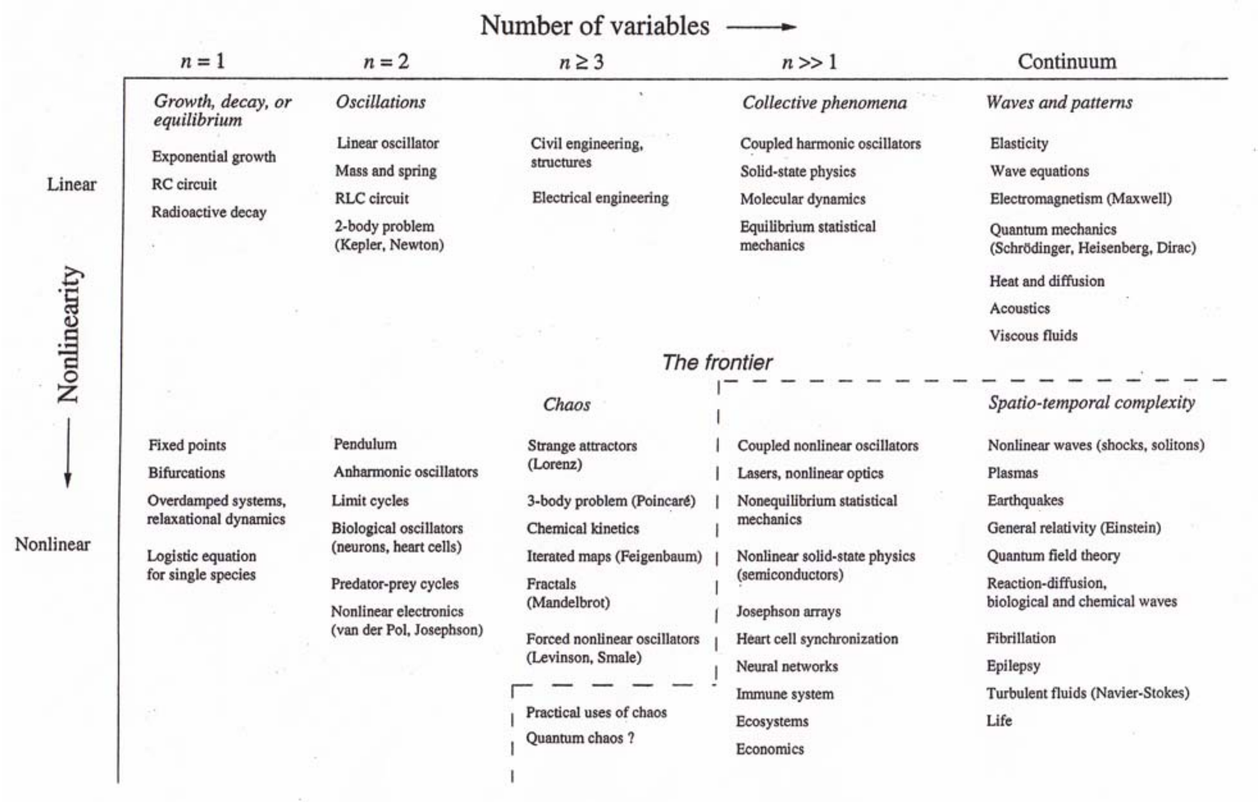
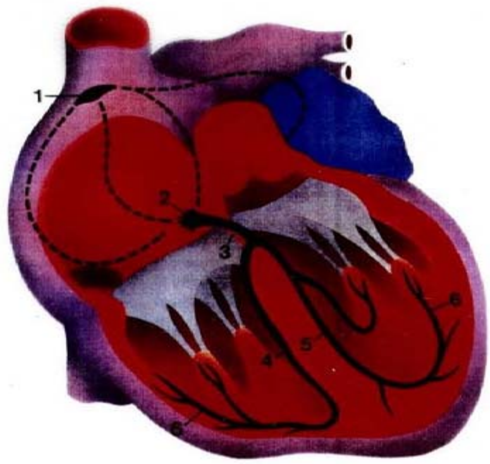
# 9. RAKENDUSED



