6. FÜÜSIKALISTE SÜSTEEMIDE MODELLEERIMINE

Põhiidee: on olemas reeglid, mis fikseerivad
põhimõtted jättes piisava vabaduse

3 astet:

1. Klassikaline dünaamika 1687

Newton, Hamilton, ...
isoleeritud osakesed, mehaanika reeglid
kriitika – ainult vastasmõju, interaktsioon?
siiski keerulised tulemused (3 keha probleem)

2. Termodünaamika – 1850 2. seadus

isoleeritud süsteemi entroopia saab ainult kasvada
kuni süsteem jõuab termodünaamilise tasakaaluni

lisandus pöördumatus (irreversibility)

kõik pöördumatud protsessid lisavad entroopiat

3. Dissipatiivsete struktuuride (hajusstruktuuride) teooria

koherentsed arenevad struktuurid

avatud süsteem ammutab energiat ümbritsevast süsteemist ja ekspordib entroopiat

I. Prigogine

Erinevus 3 tasandi vahel

1. klassikaline dünaamika

   aja sümmeetria rikkumine

   kausaalsuse printsiip

2. termodünaamika

   aja ja ruumi
   sümmeetria rikkumine

   printsiip: fluktuatsioonis
   tekib kord

3. dissipatiivsete struktuuride teooria

   vt.

   E. Jantsch The Self–Organizing Universe
   Pergamon, Oxford et al 1980

   I. Prigogine, J. Stengers."Order out of Chaos",
   Heinemann, London, 1984.

6.1 PIDEV KESKKOND

Põhi- printsiibid	Alg– oletused		Aeg ja ruum
	Jäävus– seadused		Põhi– aksioomid
Teooria	Lisa – postulaadid		Alg– tingimused, oleku– võrrandite struktuur
		Suletud süsteem	
Mudel	Lisa– oletused		Muutujate iseloom
		Oleku– võrrandite lõplik kuju	Oleku– võrrandite või vaba energia funktsiooni aproksi– meerimine
		Mate– maatiline mudel	

6.1. PIDEV KESKKOND

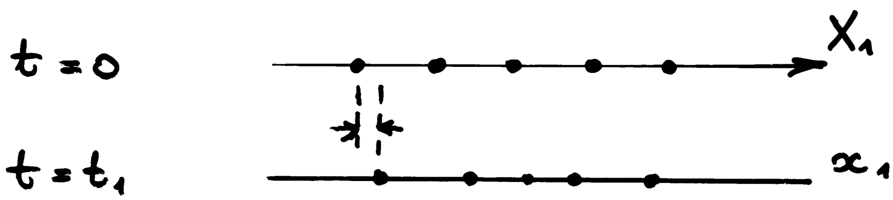
ALGOLETUSED

(i) aeg – absoluutne aeg t;

(ii) ruum – kolmemõõtmeline (3D) Eukleidese ruum;

(iii) mateeria makroskoopiline hulk – liikuvate osakeste pidev hulk, mis meelevaldsel ajahetkel $t = const$ on ruumi osas B, millel on pind A ja maht $V \neq 0$ ning positiivne mass M.

Liikumine



$$3D: x_k = x_k(X_1, X_2, X_3;\ t)$$

$$X_k = X_k(x_1, x_2, x_3;\ t)$$

$$k = 1,\ 2,\ 3$$

üheselt määratud teisendused

Tähistus: $$x_{i,k} = \frac{\partial x_i}{\partial X_k}$$

JÄÄVUSSEADUSED (lihtsustatud)

1. Massi jäävuse seadus

$$\int_V \rho_0 \, dV = \int_v \rho \, dv$$ integraalkuju

ρ_0, ρ tihedus
V, v maht

$$\frac{\partial \rho}{\partial t} + (\rho v_i)_{,i} = 0$$ diferentsiaalkuju

v_i – kiiruse komponendid

2. Impulsi jäävuse seadus (Newtoni II seadus)

$$T_{ij,i} + \rho_0(f_j - A_j) = 0$$

T_{ij} – pingetensor
f_j – mahujõud
A_j – kiirendusvektori komponendid

3. Kineetilise momendi jäävus

$T_{ij} = T_{ji}$ ehk nihkepingete paarilisus

4. Energia jäävus

$$\rho \frac{d\varepsilon}{dt} = T_{ij}\dot{E}_{ij} + Q_{K,K} + \rho_0 h$$

E_{ij} – deformatsioonitensor

$\dot{E}_{ij}$ – deformatsiooni kiirus

Q_k – soojushulk

h – lisa soojusallikas

Lisa: entroopia võrratus

$$T_{ij}\dot{E}_{ij} + \frac{1}{T} Q_k T_{,k} - \rho_0 \dot{F} - \rho_0 \dot{T} S \geq 0$$

T – temperatuur

S – entroopia

F – nn Helmholtzi vaba energia

LISAPOSTULAADID

1) Postulaat algolukorrast ajal $t = 0$:

$$\dot{U}_k = 0, \quad T_{kl} = 0, \quad Q_k = 0$$

(U_k – siire $\rightarrow$ ($x_k - X_k$))

vaba energia	$F = F_0, \quad F_0 = const.$
temperatuur	$T = T_0, \quad T_0 = const.$
vaba energia	$F = E - TS$

2) postulaat olekuvõrranditest
(mis millest sõltub)

(i) $F = F(K_1, K_2, K_3; T)$

K_i – deformatsioonitensori invariandid

T – temperatuur

(ii)

pingetensor	=	pöörduv osa	+	pöördumatu osa
		(elastne)		(dissipatiivne)

$$T^e_{kl} = \rho_0 \frac{\partial F}{\partial E_{kl}}$$ – pöörduv osa

$$T^d_{kl} = T^d_{kl}(\dot{E}_{kl})$$ – pöördumatu osa

(iii) soojusjuhtivus (Fourier' seadus)

SULETUD SÜSTEEM

$$T_{ij,i} + \rho_0 (f_i - \ddot{U}_j) = 0$$ liikumine

$$\rho_0 \, T \left(\frac{\partial^2 F}{\partial T \, \partial E_{kl}} \dot{E}_{kl} + \frac{\partial^2 F}{\partial T^2} \dot{T} \right) + T^d_{kl} \, \dot{E}_{kl} +$$

$$+ Q_{k,k} + \rho_0 h = 0$$ energia

$$Q_i = k_{ij} T_{,j}$$ Fourier' seadus

$$T_{kl} = T^e_{kl} + T^d_{kl}$$ lisapostulaat

$$F = F(K_1, K_2, K_3; \; T)$$ lisapostulaat

$$T^e_{kl} = \rho_0 \frac{\partial F}{\partial E_{kl}}$$ lisapostulaat

$$T^d_{kl} = a_{01} \cdot \dot{E}_{kl}$$ lisapostulaat (aproksimatsioon)

$$E_{ij} = \frac{1}{2} \left(U_{i,j} + U_{j,i} + U_{m,j} \cdot U_{m,i} \right)$$

deformatsioonitensor väljendatud siirde (paigutise) tuletiste kaudu

LISAOLETUSED

$U_{i,j} << 1$ väikesed deformatsioonid

$|T - T_0| \; T_0^{-1} << 1$ väike temperatuurimuutus

(NB! ohtlik oletus!)

$$F(E_{kl}, T) = F(0, T_0) +$$
$$+ \frac{\partial F(0,T_0)}{\partial E_{kl}} E_{kl} + \frac{\partial F(0,T_0)}{\partial T} T +$$
$$+ \frac{1}{2} \frac{\partial^2 F(0,T_0)}{\partial E_{kl}\, \partial E_{ij}} E_{kl}\, E_{ij} +$$
$$+ \frac{1}{2} \frac{\partial^2 F(0,T_0)}{\partial E_{kl}\, \partial T} E_{kl} (T - T_0) +$$
$$+ \frac{1}{2} \frac{\partial^2 F(0,T_0)}{\partial T^2} (T - T_0)^2 +$$
$$\cdots$$

