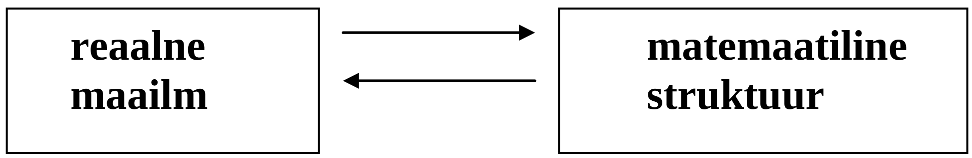
I ÜLDMÕISTED

2. Modelleerimise metodoloogia



Concise Encyclopedia on Modelling and Simulation:

Metodoloogia on teatud protseduuride hulk, mis kindlustab teatud valdkonna tüüpiliste probleemide lahendamise. Metodoloogia koosneb teatud hulgast meetoditest, teatud hulgast tegevustest (toimingutest) ning teatud hulgast seostest
meetodite ja tegevuste vahel

Näide:

taime mudel
verbaalne
visuaalne
bioloogiline
klimatoloogiline
majanduslik

Interpretatsioon:

Taim

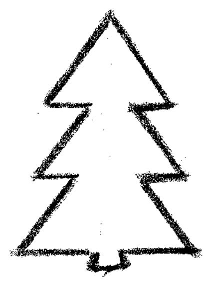
PUU

nimetus **kask**

mänd

...

verbaalne

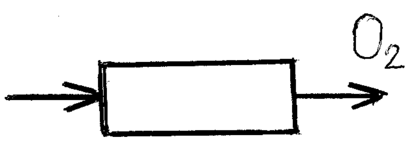


visuaalne

$$\frac{dh}{dt} = f(\ldots)$$

h **- kõrgus**

bioloogia:
kasvu mudel



klimatoloogia:
puu kui hapniku
tootja

aeg	mass
10 a	m_1
.	
.	
.	
40 a	m_4
.	
.	
.	
80 a	m_8 raieküps

Hind?

majandus

Tehnikad:
reduktsionistlik
süsteemiteoorial baseeruv

Reduktsionistlik:

Keerulised süsteemid tükeldatakse ikka väiksemateks ja väiksemateks komponentideks kuni iga komponent on piisavalt lihtne selleks, et ta funktsioon on lihtsalt modelleeritav (ja arusaadav). Iga komponendi mudeli olemasolul on keeruline süsteem modelleeritud.

A.Toffler, Sissejuhatus Prigogine & Stenger
Order out of Chaos

Üks kõige arenenum tänapäeva Lääne tsivilisatsiooni oskusi on tükeldamine (ingl. dissection): s.o. probleemide lahkamine nende kõige väiksemateks komponentideks. See oskus on meil ülihea. Nii hea, et sageli me unustame panna osad tagasi kokku tervikuks.

Süsteemiteoorial baseeruv

Keeruliste ja komplekssete struktuuride holistlik käsitlus, mis arvestab vastasmõjusid nende struktuuride käitumises ja hierarhias, s.o. süsteemide süsteemid.
Üksikkomponentide käitumine pole alati individuaalselt modelleeritud (või isegi mõistetav)

holistlik – terviklik süsteemi käsitlev
universum on terviklik süsteem)

süsteemiteooria kohaselt

jäik (hard) süsteem

paindlik (soft) süsteem

jäik → vaatlejast sõltumatu
füüsikaline
paindlik → vaatlejast sõltuv
sotsiaalsed

Eetika: tuumarelv
bioloogiline relv
inimkatsed

Kes on inimene? Vercors “Ebaloomad”

inimesel on usk millessegi
jumal
ebajumal

Modelleerimise blokk-skeemid (näited loengus)

LEONARDO DA VINCI

Motion is an accident born from the inequality of weight or force.
Force is the cause of motion, motion is the cause of force.

1) Observe the phenomenon and list quantities having numerical magnitude that seems to influence it.

2) Set up *linear* relations among pairs of these quantities as are not obviously contradicted by experience.

3) Propose these rules of three trial by experiment.