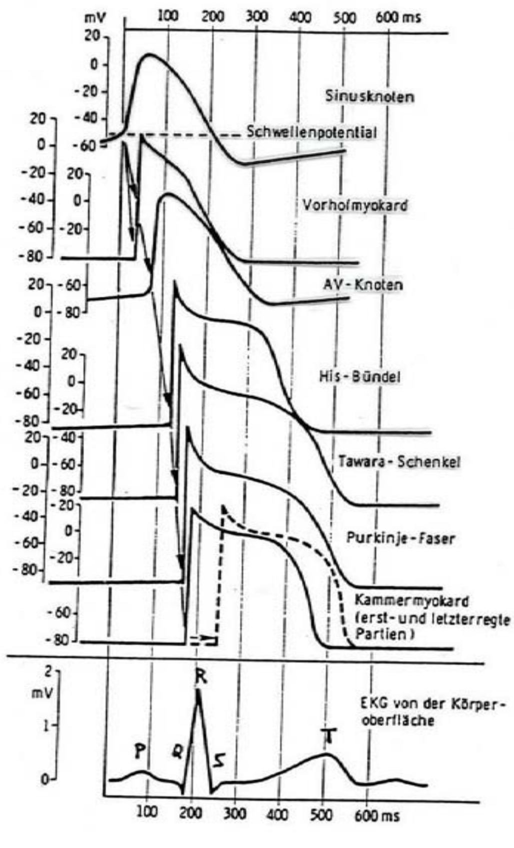
# Südame erutussüsteem

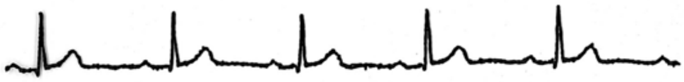



1. SA-node
2. AV-node
3. His bundle

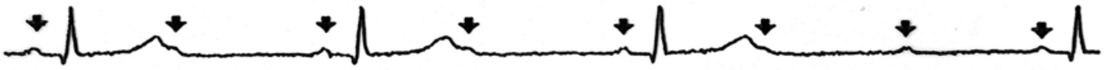
4.–5. Bundle branches

6. Purkinje fibres



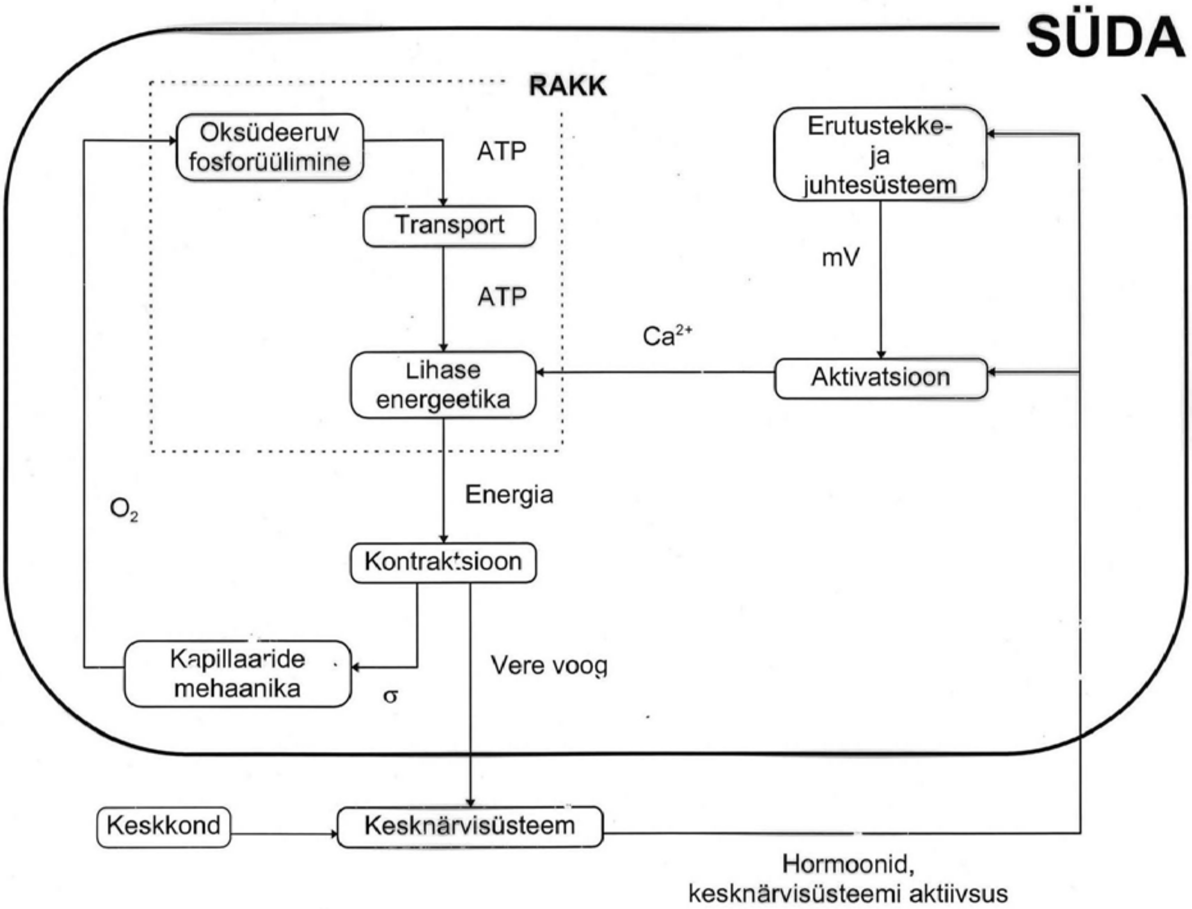


1. Normally beating heart



2. Mobitz2 — second order A-V block with constant $PQ$-time

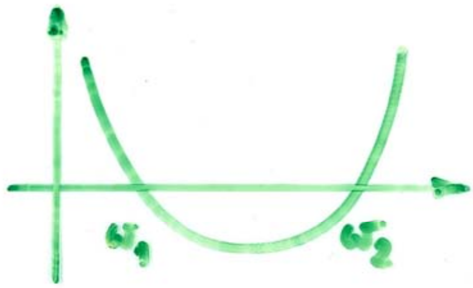
3. Wenckebach — second order AV block with progressively increasing $PQ$-time



Derivation of the NPE:
Engelbrecht J. *An Introduction to Asymmetric Solitary Waves.* Longman. London & Harlow, 1991.

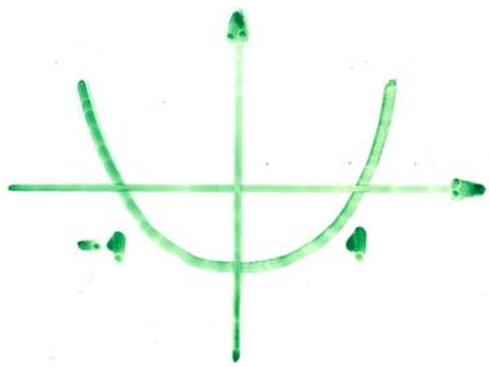
Nerve pulse equation:

$$\ddot{w} + \varepsilon_w (w - w_1)(w - w_2)\dot{w} + w = 0, \;\; w_1 w_2 > 0.$$



Van der Pol equation:

$$\ddot{u} + \varepsilon_u (u^2 - 1)\dot{u} + u = 0.$$



Our case:

$$\ddot{w} + \varepsilon (w - w_1)(w - w_2)\dot{w} + w = I \sum_{k=-\infty}^{\infty} \delta\,(t - kT)\,,$$

where

$$\varepsilon = 3.265, \;\; w_1 = 0.5, \;\; w_2 = 1.9, \;\; T = \frac{2\pi}{\omega}.$$

NPE driven by Dirac delta spikes:

$$\ddot{w} + \varepsilon(w - w_1)(w - w_2)\dot{w} + w = I \sum_{k=-\infty}^{\infty} \delta\left(t - \frac{2k\pi}{\omega}\right)$$

R.h.s. given as a Fourier series:

$$I \sum_{k=-\infty}^{\infty} \delta\left(t - \frac{2k\pi}{\omega}\right) = I_0 + 2I_0 \sum_{n=1}^{\infty} \cos n\omega t,$$

where

$$I_0 = \frac{I\omega}{2\pi}$$

The NPE after a transformation and truncation of r.h.s.