Harilik rittaarendus tasakaaluoleku $(0, \; T_0)$ suhtes, nn. MacLaureni rida

Tuletised on konstandid!

$$\rho_0 F = \frac{1}{2} \lambda K_1^2 + \mu K_2 + \quad \text{(kõrgemat järku liikmed) +}$$
$$+ \kappa K_1 (T - T_0) + \kappa_1 (T - T_0)^2$$

$$K_1 = E_{ii} \qquad K_2 = E_{ij} E_{ij} \qquad \left. \begin{matrix} \lambda, \; \mu \\ \kappa, \; \kappa_1 \end{matrix} \right\} \text{konstandid}$$

MATEMAATILINE MUDEL

$$\rho_0 \ddot{U}_n - C_{nklm} U_{k,lm} - D_{nklm} \dot{U}_{k,lm} = B_{nk} T_{,k}$$

$$g\dot{T} + H_{mn} \dot{U}_{m,n} = Q_{k,k}$$

$$Q_n = kT_{,n}$$

$\rho_0, \quad C_{nklm}, \quad D_{nklm}, \quad B_{nk}, \quad g, \quad H_{mn}, \quad k$

konstandid lihtsaimal juhul

kuid võivad olla funktsioonid

$$\left(X_i, \ t, \ U_i, \ T\right)$$

OLULINE:

Jäävusseadused + olekuvõrrandid

määratakse eksperimendist

ELEKTRODÜNAAMIKA

E – elektriväli

J – vool

P – polarisatsioon

H – magnetväli

M – magneetumine

$\vec{v}$ – kiirus

q_e – laeng

D = E + P

B = H + M

$$M = M + \frac{1}{c} v \times P$$

$$H = H - \frac{1}{c} v \times D$$

$$B = B - \frac{1}{c} v \times E$$

$$\Im = J - q_e v$$

$$E = E + \frac{1}{c} v \times B$$

Maxwelli võrrandid

$$\nabla \cdot D - q_e = 0$$

$$\nabla \times E + \frac{1}{c}\frac{\partial B}{\partial t} = 0$$

$$\nabla \cdot B = 0$$

$$\nabla \times H - \frac{1}{c}\frac{\partial D}{\partial t} = \frac{1}{c} J$$

$$\frac{\partial q_e}{\partial t} + \nabla \cdot J = 0$$

Termomehaanikalised jäävusseadused

mass $$\dot{\rho} + \rho \nabla \cdot \mathrm{v} = 0$$

impulss $$t_{kl,k} + \rho\left(f_l - \dot{v}_l\right) + F_l^E = 0$$

moment $$t_{[kl]} = E_{[k}P_{l\,]} + B_{[k}M_{l\,]}$$

energia $$\rho\left(\dot{\psi} + \dot{\theta}\,\eta + \theta\,\dot{\eta}\right) - t_{kl}\,v_{l,k} - \nabla \cdot q - \rho\,h + \\ + P_k\,\dot{E}_k + M_k\,\dot{B}_k - \mathfrak{I}_k\,E_k = 0$$

entroopia $$\rho\,\gamma \equiv \rho\,\dot{\eta} - \nabla \cdot \left(q/\theta\right) - \left(\rho\,h/\theta\right) \geq 0$$

6.2 MEHAANIKA

JÄIGA KEHA MEHAANIKA

Jäävusseadused:

1. Energia on jääv

$$\underset{}{E} = \underbrace{U}_{\text{potentsiaalne}} + \underbrace{K}_{\text{kineetiline}}$$

energia

2. Impulss **P** on jääv

3. Masskeskpunkti kiirus **V** on jääv

koordinaat $$R = \sum_i m_i \; r_i \; / \sum_i m_i$$

$$P = \sum_i m_i \; v_i = M \frac{dR}{dt} = M \; V$$

4. Kineetiline moment **L** on jääv

10 jäävusseadust

energia	1	
impulss	3	komponenti
kiirus	3	komponenti
kineetiline moment	3	komponenti
	10	

Jäiga ja deformeeruva keha võrdlus

	JÄIK	DEFORMEERUV
Massi jäävus	Tagatud definitsiooni kohaselt	$\frac{\partial\rho}{\partial t} + (\rho v_i)_{,i} = 0$
Liikumis-hulga jäävus	$\frac{d\vec{k}}{dt} = \vec{F}$	$T_{ij,i} - \rho a_i = 0$
Kineetilise momendi jäävus	$\frac{d\vec{h}_o}{dt} = \vec{M}$	$T_{ij} = T_{ji}$
Energia	$\frac{dK}{dt} = \frac{dW^e}{dt} + \frac{dW^i}{dt}$	$\rho\frac{d\varepsilon}{dt} = \frac{1}{2}(v_{r,p} + v_{p,r})T_{pr} + Q_{p,p}$
Olekuvõrrand	Keha on jäik	$T_{ij} = T_{ij}(u_{m,n})$

Descartes'i kooordinadid

W^e välisjõudude töö

W^i sisejõudude töö

K kineetiline energia

6.3 ÜLDISED IDEED

Hamiltoni funktsioon

$$E = T + U = H$$

kineetiline energia potentsiaalne energia Lagrange'i funktsioon

$$L = \sum_i p_i \ \dot{q}_i - U$$

q_i – üldistatud muutuja

p_i – sellele vastav impulss

liikumisvõrrandid

$$\begin{cases} \dfrac{dp_i}{dt} + \dfrac{\partial H}{\partial q_i} = 0 \\ \dfrac{dq_i}{dt} - \dfrac{\partial H}{\partial p_i} = 0 \end{cases}$$

Sisemuutujad

Vaadeldavad (observable) ehk mõõdetavad muutujad

- siire
- deformatsioon
- pinge / surve
- temperatuur
- ….

Sisemuutujad (internal variables)

- termodünaamilised parameetrid
- fenomenoloogilised parameetrid

e. muutujad

	Mõõdetavad muutujad	Sise-muutuja(d)
nemaatilised kristallid	surve	suunavektor
plastsus-teooria	deformatsioon	purunemis-parameeter
närviimpulss	potentsiaal (s.o. pinge)	ioonvoolu parameetrid

Klassikaline:

Mõõdetavad muutujad	Sise- muutuja(d)
kehtivad jäävusseadused, alluvad inertsile	ei allu inertsile
liikumisvõrrandid	difusioonvõrrandid kineetilised võrrandid
vaja teada Helmholtzi vaba energiat F	vaja teada dissipatsioonipotentsiaali
u	w
$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = f(u, w)$	$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial x^2} + g(w, u)$

Sisemutujate inerts? Vt. näide

Modifitseeritud:

Mõõdetavad muutujad	Sise- muutuja(d)
	alluvad inertsile
	$\frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w}{\partial x^2} + g(w, u)$

Vt. mikrostruktuuriga materjalid (Berezovski, Engelbrecht)