ISAAC NEWTON “PRINCIPIA”, 1687

I Every body continues in its state of rest, or of uniform motion straight ahead, unless it be compelled to change that state by forces impressed upon it.

II The change of motion is *proportional* to the motive force impressed, and it takes place along the right line in which that force is impressed.

III To an action there is always a contrary and equal reaction; or, the mutual actions of two bodies upon each other are always equal and directed to contrary parts

I Iga keha säilitab oma oleku kas paigalseisu või ühtlase sirgjoonelise liikumise kujul seni, kuni temale rakenduvad jõud seda olekut ei muuda.

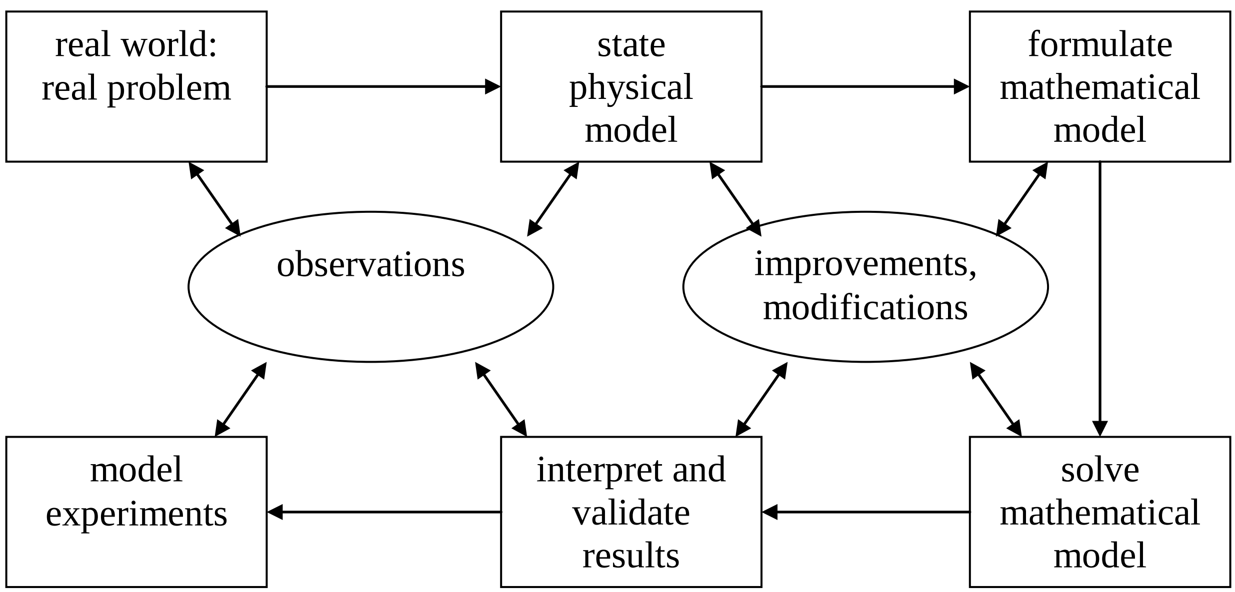
II Liikumishulga muutus on võrdeline kehale mõjuva jõuga ning toimub samas suunas mõjuva jõuga.

III Jõud esinevad ainult paariti: iga mõjuga kaasneb alati niisama suur, kuid vastassuunaline vastumõju.

1. On soovitav probleemsituatsiooni kirjeldada nii üldiselt kui üksikult nii palju kui võimalik ilma matemaatikata
   Seega: alustada matemaatilisest mudelist ja suruda see peale probleemile – vale!