$$\ddot{v} + \varepsilon(v - v_1)(v - v_2)\dot{v} + v = 2I_0 \cos \omega t,$$

where

$$v = w - I_0,\ v_1 = w_1 - I_0,\ v_2 = w_2 - I_0.$$

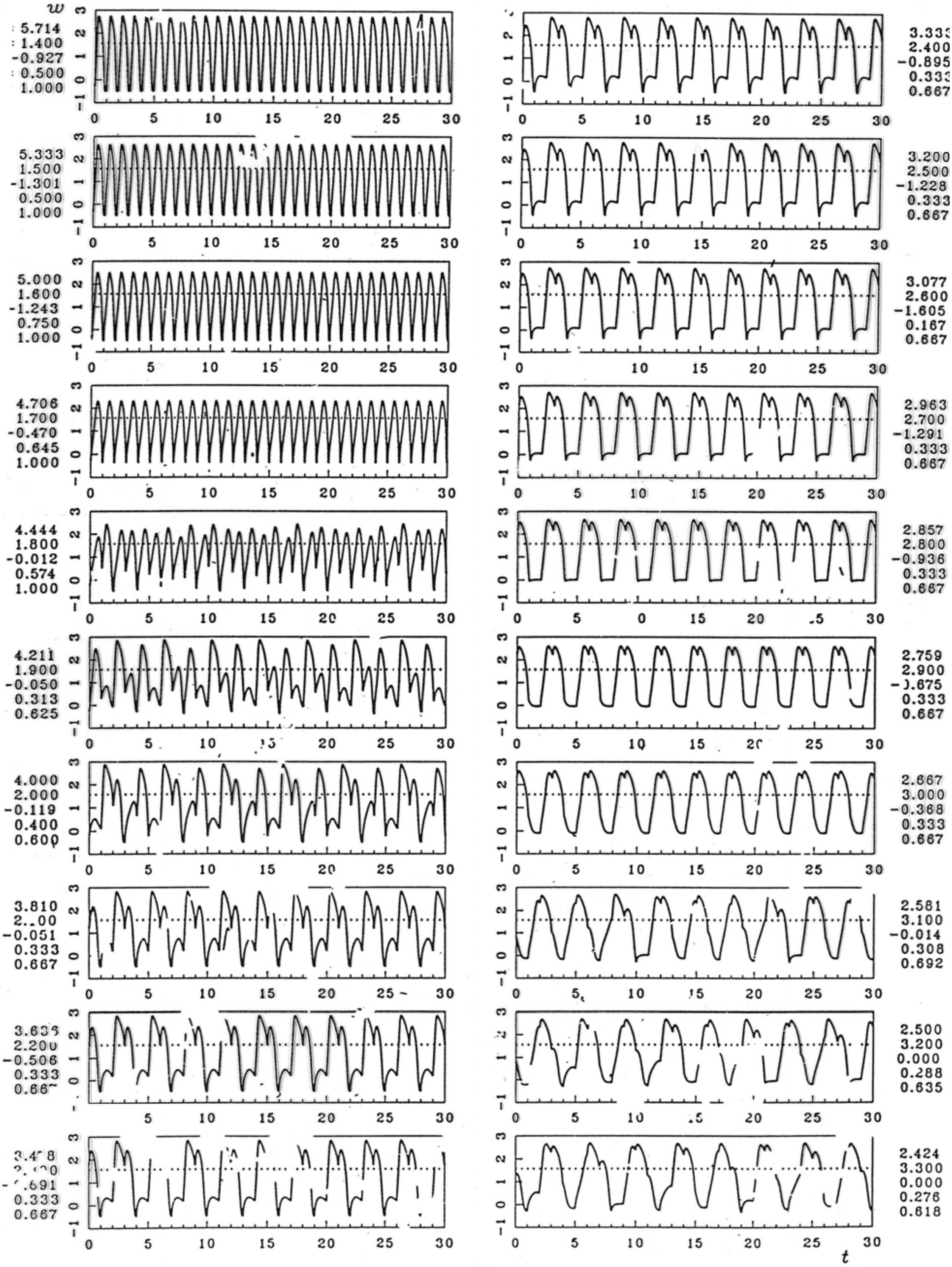
For constant $\omega$ and increasing $I$ we expect

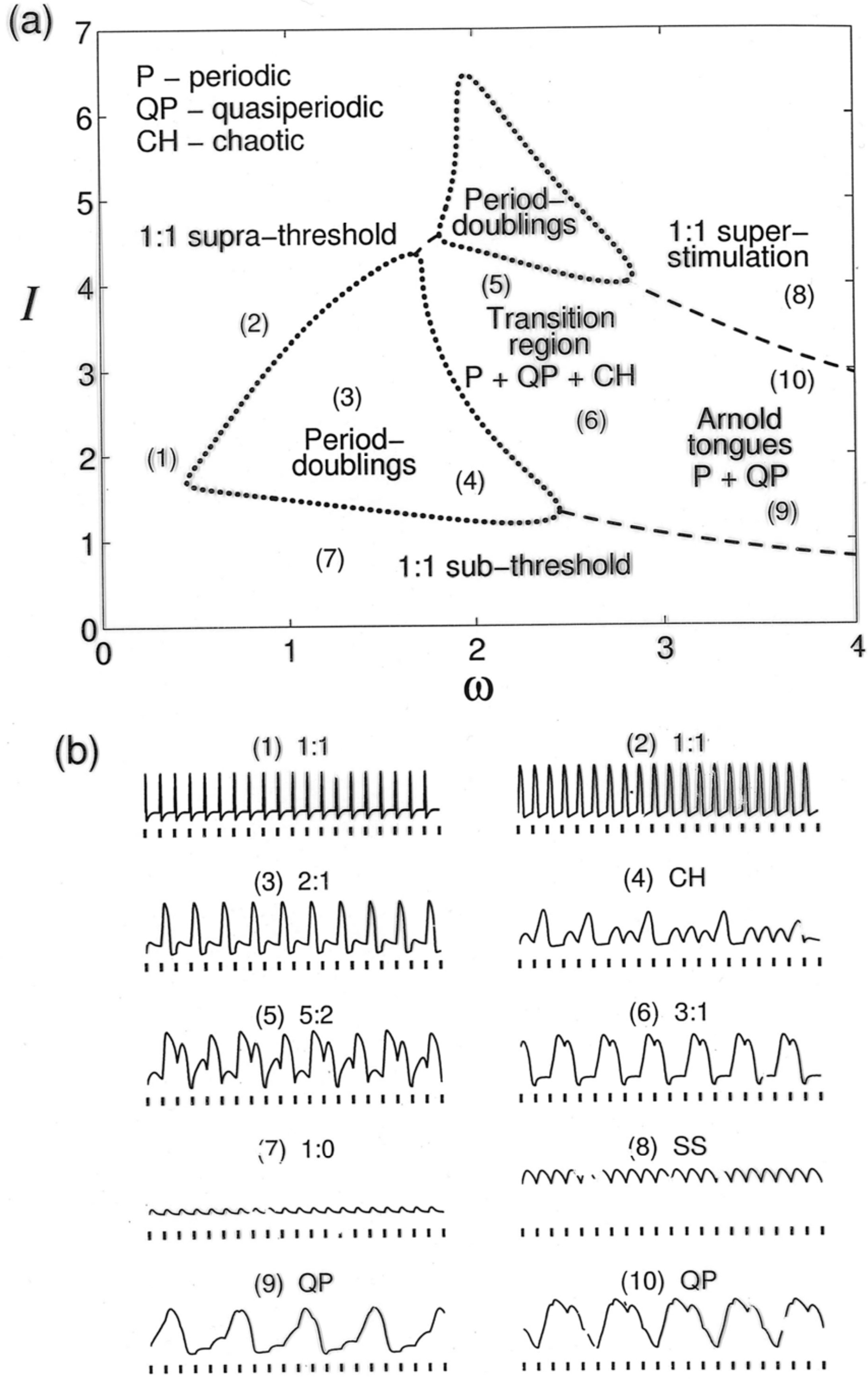
(a) $I_0 = w_1$, Neimark-Sacker;

(b) $I_0 = \dfrac{w_1 + w_2}{2}$, symmetric van der Pol equation — no inversion-symmetric solutions of even period (inversion-symmetry: $F(\ddot{v}, \dot{v}, v, t) = -F(-\ddot{v}, -\dot{v}, -v, t + T/2)$);

