STRESS WAVE PROPAGATION IN FUNCTIONALLY GRADED MATERIALS

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Abstract

The propagation of stress waves in functionally graded materials (FGMs) is studied numerically by means of the composite wave-propagation algorithm. Two distinct models of FGMs are considered: i) a multilayered metal-ceramic composite with averaged properties within layers; ii) randomly embedded ceramic particles in a metal matrix with prescribed volume fraction. The numerical simulation demonstrates the applicability of that algorithm to the modelling of FGMs without any averaging procedure. The analysis based on simulation shows significant differences in the stress wave characteristics for the distinct models that can be used for optimizing the response of such structures to impact loading.

Introduction

Studies of the evolution of stresses and displacements in FGMs subjected to quasistatic loading [1], [2] show that the utilization of structures and geometry of a graded interface between two dissimilar layers can reduce stresses significantly. Such an effect is also important in case of dynamical loading where energy-absorbing applications are of special interest. One-dimensional stress wave propagation in FGMs is discussed in [3], [4], [5]. The impact response of graded metal-ceramic plates in two dimensions is examined in [6]. The response of FGM plates excited by impact loads in three dimensions is studied numerically in [7]. In these studies, FGMs are approximated by multilayered media, and material properties for each layer are assumed to be constant or to be expressed by linear or quadratic functions within a layer. A difficulty in such an approach is the accurate estimation of material properties depending on the functionally varying volume composition of constituent particles. The FGMs are characterized not only by the presence and appearance of compositional gradients but also by the sophisticated behavior of the FGM components. In the simplest case, the structure of a FGM can be represented by the model-like system of a matrix with embedded particles.

In what follows, computations are performed for two distinct models of FGMs: (i) a multilayered metal-ceramic composite with averaged properties within layers; (ii) randomly embedded ceramic particles in a metal matrix with prescribed volume fraction. The main aim of the paper is to compare the time evolution of the field quantities in these two models. We focus our attention on the case of dynamic loading of a plate where the wavelength of stress pulse is comparable with the plate thickness. This means that the wavelength is much larger than the size of inclusions. In addition, the rise time of the applied stress pulse is much larger than the ratio of the reinforcement dimension to the fastest wavespeed in the reinforcing material.

Formulation of the problem

A two-dimensional problem of the impulsive loading of a plate of thickness h and length $L \gg h$ is considered. The load is applied transversally at the central region of the plate upper surface of length a $(a < h)$ (Fig. 1). The material of the plate is assumed to be compositionally graded along the thickness direction. The gradation is described in terms of the volume fraction of a ceramic reinforcing phase within a metal matrix. Following [6], three timescales of the problem can be introduced:

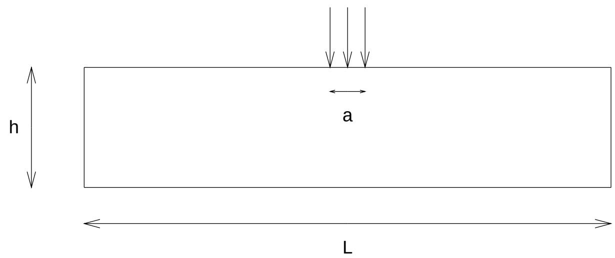

Figure 1: The geometry of the problem.

$$t_0 = \frac{h}{c_f}, \qquad t_b = \frac{L}{c_f}, \qquad t_r, \tag{1}$$

where c_f is the speed of the fastest longitudinal wave, and t_r is the shortest rise time, corresponding to the applied loading. Both metal and ceramics are assumed to behave as linear isotropic elastic media. The governing equations of the problem can be represented in terms of stresses and velocities in a rather simple way [8], [9]:

$$\rho_0(\mathbf{x}) \frac{\partial v_i}{\partial t} = \frac{\partial \sigma_{ij}}{\partial x_j}, \tag{2}$$

$$\frac{\partial \sigma_{ij}}{\partial t} = \lambda(\mathbf{x}) \frac{\partial v_k}{\partial x_k} \delta_{ij} + \mu(\mathbf{x}) \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right), \quad (3)$$

where t is time, x_j are spatial coordinates, v_i are components of the velocity vector, σ_{ij} is the Cauchy stress tensor, ρ is the density, λ and μ are the Lamé coefficients, δ_{ij} is the Kronecker delta. The indicated explicit dependence on the point $\mathbf{x}$ means that the properties of graded materials are reflected in $\rho(\mathbf{x})$, $\lambda(\mathbf{x})$, and $\mu(\mathbf{x})$. Assuming that the plate is at rest for $t \leq 0$, system of equations (2), (3) must be solved under the following initial conditions:

$$u_i(\mathbf{x}, 0) = 0, \qquad \sigma_{ij}(\mathbf{x}, 0) = 0. \quad (4)$$

The upper surface of the plate is subjected to a stress pulse given by

$$\sigma_{22}(x, 0, t) = \sigma_0 sin^2(\pi(t - 2t_r)/2t_r), \quad (5)$$

$$-a/2 < x < a/2, \quad 0 < t < 2t_r.$$

Other parts of upper and bottom surfaces are stress-free, lateral boundaries are assumed to be fixed.