2. Probleemsituatsiooni üksikasjaline kirjeldus annab ülevaate muutujatest, parameetritest, ka seostest nende vahel

3. Ainult juhul, kui situatsioonil on selgelt eristatavad alamosad, võiks kõne alla tulla tükeldamine
   - agregatsioon – vt hiljem meetodid
   - agregatsioon nagu teisedki meetodid on hästi põhjendatud matemaatiliselt



Iseärasused tänapäeval:

- inimene - masin: kommunikatsioonibarjäär on kadunud;
- numbriliste meetodite lai valik;
- odav arvutiaeg;

NB! Näited: 1) Sci-fiction kuidas modelleerida mõtteliselt materiaalseid asju

2) Joonistamismäng vilets/hea seletus

1.2.1 GUIDELINES ON FORMULATING THE PROBLEM

1. Define the problem and give its history and its causes.
2. State the objectives and the constraints.
3. Are you sure that this is the problem you want to solve? Why you want to solve the problem?
4. Is there anyone who needs the solution? Are you sure you need the solution? Why?
5. Are there other related problems, perhaps easier, which should be solved first? List them.
6. What is the solution needed for?
7. What effect would it have?
8. How would you implement it?
9. How much would it cost to solve the problem? What are the resources available?
10. How much benefit would there be from the solution?
11. If the problem is ignored, will it terminate over time?
12. Get outside the problem and look at it. Is it significant?
    What is your vantage point for this judgment?
13. What are the stable solutions of the problem?
14. Can change in law or administration eliminate the problem?
15. Can the problem also be viewed as someone else´s problem?
    Perhaps you can engage his cooperation in modeling a solution?

1.2.2. GUIDELINES ON HOW TO SOLVE THE PROBLEM

1. Does the problem have a solution?
2. Give all alternative solutions: are they exhaustive? How do you demonstrate this?
3. Give an optimal or near-optimal solution.
4. Give an average solution.
5. Give an approximate solution.
6. Start at both ends: the raw data and a hypothesized answer and move toward the middle to develop justification.
7. Start in the middle and move towards the ends.
8. Embed the problem in a larger context and solve it.
9. Abstract the problem to a simpler formulation.
10. Can you derive the solution from that of a related problem?
11. Simulate the problem in search of solution.
12. Construct a working hypothesis.

13. Develop and test the hypothesis.
14. Define the utilities and the payoffs in the process being studied.
15. How sensitive is the solution to changes in the data?
16. What are the invariants of the problem as reflected in the solution?
17. Update feedback of the implemented solution onto the problem under study.
18. Analyze the faulty solutions of the problem to get a better understanding of the preferred ones.

1.2.3. IMPACT OF THE SOLUTION

1. How can you communicate the problem and its solution to others? What is the most effective way to convince different people of your solutions?
2. What is the impact of the solution on people, things, etc.?
3. What people should be involved in implementing the solution?
4. What personnel commitments, organizational structure, and equipment are needed to find the solution and , in particular, to implement it?
5. What happens to the organization after the problem has been solved and the solution implemented?
6. Can the organization solve other problems?
7. How should people be motivated to solve the problem?
8. What are the sanctions on, and threats to, the individuals and organizations involved?
9. What is the moral impact of the problem and its solution on people?
10. Will there be a chain reaction, either creating new problems or solving other problems, as a result of this solution?
11. Are there residual problems which must now be solved?