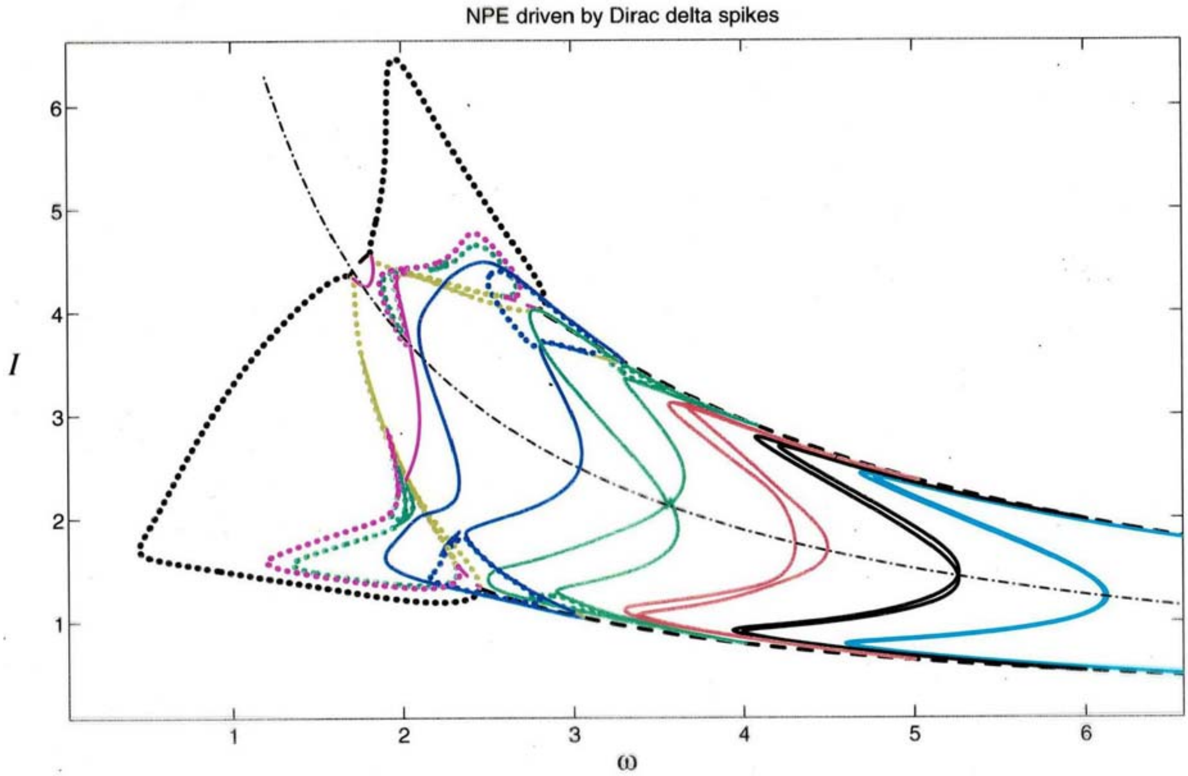
(c) $I_0 = w_2$, inverse Neimark-Sacker.