In order to compute the overall strain/stress distributions in FGMs, one needs the appropriate estimates for properties of the graded layer, such as the Young's modulus, Poisson's ratio, etc. A large number of papers on the prediction of material properties of FGMs has been published (see, for example, [10], [11], [12]). In most of these studies the averaging methods have been used which are simple and convenient to predict the overall thermomechanical response and properties, however, owing to the assumed simplifications, the validity of simplified models in real FGMs is affected by the corresponding detailed microstructure and other conditions. Still, the averaging methods may be selectively applied to FGMs subjected to both uniform and non-uniform overall loads with a reasonable degree of confidence.

For the comparison of the two distinct models of FGMs, it is sufficient to employ the linear rule of mixtures for the Young's modulus and Poisson's ratio of the graded layer in the multilayered model of metal-ceramic composite with averaged properties within layers. According to the linear rule of mixtures, the simplest estimate of any material property, a property $P(\mathbf{x})$ at a point $\mathbf{x}$ in dual-phase metal-ceramic materials is approximated by a linear combination of volume fractions V_m and V_c and individual material properties of metal and ceramic constituents P_m and P_c:

$$P(\mathbf{x}) = P_m V_m(\mathbf{x}) + P_c V_c(\mathbf{x}). \quad (6)$$

The same individual material properties without any averaging are used in the model with randomly embedded ceramic particles in a metal matrix.

Computational results

An improved composite wave propagation scheme [8], [9] is applied for the numerical solution. We con-

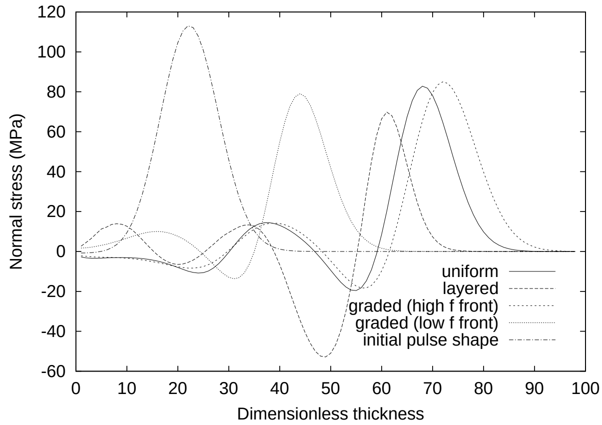

Figure 2: Normal stress distribution along the centerline in metal-ceramic composite with ceramic reinforcement at $3\ \mu s$ (multilayered model with averaged properties within layers).

sider four possible forms of the ceramic particulate reinforcement volume fraction variation along the thickness: uniform, layered, and graded with two different distributions of volume fraction $f = V_c$, where the elastic properties of the metal matrix and ceramic reinforcement are the following [6]: Young's modulus $70\ GPa$ and $420\ GPa$, Poisson's ratio 0.3 and 0.17, and density $2800\ kg/m^3$ and $3100\ kg/m^3$, respectively. These structures are examined as the Cases A, B, C and D by [6] in the axisymmetric case. Simultaneously, the same structures are represented by randomly embedded ceramic particles with the corresponding volume fraction. The same algorithm was applied for the numerical simulation of stress wave propagation in both models. One of the issues of interest in the use of layered or graded structures is the effect of the layering or gradation on maxima of stresses and their distributions. Therefore, we consider the normal stress distribution along the centerline of the plate where maximal values of stress are expected.

The normal stress distribution along the centerline for the multilayered model with averaged properties within layers is shown in Fig. 2. Since the results are given at the same instant of time, the difference in the position of normal stress profiles characterizes the corresponding difference in the speed of the stress wave in each structure. The maximal tensile stresses are considerably higher for the layered case. The huge tensile stresses in the layered material dictates its rejection in order to provide the improvement of the integrity of the structure under dynamic loading. The situation is changed for the model of randomly embedded particles (Fig. 3). The

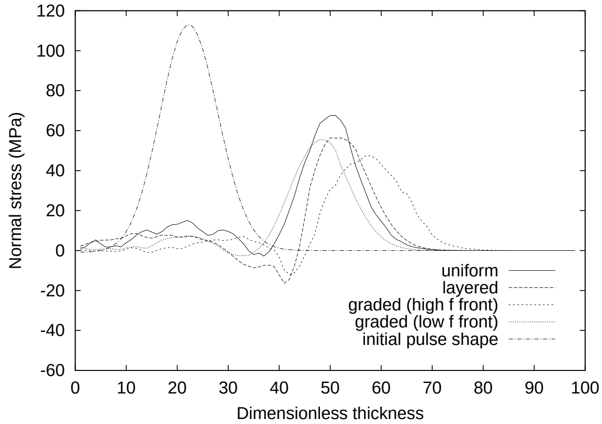

Figure 3: Normal stress distribution along the centerline in metal-ceramic composite with ceramic reinforcement at $3\ \mu s$ (model with randomly embedded particles).

amplitude of the tensile stresses decreases significantly, especially for uniform and low f front grading. It seems that the latter structure is the best choice for the damping both the compressional and tensile stresses. This result is not so obvious in the case of the multilayered model with averaged properties within layers.

Final remarks

Theoretical prediction of dynamic behaviour of FGMs depends on how well their properties are modelled in computer simulations. In this paper, we applied the composite wave-propagation algorithm [8], [9] to compare the models of discrete layers with averaging the material properties and of randomly embedded ceramic particles in a metal matrix. The results of performed numerical simulations of stress wave propagation in FGMs show a significant difference between characteristics of wave fields in the distinct models, though the overall wavefronts picture seems similar in both cases. This means that a model of FGM without averaging of material properties can give a more detailed information about the dynamic behavior of a chosen structure, which may be used in its optimization for a particular situation.

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