Põhinõuded olekuvõrranditele

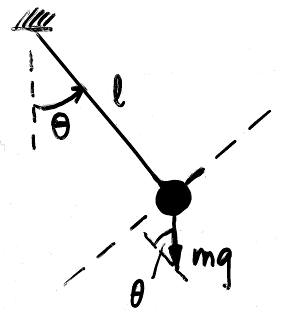
Põhjuslikkus	Causality	
Määratus	Determinism	
Võrdesitatus	Equipresence	kõik muutujad võrdsõltuvad
Objektiivsus	Objectivity	taustsüsteem ei tohi muuta
Aja pööratavus	Time reversal	$t \rightarrow -t$?
Keskkonna invariantsus	Material invariance	
Lokaalsuse printsiip	Neighbourhood	kauge ruum ei mõju ?
Mälu printsiip	Memory	kauge aeg ei mõju ?
Sobivus	Admissibility	olekuvõrrandid ei ole vastuolus jäävusseaduste ja entroopia tingimustega

ÜLDINE JÄÄVUSSEADUS

Muutuja kogus antud ajahetkel	=	muutuja kogus eelmisel ajahetkel
	+	kogus, mis on loodud ajaintervalli jooksul
	–	kogus, mis on hävitatud ajaintervalli jooksul
	+	juurdevool süsteemi

antud ajahetk	–	eelmine ajahetk	=	aja intervall

Näide: Pendel



Liikumisvõrrand

$$ml\ddot{\theta} + mg\ \sin\theta = 0$$

$$\ddot{\theta} + k^2 \sin\theta = 0$$

$$k^2 = \frac{g}{l}$$

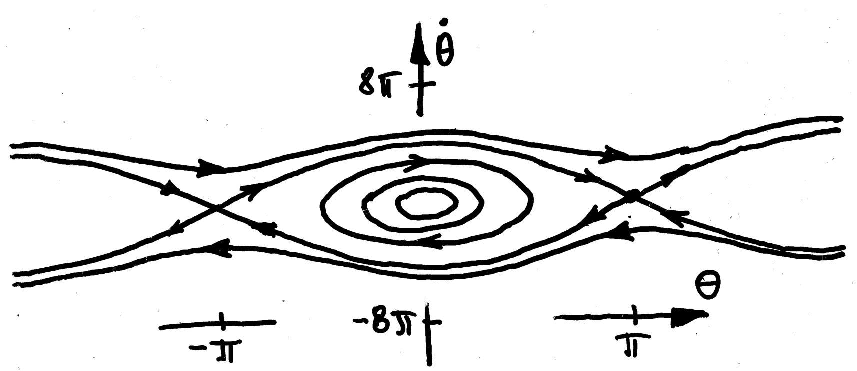
$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \ldots$$

$$\ddot{\theta} + k^2\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \ldots\right) = 0$$

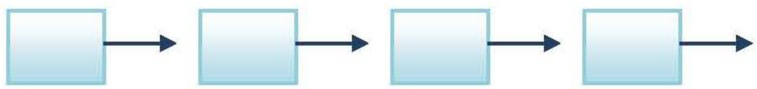
$$\ddot{\theta} + k^2\theta = 0$$

$$T \cong 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \ldots\right)$$

φ^0	$\frac{1}{16}\theta_0^2$	$11/3072\ \theta_0^4$
10^0	0.002	$3\cdot 10^{-6}$
20^0	0.0076	$1\cdot 10^{-4}$
30^0	0.039	$1.4\cdot 10^{-3}$



Transpordivoogude modelleerimine

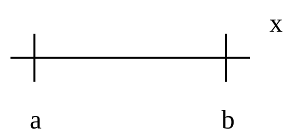


$$\text{kiirus} = \frac{\text{tee pikkus}}{\text{aeg}} \qquad v = \frac{dx}{dt}$$

$$\text{voog}: \quad q = \frac{\text{arv}}{\text{tunnis}} \qquad\qquad \text{tihedus}: \quad \rho = \frac{\text{arv}}{\text{km kohta}}$$

voog = tihedus x kiirus $\quad q = \rho \cdot v$

voo jäävus intervallis



arv intervallis: $\quad N = \int_a^b \rho(x,t)dx$

muutus: $\quad \dfrac{dN}{dt} = q(a,t) - q(b,t)$

jäävus: $\quad \dfrac{d}{dt}\int \rho(x,t)dx = q(a,t) - q(b,t)$

siit $\quad \dfrac{\partial \rho}{\partial t} + \dfrac{\partial q}{\partial x} = 0 \qquad$ ehk $\quad \dfrac{\partial \rho}{\partial t} + \dfrac{\partial}{\partial x}(\rho v) = 0$

Pidev keskkond,

massi jäävus $\quad \dfrac{\partial \rho}{\partial t} + \dfrac{\partial}{\partial x}(\rho v) = 0$

Balance laws

Canonical (material) momentum balance

$$\left.\frac{\partial \mathbf{P}}{\partial t}\right|_{\mathrm{x}} - \mathrm{Div}_R \mathbf{b} = \mathbf{f}^{int} + \mathbf{f}^{ext} + \mathbf{f}^{inh}$$

P – material momentum, **b** – material Eshelby stress,
material inhomogeneity force – f^{inh},
material external (body) force – f^{ext},
material internal force – f^{int}.

Energy conservation

$$\left.\frac{\partial(\mathrm{S}\theta)}{\partial t}\right|_{\mathrm{x}} - \nabla_R \cdot \mathbf{Q} = \mathrm{h}^{int}, \;\; \mathrm{h}^{int} := \mathbf{T} : \dot{\mathbf{F}} - \left.\frac{\partial W}{\partial t}\right|_{\mathrm{x}}$$

the second law

$$\mathrm{S}\dot{\theta} + \mathbf{S} \cdot \nabla_{\mathrm{R}}\, \theta \le \mathrm{h}^{int} + (\theta \mathbf{K}) \quad \mathrm{S}\dot{\theta} + \mathbf{S} \cdot \nabla_R \le \mathrm{h}^{int} + (\theta \mathbf{K})$$

S – the entropy flux, S – the entropy density per unit reference volume,
θ – absolute temperature, **K** – extra entropy flux,
T – the first Piola – Kirchhoff tensor, **F** – deformation gradient.

Internal variables (1)

Observable – strains, displacements, etc.
internal – describe the internal structure of the material

see Maugin, Muschik, 1994

- damage parameter
- orientation of liquid crystals
- dislocations

etc.

In this formalism : internal variables might not be inertial

Internal variables (2)

Microstructured solids

Single / dual variables

Berezovski, Engelbrecht, Maugin, 2008

Dual variables α, β — second order tensors

Free energy W

$$W = \overline{W}\left(\mathbf{F}, \theta, \boldsymbol{\alpha}, \nabla_R \boldsymbol{\alpha}, \boldsymbol{\beta}, \nabla_R \boldsymbol{\beta}\right)$$

Internal variables (3)

From dissipation inequality

$$\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} = \mathrm{L} \begin{pmatrix} \tilde{A} \\ \tilde{B} \end{pmatrix} = \begin{pmatrix} \mathrm{L}^{11} & \mathrm{L}^{12} \\ \mathrm{L}^{21} & \mathrm{L}^{22} \end{pmatrix} \begin{pmatrix} \tilde{A} \\ \tilde{B} \end{pmatrix}$$

L – depend on state variables

$\tilde{A}, \tilde{B}$ related to $\bar{W}$

A simple non-dissipative process

$$\dot{\boldsymbol{\alpha}} = \mathrm{L}^{12}\, \tilde{\mathcal{B}}\, , \quad \dot{\boldsymbol{\beta}} = -\mathrm{L}^{12}\, \tilde{\mathcal{A}}$$

A special case $\overline{\mathrm{W}}$ independent of $\nabla_{R}\, \beta$

$$\ddot{\boldsymbol{\alpha}} = \left(\mathrm{L}^{12} \cdot \mathrm{L}^{12}\right) \tilde{\mathcal{A}}$$

inertia account

Mindlin model

Mindlin model

Mindlin model	Internal variable
two balance laws	one balance law dissipation inequality

$$\rho_0 u_{tt} = a u_{xx} + A \psi_x$$
$$I \psi_{tt} = C \psi_{xx} - A u_x - B \psi$$