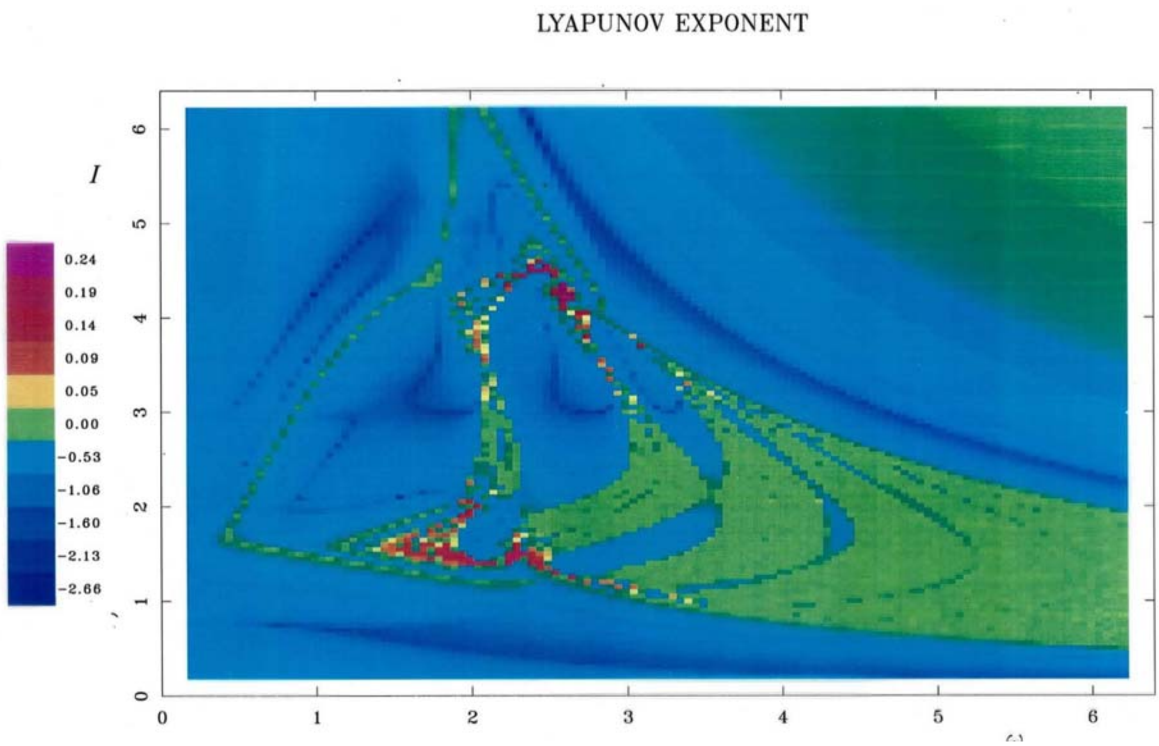
$I\,\omega = 8.00$

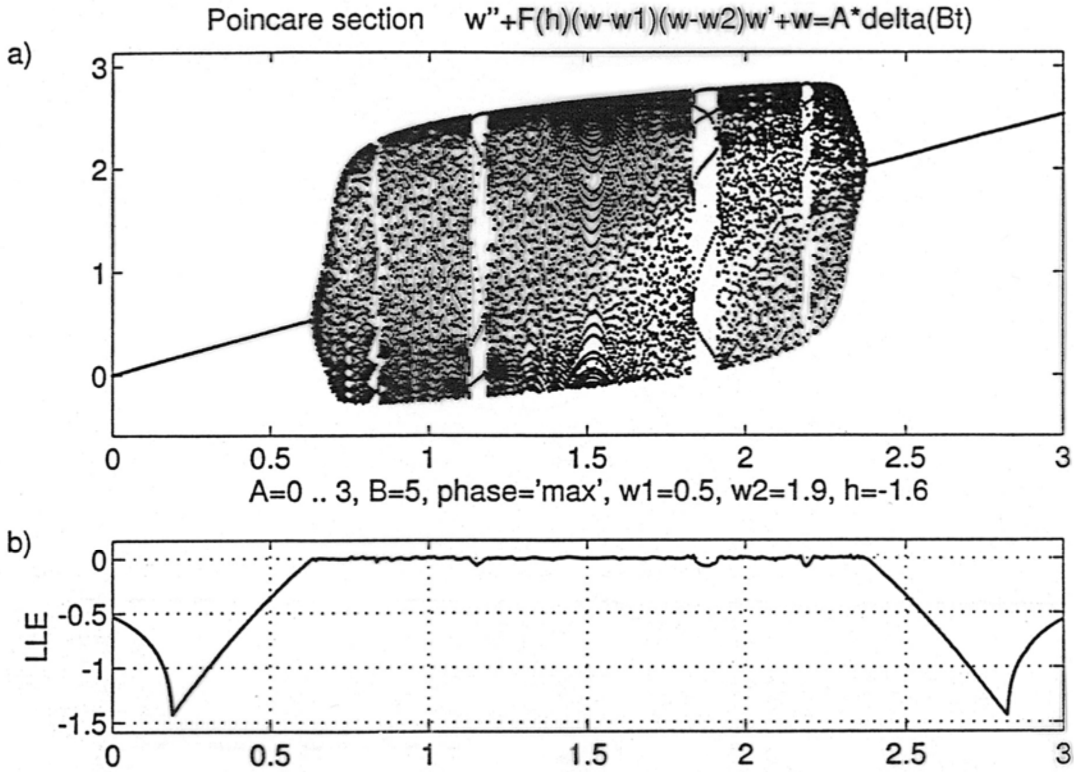

w
5.714
1.400
-0.927
0.500
1.000
5.333
1.500
-1.301
0.500
1.000
5.000
1.600
-1.243
0.750
1.000
4.706
1.700
-0.470
0.645
1.000
4.444
1.800
-0.012
0.574
1.000
4.211
1.900
-0.050
0.313
0.625
4.000
2.000
-0.119
0.400
0.600
3.810
-0.051
0.333
0.667
2.200
-0.506
0.333
0.333
0.667
2.400
-0.895
0.667
3.200
2.500
-1.228
0.333
0.667
3.077
2.600
-1.605
0.167
0.667
2.963
2.700
-1.291
0.333
0.667
2.857
2.800
-0.936
0.333
0.667
2.759
2.900
0.333
0.667
2.667
3.000
-0.368
0.333
0.667
2.581
3.100
-0.014
0.308
0.692
2.500
3.200
0.000
0.288
0.635
2.424
3.300
0.000
0.276
0.618
t

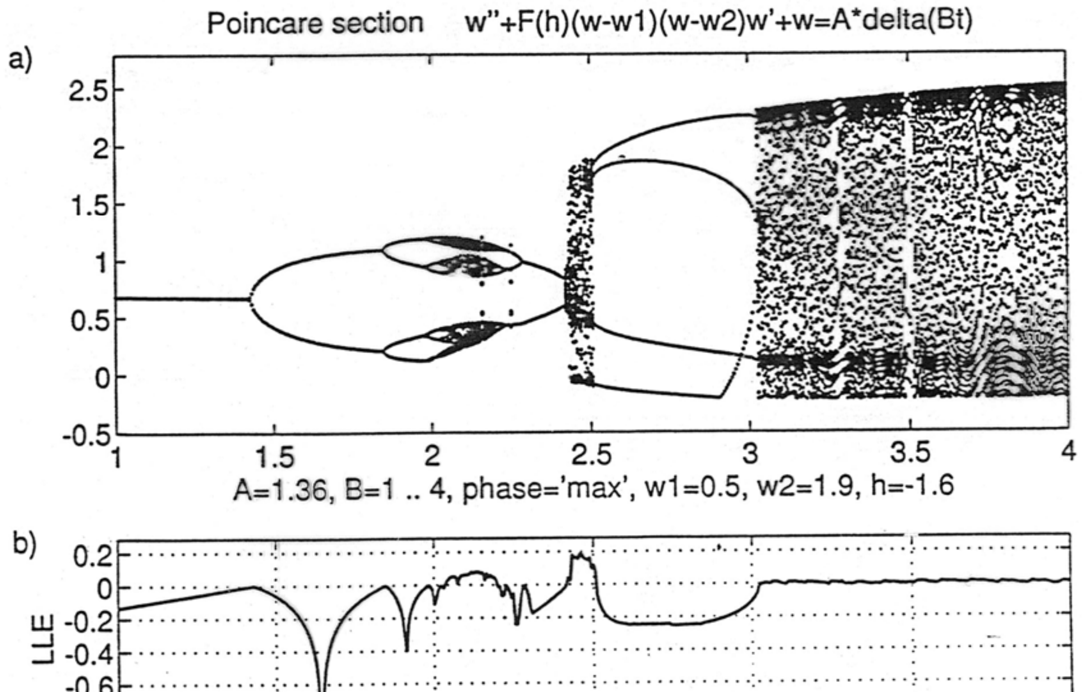


(a)
P – periodic
QP – quasiperiodic
CH – chaotic
1:1 supra–threshold
Period–doublings
1:1 super–stimulation
(5)
Transition region
P + QP + CH
(8)
(2)
(3)
(10)
(6)
Arnold tongues
P + QP
(1)
Period–doublings
(4)
(9)
(7)
1:1 sub–threshold
I
ω
(b)
(1) 1:1
(2) 1:1
(3) 2:1
(4) CH
(5) 5:2
(6) 3:1
(7) 1:0
(8) SS
(9) QP
(10) QP

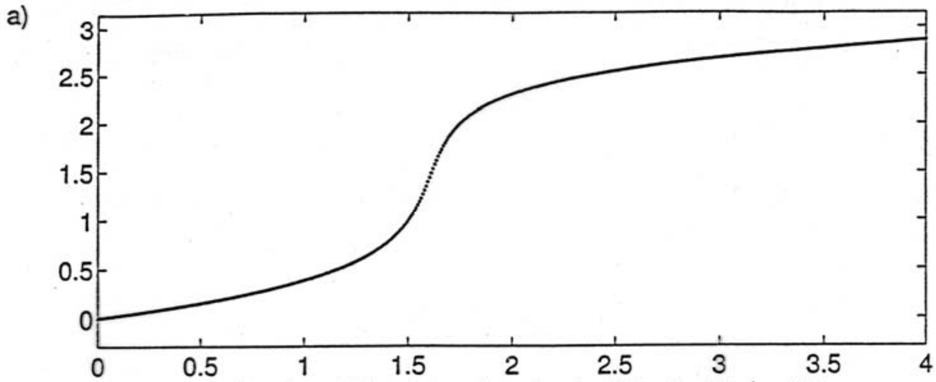

NPE driven by Dirac delta spikes
I
ω
1
2
3
4
5
6


LYAPUNOV EXPONENT
I
0.24
0.19
0.14
0.09
0.05
0.00
-0.53
-1.06
-1.60
-2.13
-2.66
0
1
2
3
4
5
6

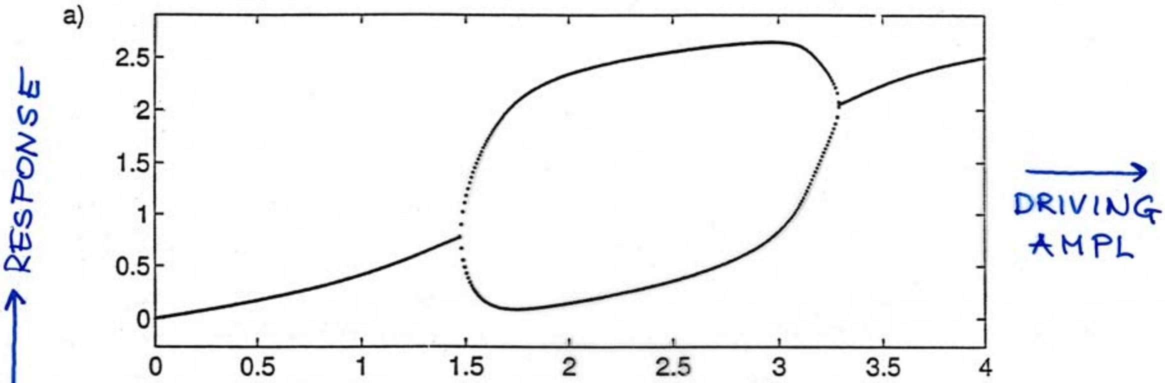

Poincare section w''+F(h)(w-w1)(w-w2)w'+w=A*delta(Bt)
a)
A=0 .. 3, B=5, phase='max', w1=0.5, w2=1.9, h=-1.6
b)
LLE


