix. Numerical difficulties concerning rapidly-varying properties of the medium are overcome by using the composite wave-propagation algorithm, because every discontinuity in the parameters is taken into account by solving the Riemann problem at each interface between discrete elements. The reflection and transmission of waves at each interface are handled automatically.

Here these studies are extensively enlarged. Several possible forms of the ceramic reinforcement volume fraction along the thickness are considered. First, the cases examined in [7] for the axisymmetric case are studied: uniform, layered and graded with two different distributions of volume fraction. Second, more complicated cases are analysed: multi-layer coated, partly layered medium with uniform and randomly distributed layering. The influence of the thickness of graded layers on dispersion of waves is clarified.

Typical examples of contour plots showing the full displacement fields are presented in Fig. 1 for the same density distribution in different models. Computations performed under the same conditions for distinct models of the same FGM: L=49 mm, h=24.5 mm, a=12.25 mm, maximal amplitude of loading was equal to 125 MPa. The difference in the propagation speed of stress wave is clearly seen whereas the particle volume fraction was distributed similarly in both models. In addition, we have the distortion of the symmetrical shape of the wavefronts in the model with randomly embedded particles due to random particle distribution.

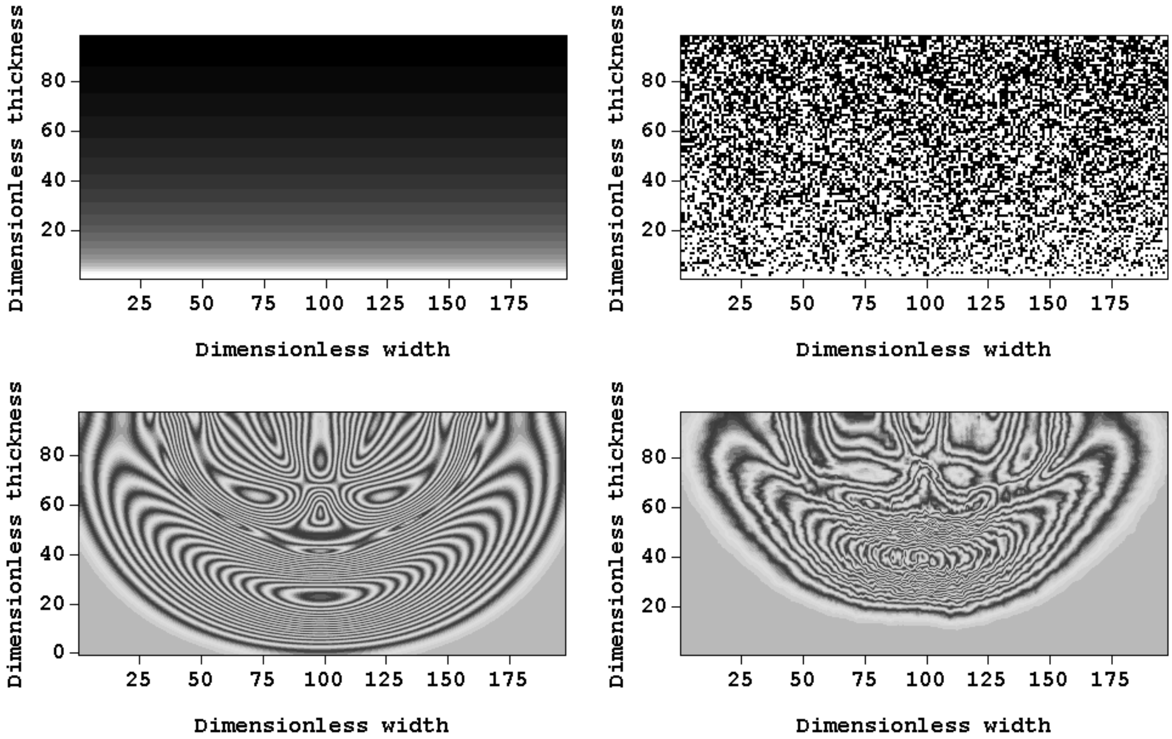

Fig.1. Density distributions and wavefronts in metal-ceramic composite for different models.

CONCLUSIONS

Theoretical prediction of dynamic behaviour of FGMs depends on how well their properties are modelled in computer simulations. While many averaging models of the properties of FGMs are widely accepted, a more natural model of a matrix with randomly embedded particles is seldom used because of numerical difficulties in the case of rapidly-varying properties of the medium. In this paper, we applied the composite wave-propagation algorithm to compare the models of discrete layers with averaging the material properties and of randomly embedded ceramic particles in a metal matrix.

The results of performed numerical simulations of stress wave propagation in FGMs show a significant difference between characteristics of wave fields in the distinct models. This means that a model of FGM without averaging of material properties can give more detailed information about the dynamic behaviour of a chosen structure. In addition, using this information for the nondestructive evaluation (NDE) of material properties is proposed; these studies are in progress.

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