Ψ – microdeformation	Ψ – internal variable

Hierarchy waves

System of two 2nd order equations $\rightarrow$ one 4 th order equation

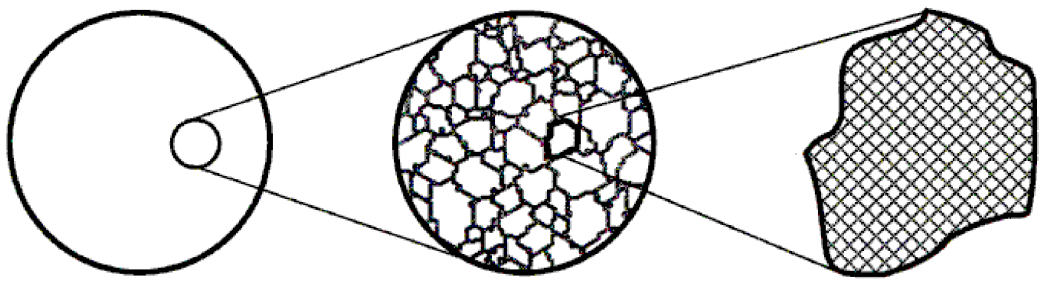
$$u_{tt} = \left(c_0^2 - c_A^2\right) u_{xx} - p^2 \underbrace{\left(u_{tt} - c_0^2 u_{xx}\right)_{tt}} + p^2 c_1^2 \underbrace{\left(u_{tt} - c_0^2 u_{xx}\right)_{xx}}$$

Simplified (slaving principle)

$$u_{tt} = \left(c_0^2 - c_A^2\right) u_{xx} - p^2 c_A^2 \underbrace{\left(u_{tt} - c_1^2 u_{xx}\right)_{xx}}$$

Multiple scales

macrostructure microstructure 1 microstructure 2



Multiple scales

$$u_{tt} = \left(c_0^2 - c_A^2\right) u_{xx} + p_1^2 c_{A1}^2 \underbrace{\left[u_{tt} - \left(c_1^2 - c_{A2}^2\right) u_{xx}\right]_{xx}} -$$

$$- p_1^2 c_{A1}^2 \; p_2^2 c_{A2}^2 \underbrace{\left(u_{tt} - c_2^2 u_{xx}\right)_{xxxx}}$$

Nonlinearities

free energy W – cubic

model

$$\rho u_{tt} = \alpha u_{xx} + N u_x u_{xx} + A \psi_x$$

$$I \psi_{tt} = C \psi_{xx} + M \psi_x \psi_{xx} - A u_x - B \psi$$

hyperbolic wave equation → two waves
evolution equation → one-wave equations

Separation techniques:

Taniuti, Nishihara 1977, 1983
Jeffrey, Kawahara 1982
Engelbrecht 1983

Evolution equation

KdV $$u_\tau + kuu_\xi + du_{\xi\xi\xi} = 0$$

mKdV $$u_\tau + [P(u)]_\xi + du_{3\xi} + bu_{5\xi} = 0$$

$$P(u) = -\frac{1}{2}u^2 + \frac{1}{4}u^4$$

microstructured solids $$u_\tau + qvv_\xi + dv_{\xi\xi\xi} + r\left(v_\xi^2\right)_{\xi\xi} = 0$$

Näide : LUMI JA SUUSATAMINE

Lume struktuur

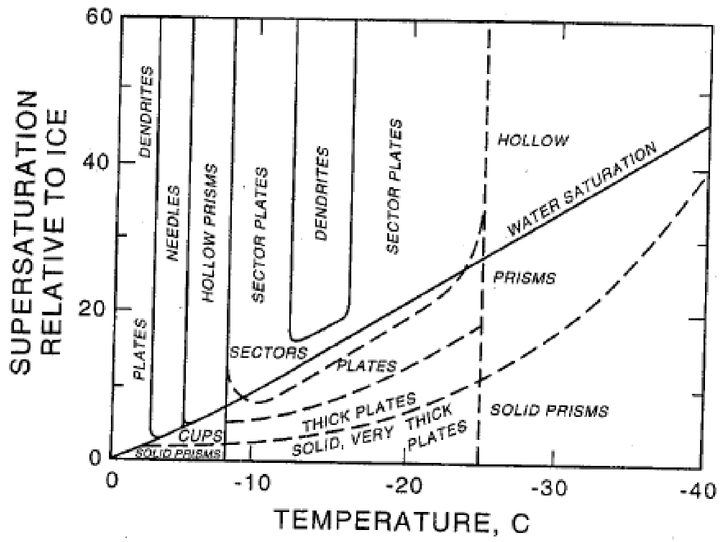


FIGURE 2.3. Atmospheric snow crystal types given as functions of the temperature and supersaturation relative to a flat ice surface at the moment of their formation. (From J. B. Mason, 1971, *The Physics of Clouds*, 2nd ed., Oxford, UK: Clarendon Press. By permission of Oxford University Press.)

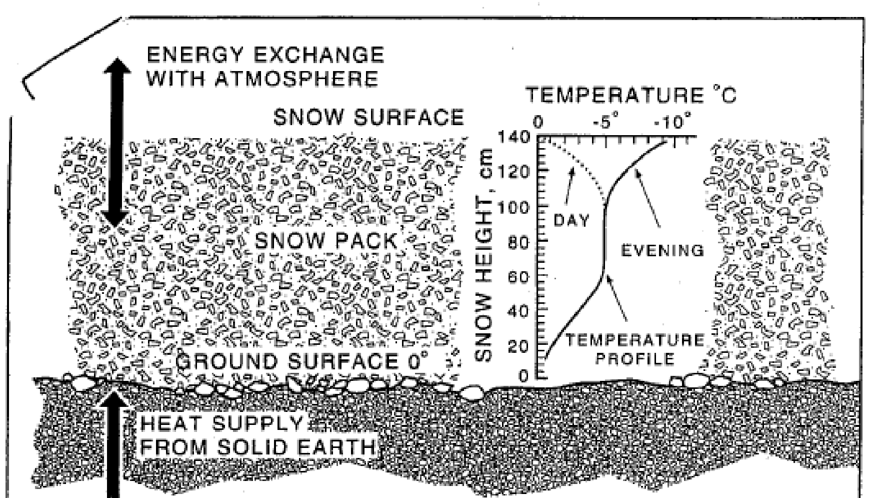


FIGURE 2.8. Temperature profile of a snowpack. Temperature gradients exist at the top and at the bottom, near the surface of the snowpack and at the ground surface. In the middle, the snowpack is at an equitemperature state (Perla and Martinelli, 1979).