Poincare section w''+F(h)(w-w1)(w-w2)w'+w=A*delta(Bt)
a)
A=1.36, B=1 .. 4, phase='max', w1=0.5, w2=1.9, h=-1.6
b)
LLE

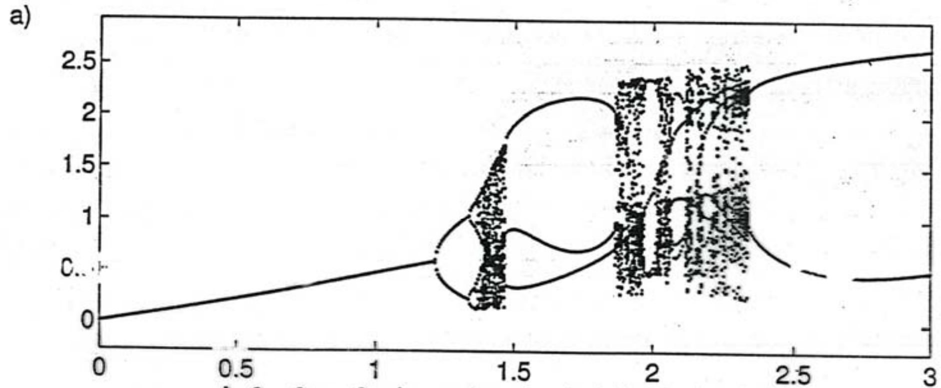
Poincare section $w''+F(h)(w-w1)(w-w2)w'+w=A*delta(\omega t)$



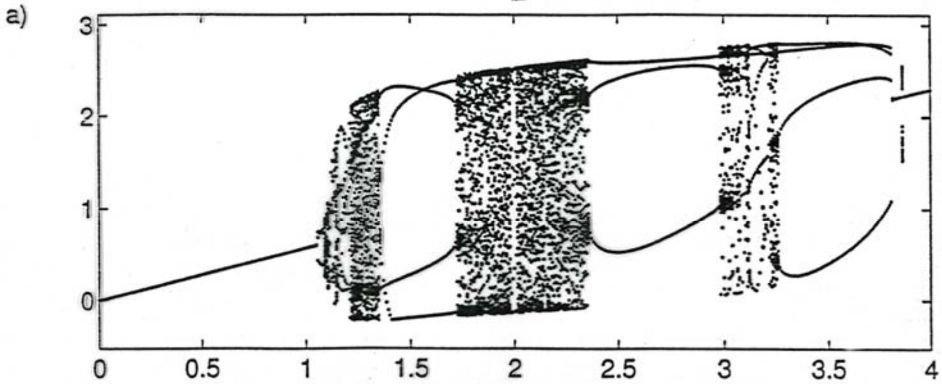
A=0 .. 4, ω=0.2, phase='max', w1=0.5, w2=1.9, h=-1.6