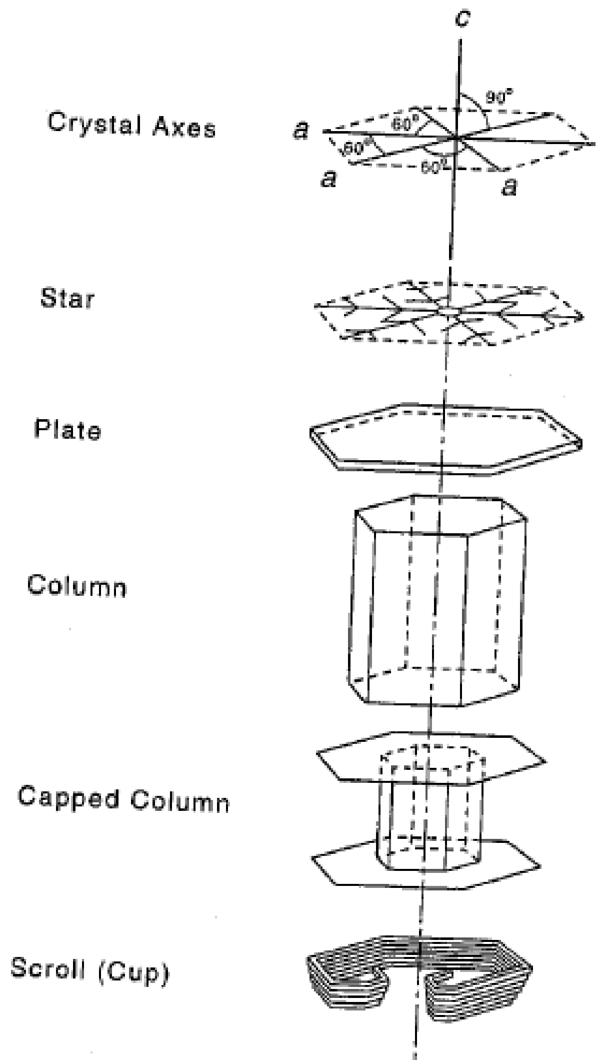


FIGURE 2.2. Principal types of atmospheric snow crystals shown in relation to the crystal axes of ice (Perla and Martinelli, 1979).

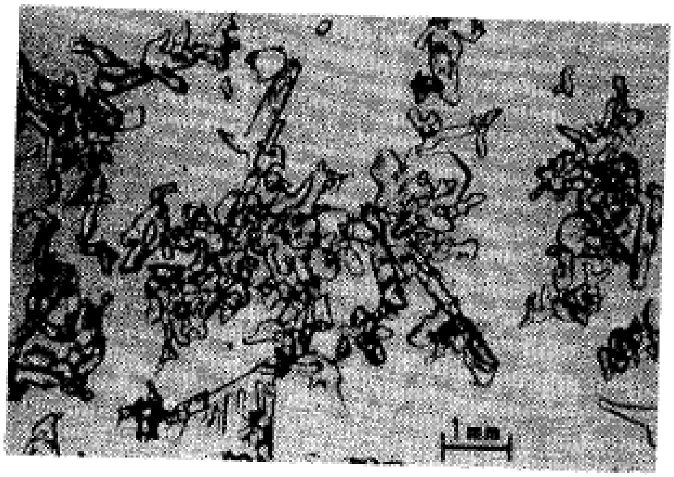
(a) 

FIGURE 2.6. Equitemperature (ET) metamorphism of newly fallen snow. The faceted, dendritic arms of the new snow grains in the left frame have disappeared in the right frame, metamorphosing into rounded, sintered grains. (Reprinted with permission from Colbeck *et al.*, 1990. Photographs courtesy of E. Akitaya.)

 (b)

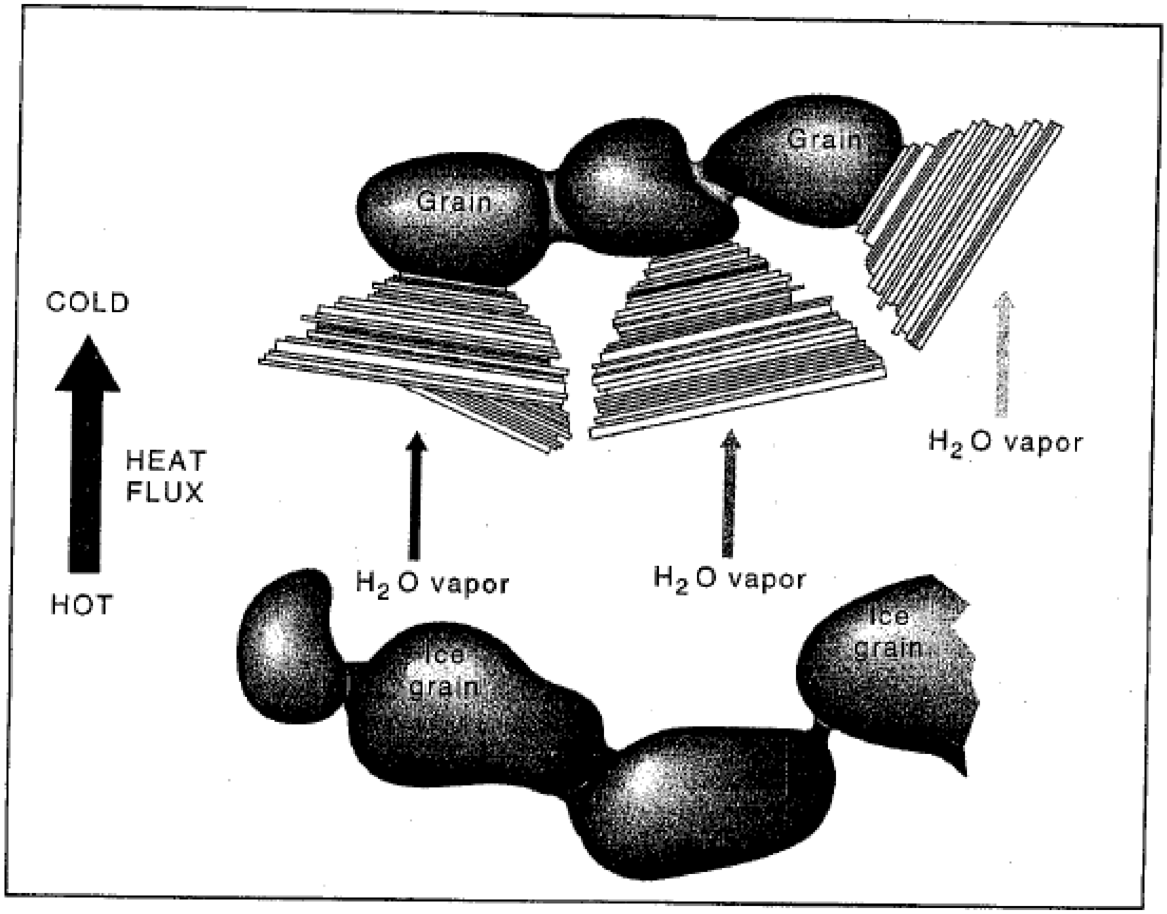


FIGURE 2.9. The transport by convection of water vapor from the warm, sintered grains below to the colder grains above. Condensation forms the prismatic laminar facets (Perla and Martinelli, 1979).

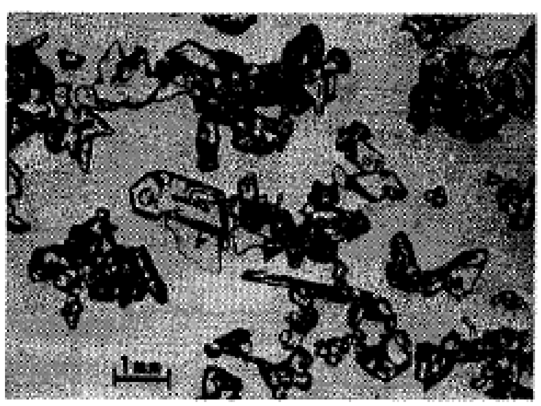
(a)   (b)

FIGURE 2.10. Temperature-gradient (TG) snow in successive stages of growth. Notice the change in scale from (a) to (b). (Reprinted with permission from Colbeck *et al.*, 1990. Photographs courtesy of K. Izumi.)

Suusa mudel

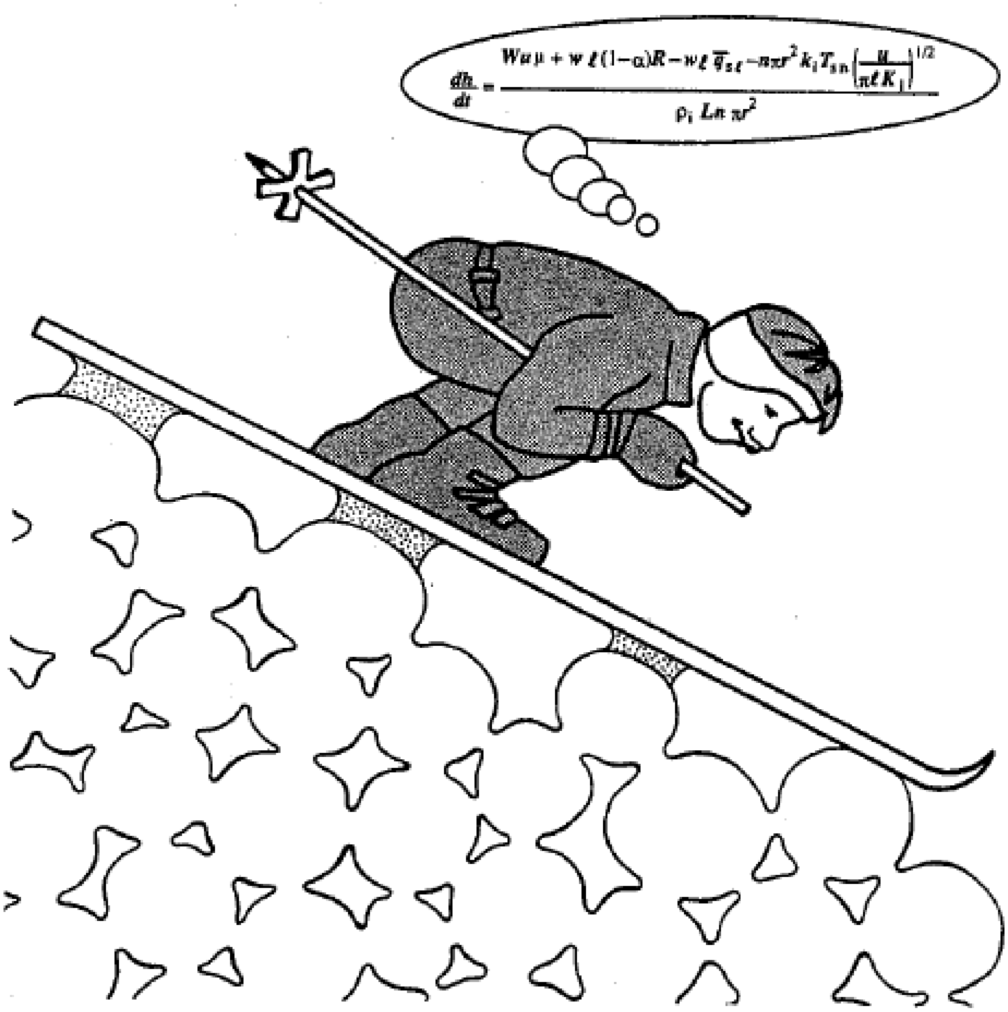


FIGURE 2.1. This skier heads down the hill, his skis lubricated by a film of water that forms under his skis. In his thoughts he mulls over a mathematical formula that we will discuss later in Chap. 8 on snow friction processes. (Colbeck, 1992. Drawn by Marilyn Aber, CRREL.)

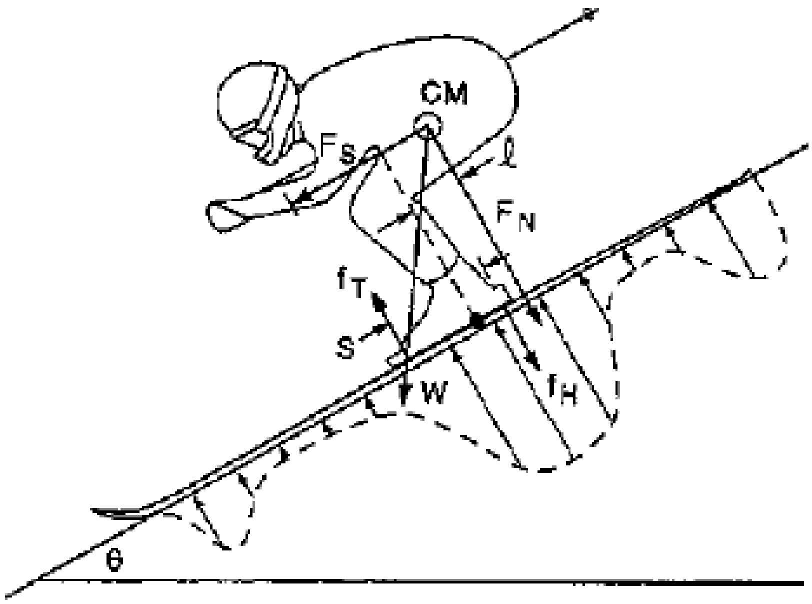


FIGURE 3.2. Forces acting on the ski through the boot.

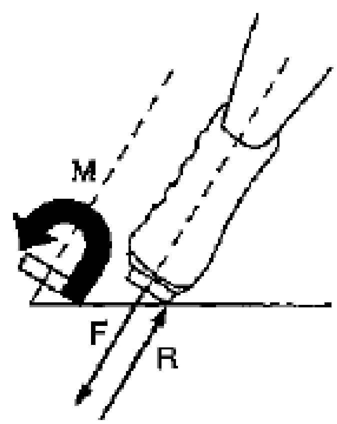


FIGURE 3.3. The lateral moment generated when the ski is set on edge.

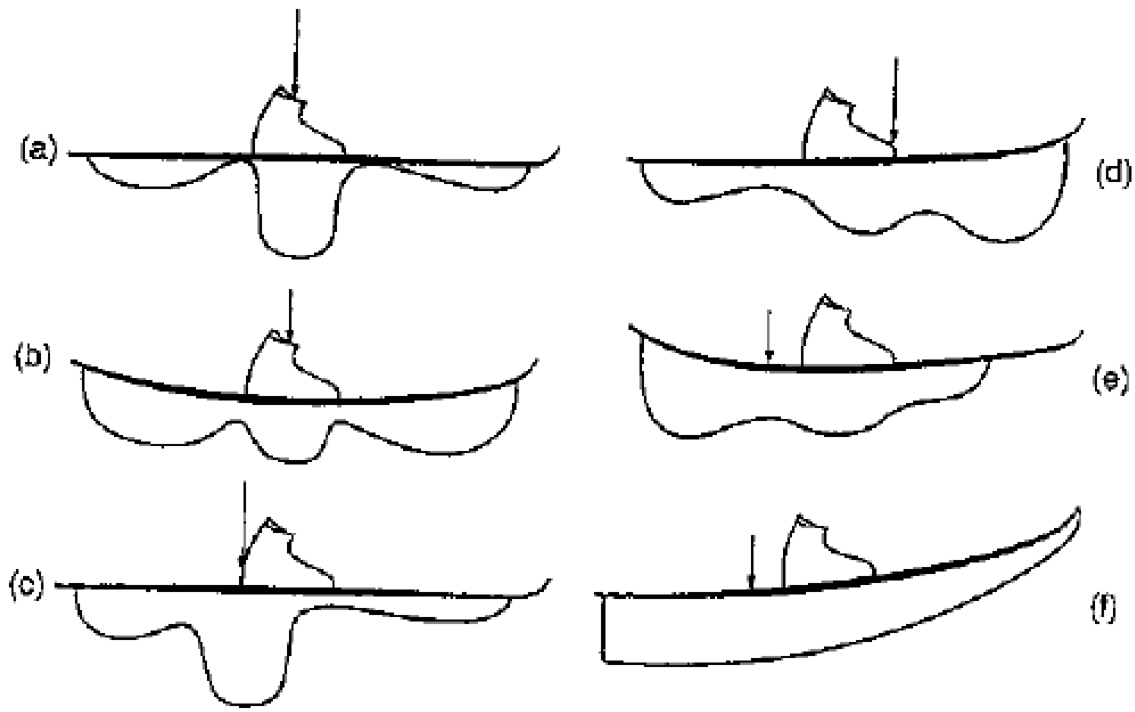


FIGURE 3.8. Bottom load distributions for six skis with different stiffness properties on snow beds of varied deformation.