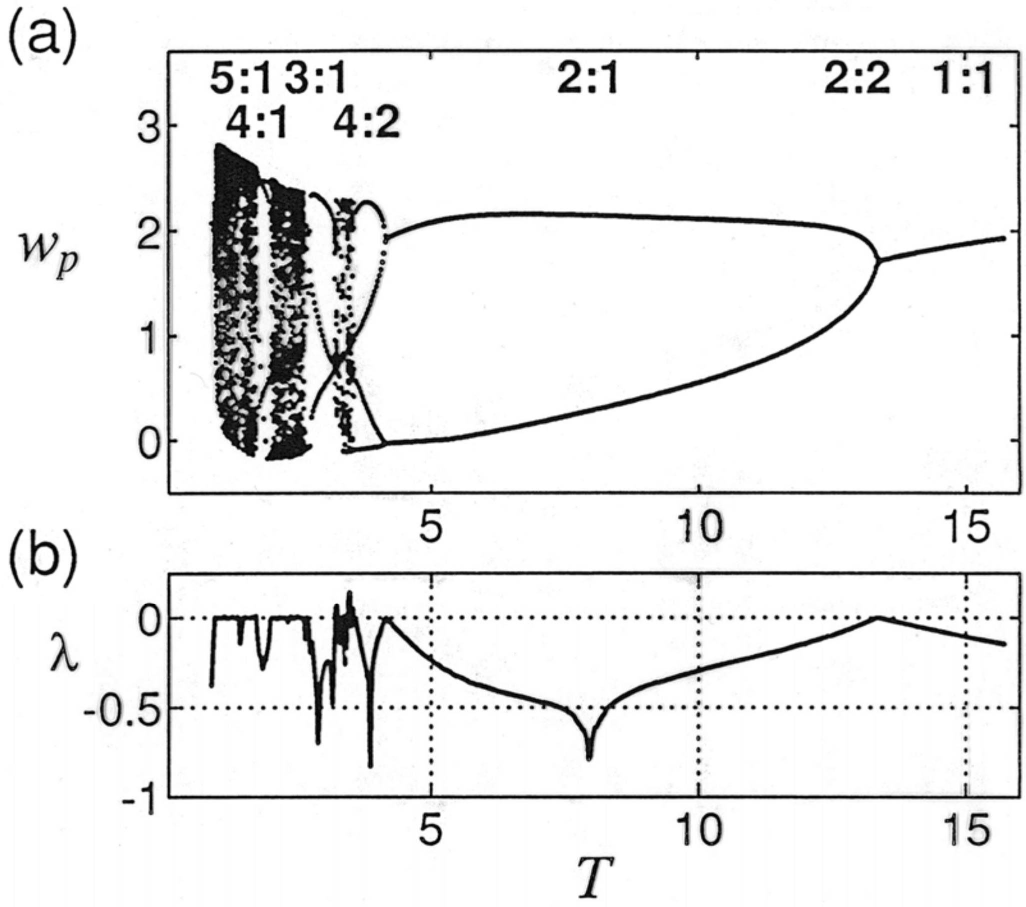

A=0 .. 4, ω=1, phase='max', w1=0.5, w2=1.9, h=-1.6



A=0 .. 3, ω=2, phase='max', w1=0.5, w2=1.9, h=-1.6



A=0 .. 4, B=3, phase='max', w1=0.5, w2=1.9, h=-1.6




1 : 1 → 2 : 2, 2 : 1 → 4 : 2 → chaos → 3 : 1 →

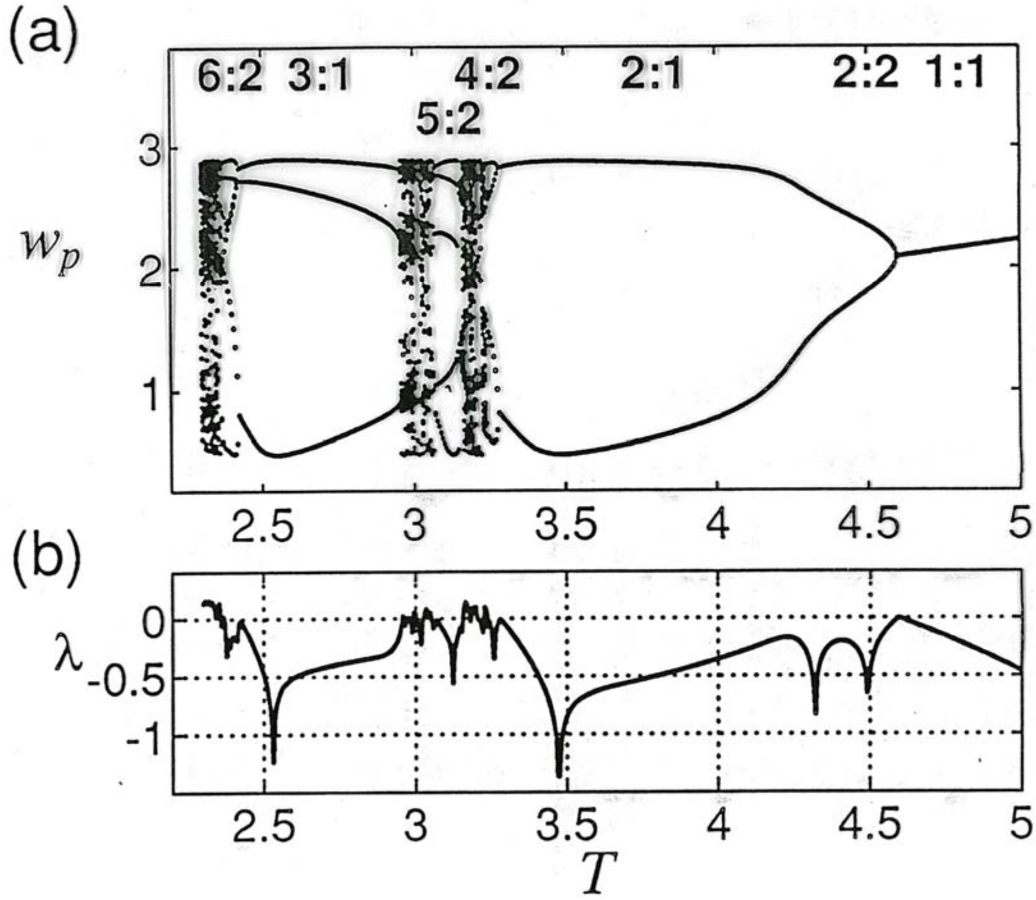
6 : 2, quasiperiodic, 4 : 1, quasiperiodic → etc