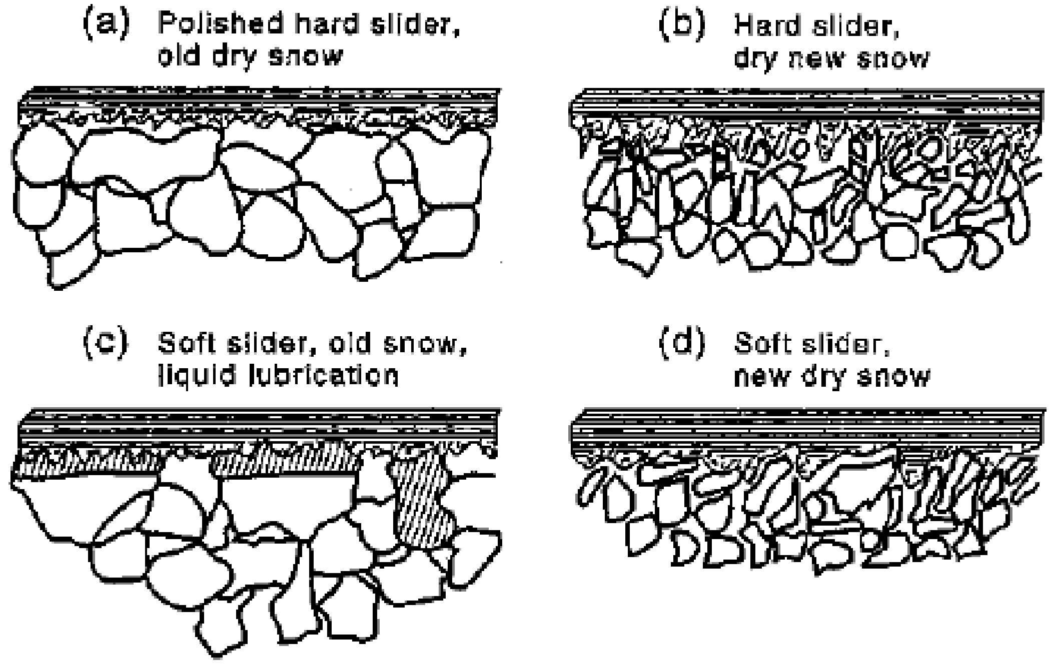


FIGURE 8.3. Four examples of the interactions that may occur at the interface of the ski bottom and the surface of the snow.

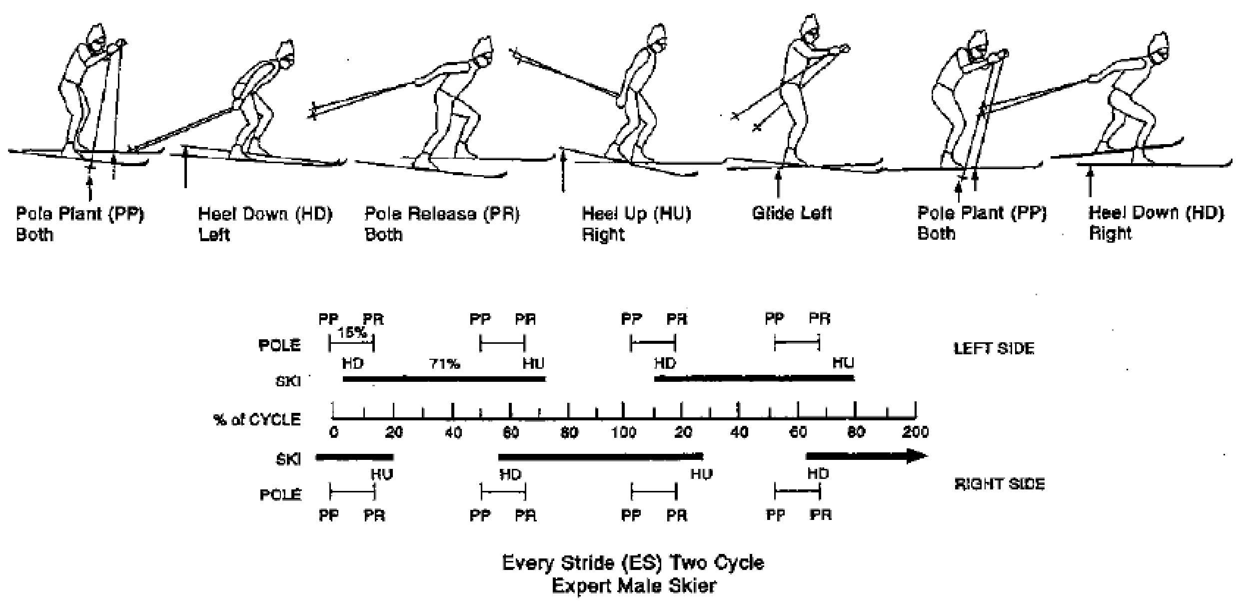
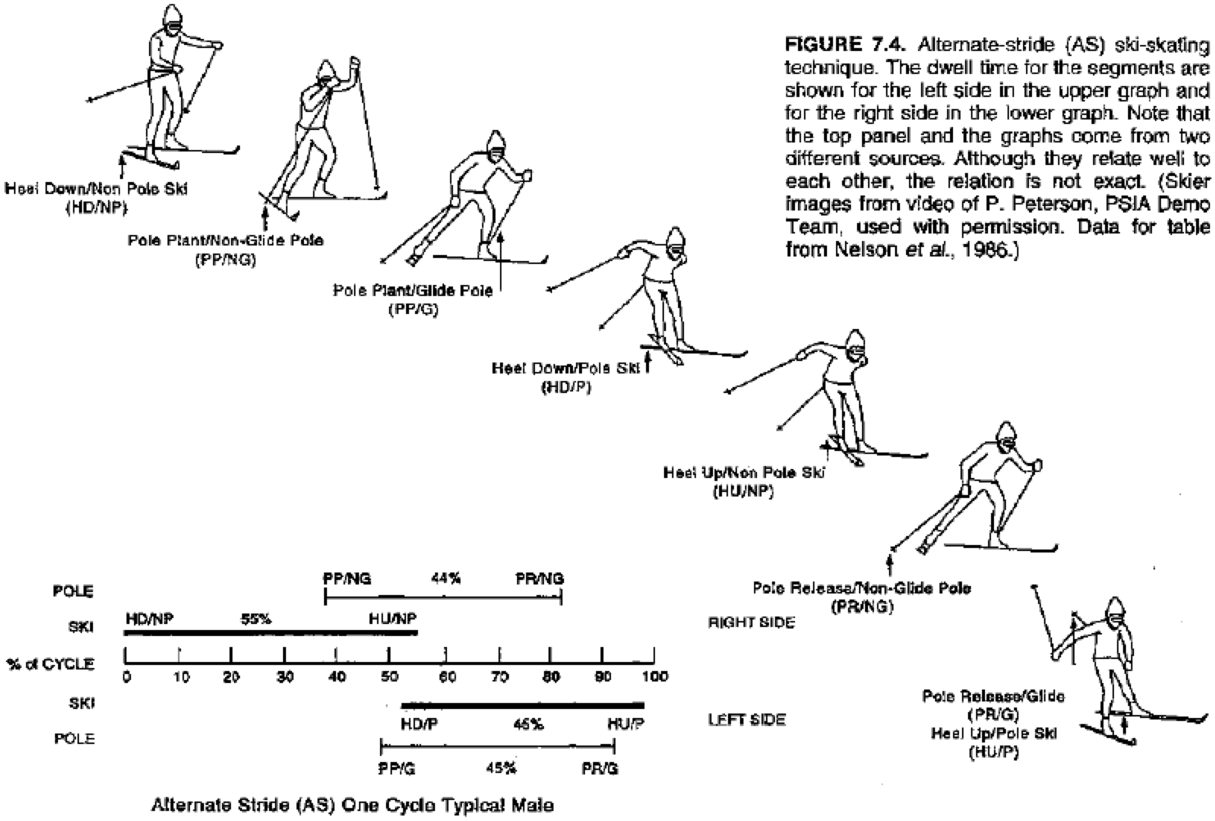


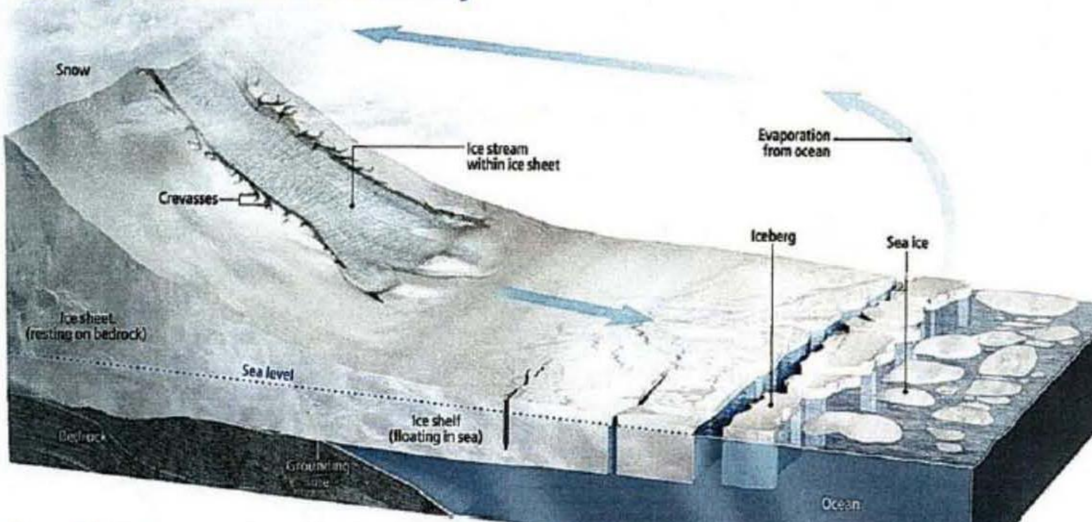
FIGURE 7.3. Every-stride (ES) ski-skating technique. The top panel shows the maneuvers for one half cycle. The lower graph shows the dwell time for each segment. Note that the top panel and the graphs come from two different sources. Although they relate well to each other, the relation is not exact. (Skier images from video of P. Peterson, PSIA Demo Team, used with permission. Data for the graph from Nelson *et al.*, 1986.)

FIGURE 7.4. Alternate-stride (AS) ski-skating technique. The dwell time for the segments are shown for the left side in the upper graph and for the right side in the lower graph. Note that the top panel and the graphs come from two different sources. Although they relate well to each other, the relation is not exact. (Skier images from video of P. Peterson, PSIA Demo Team, used with permission. Data for table from Nelson *et al.*, 1986.)



[ICE MOVEMENT 101]

Steady State in a Frozen Country



▲ Schematic diagram shows how an ice sheet flows downslope across its underlying bedrock toward the sea, while snowfall from evaporated seawater replenishes part or all of the ice mass at its surface. Most of the ice flowing from the continental interior is carried to the sea by ice streams, relatively fast-moving conveyor belts of ice that break away from the surrounding sheet; the ice sheet travels seaward as well, albeit much more slowly. Once the base of the moving ice leaves its "grounding line," the floating ice is called an ice shelf, and it displaces a mass of water equal to its weight, raising the sea level accordingly. Throughout most of the past several millennia those processes did not raise sea level or shrink ice sheets because seawater evaporation and inland snowfall roughly balanced the discharge of ice into the sea.

Elastsusteooria 3D

$$(\lambda+2\mu)u_{1,11}+(\lambda+\mu)(u_{2,21}+u_{3,31})+\mu(u_{1,22}+u_{1,33})-\rho_0\frac{\partial^2 u_1}{\partial t^2}=0$$

$$(\lambda+2\mu)u_{2,22}+(\lambda+\mu)(u_{3,12}+u_{3,32})+\mu(u_{2,11}+u_{2,33})-\rho_0\frac{\partial^2 u_2}{\partial t^2}=0$$

$$(\lambda+2\mu)u_{3,33}+(\lambda+\mu)(u_{1,13}+u_{2,23})+\mu(u_{3,11}+u_{3,22})-\rho_0\frac{\partial^2 u_3}{\partial t^2}=0$$

$u_1 \quad u_2 \quad u_3$ – siirde komponendid

$x_1 \quad x_2 \quad x_3$ – koordinaadid

Navier- Stokes'i võrrandid hüdrodünaamikas, 3D

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]=-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right]+F_x$$

$$\rho\left[\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right]=-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right]+F_z$$

$$\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right]=-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right]+F_y$$

u, v, w – kiiruse komponendid

x, y, z – koordinaadid

p – surve

μ – viskoosus

F_x, F_y, F_z – välisjõud

$$\rho\left[\frac{\partial U}{\partial t}+U\cdot\nabla U\right]=-\nabla p+\mu\nabla^2 U+F$$

Magnetvoo jäävus

$$\oint_{\partial V} \mathbf{B} \cdot \mathbf{da} = 0$$

Laengu jäävus

$$\frac{d}{dt}\int_V q_e \, dv + \frac{\delta}{\delta t}\int_\sigma w_e \, da = -\int_{\partial V} \mathfrak{I} \, da - \oint_{\partial\sigma} b \cdot K \, ds$$

Gaussi seadus (elektrivoog)

$$\oint_{\partial V} \mathbf{D} \cdot \mathbf{da} = \int_V \mathbf{q}_e \, \mathbf{dv} + \int_\sigma \mathbf{w}_e \, \mathbf{da} + \oint_{\partial\sigma} \mathbf{b} \cdot \hat{\pi} \, \mathbf{ds}$$

Faraday seadus (elektromotoorne jõud)

$$\frac{1}{c}\frac{d}{dt}\int_S B \cdot da + \oint_{\partial S} E \cdot dx = 0$$

Ampère'i seadus (magnetiline induktsioon)

$$-\frac{1}{c}\frac{d}{dt}\int_S D \cdot da + \oint_{\partial S} H \cdot dx = \frac{1}{c}\int_S \mathfrak{I} \cdot da - \frac{1}{c}\int_\gamma b \cdot K \, ds$$

NB! A.C. Eringen, G.A. Maugin
Electrodynamics of Continua I, II
Springer, New York et al, 1990