Difference: 4 : 1 becomes quasiperiodic

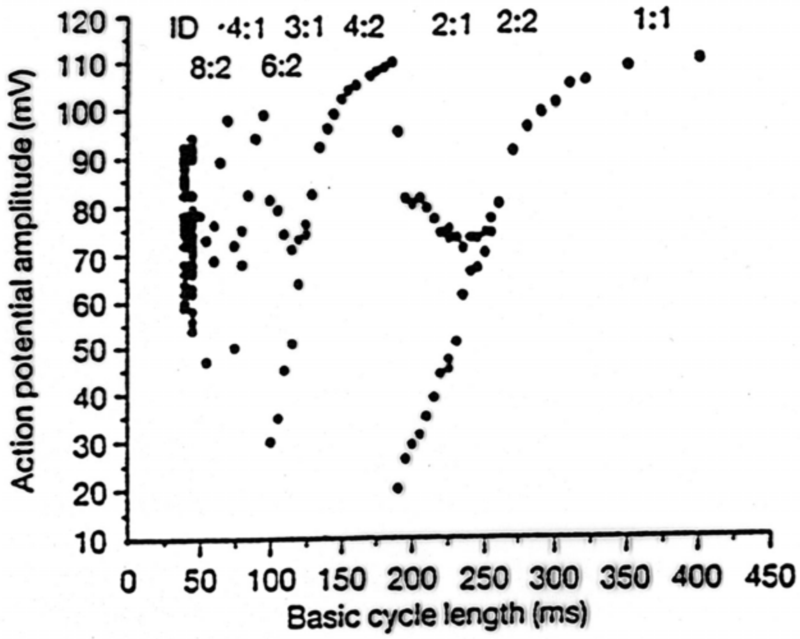
via saddle-node bifurcation

Chialvo et al.: period doubling

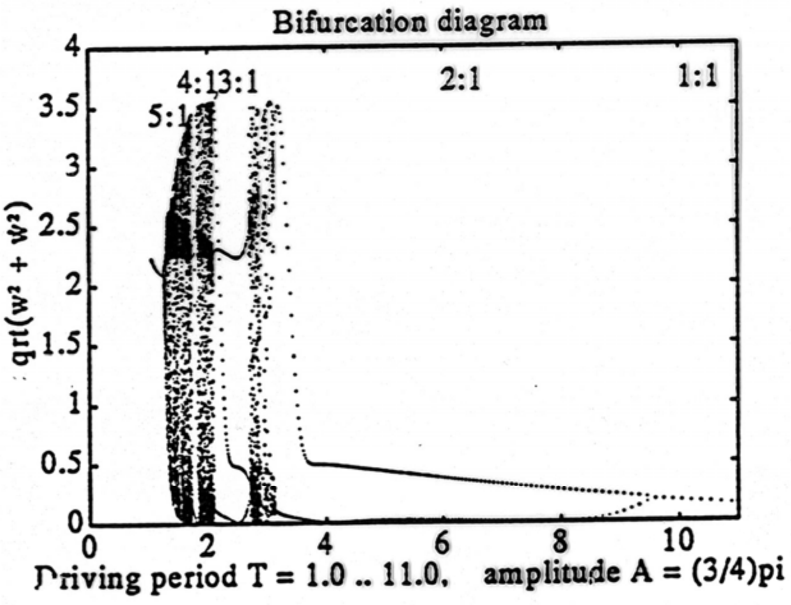
Here, NPE describes earlier development



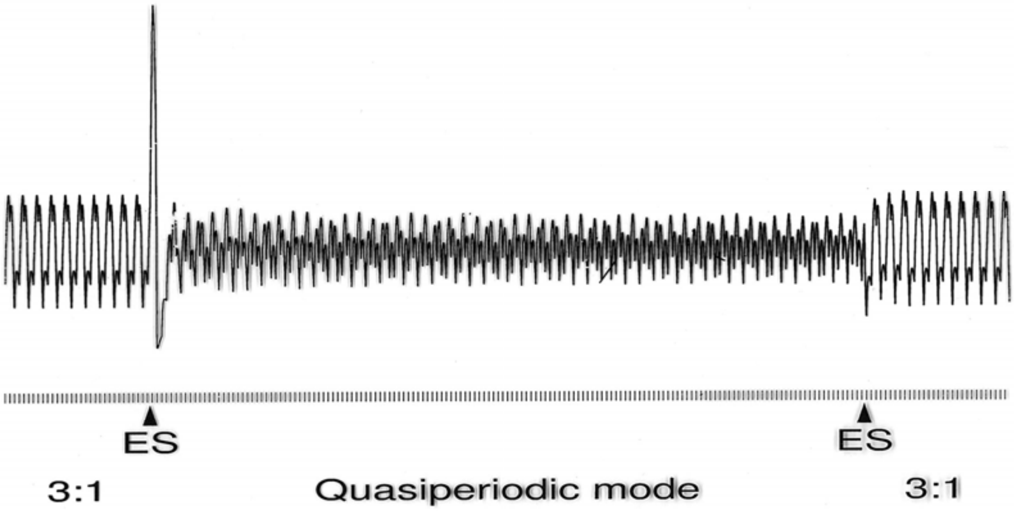

$A \sum \delta(t - nT), \quad A = 4.0 \quad (2.5 \times \text{threshold})$



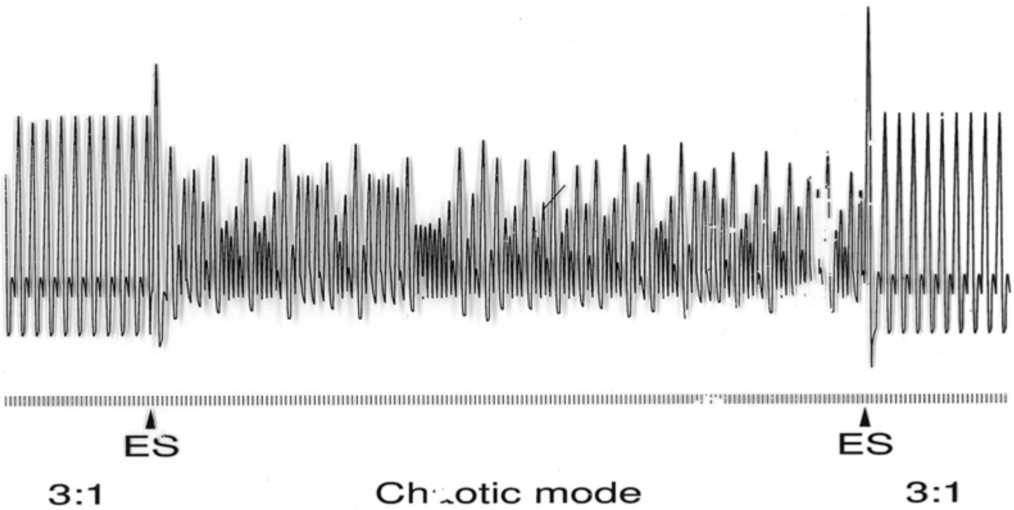

Experimental action potential from a driven sheep cardiac Purkinje fibre.



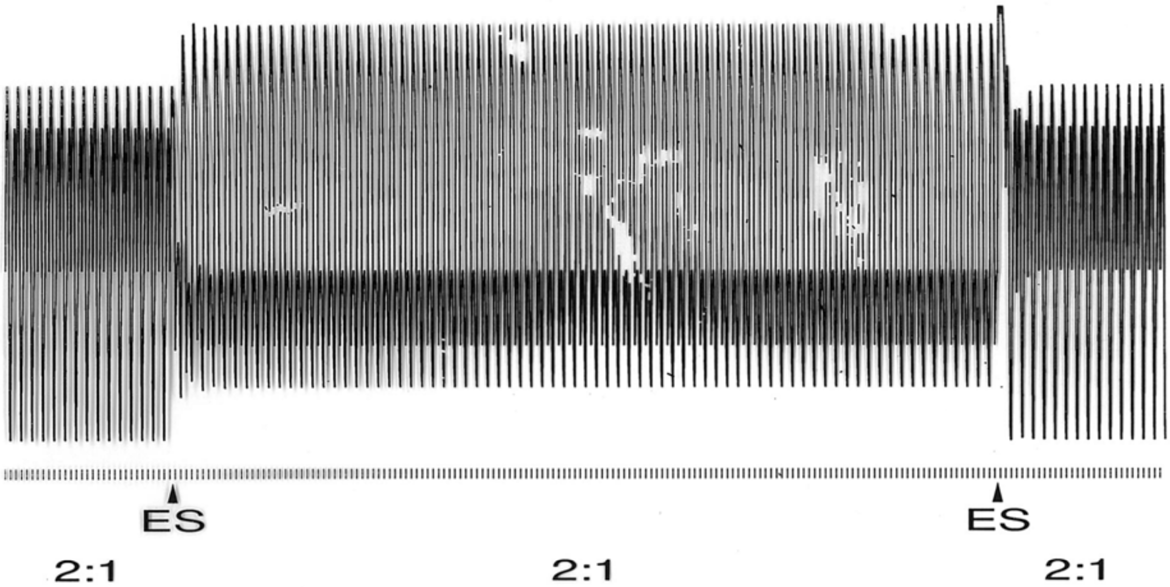

Action potential calculated from the driven nerve pulse equation.



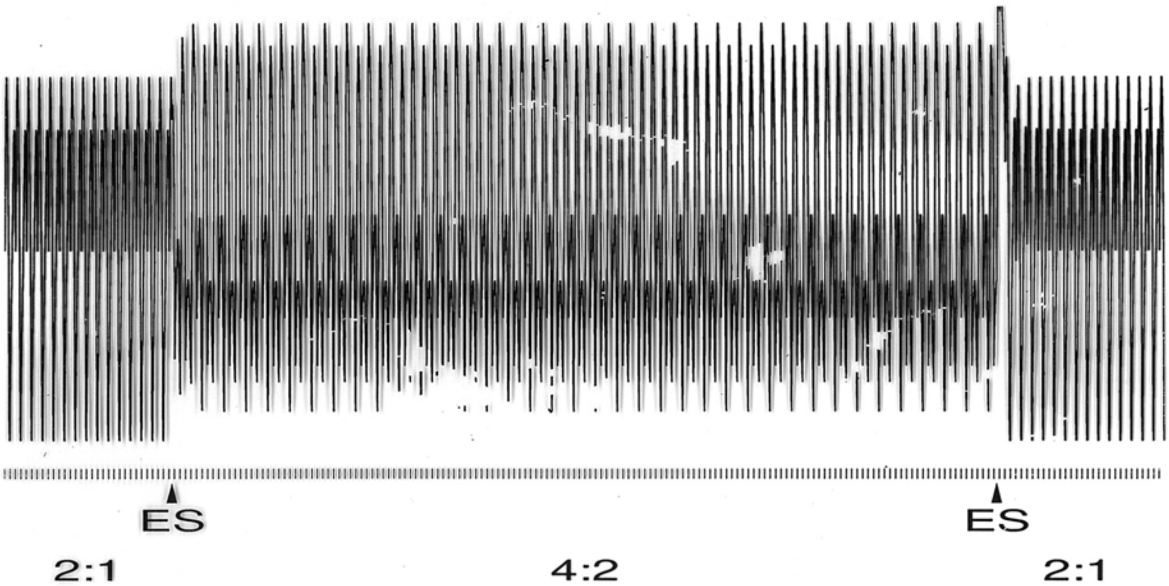

Saddle-node ← Bistability → Quasiperiodic blue sky catastrophe




Saddle-node ← Bistability → Chaotic blue sky catastrophe




Saddle-node ⟵ Bistability ⟶ Saddle-node




Saddle-node ⟵ Bistability ⟶ Saddle-node