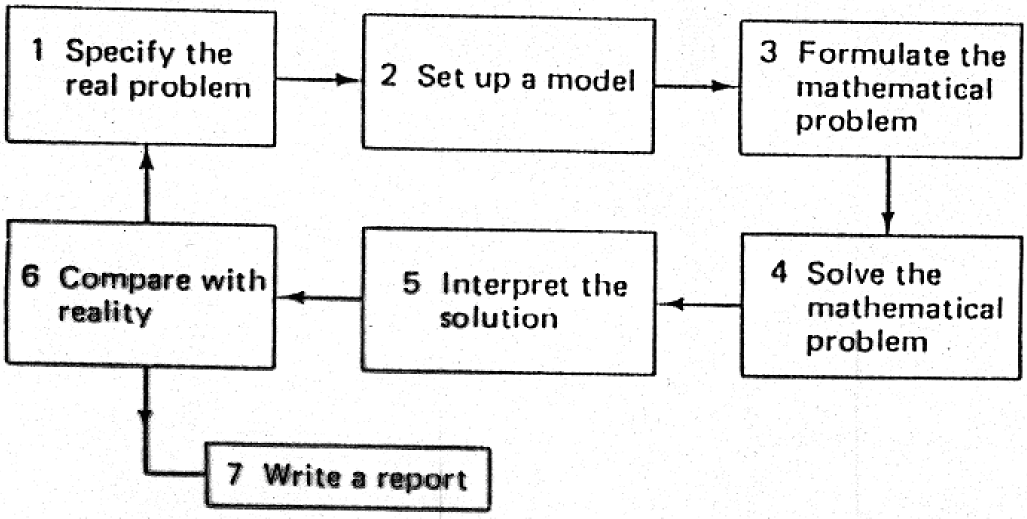
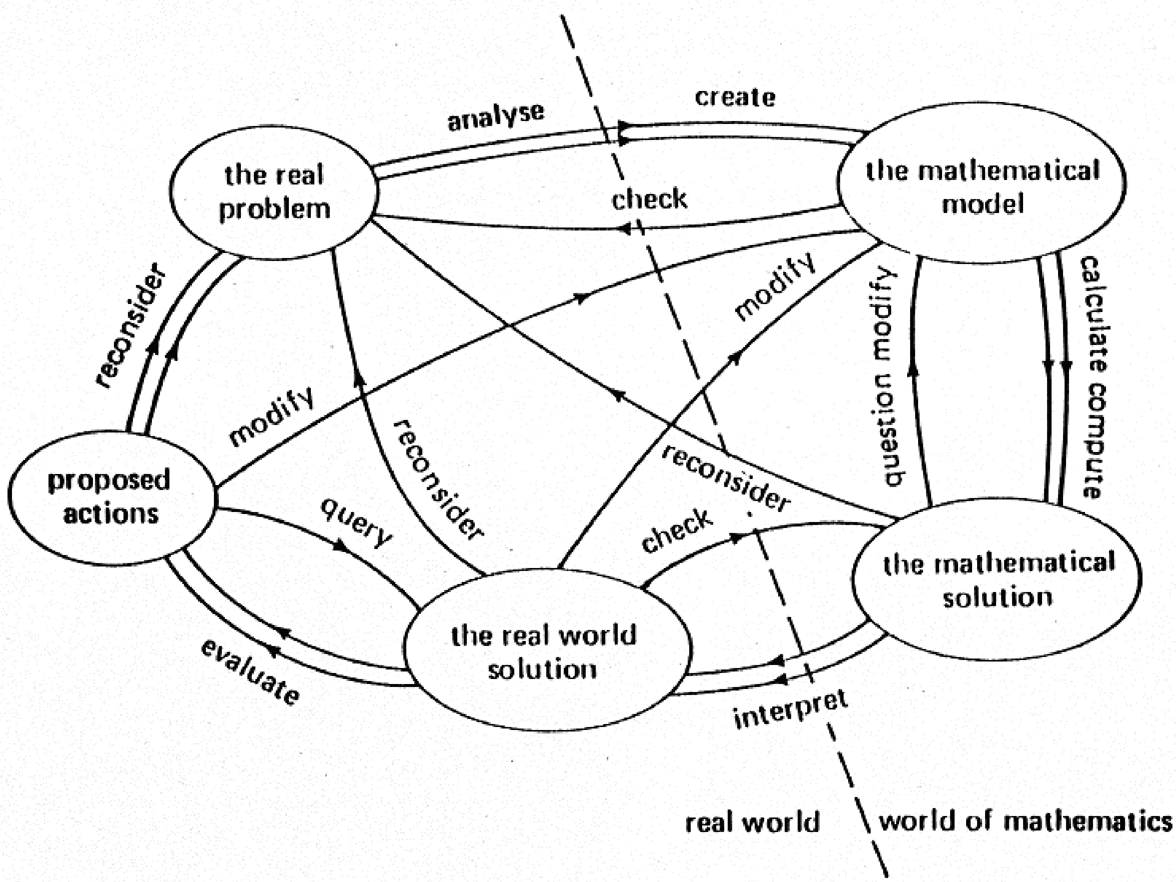


Fig. 2.2. The modelling methodology of Penrose (1978).

Fig. 2.3. A richly linked model of mathematical modelling.



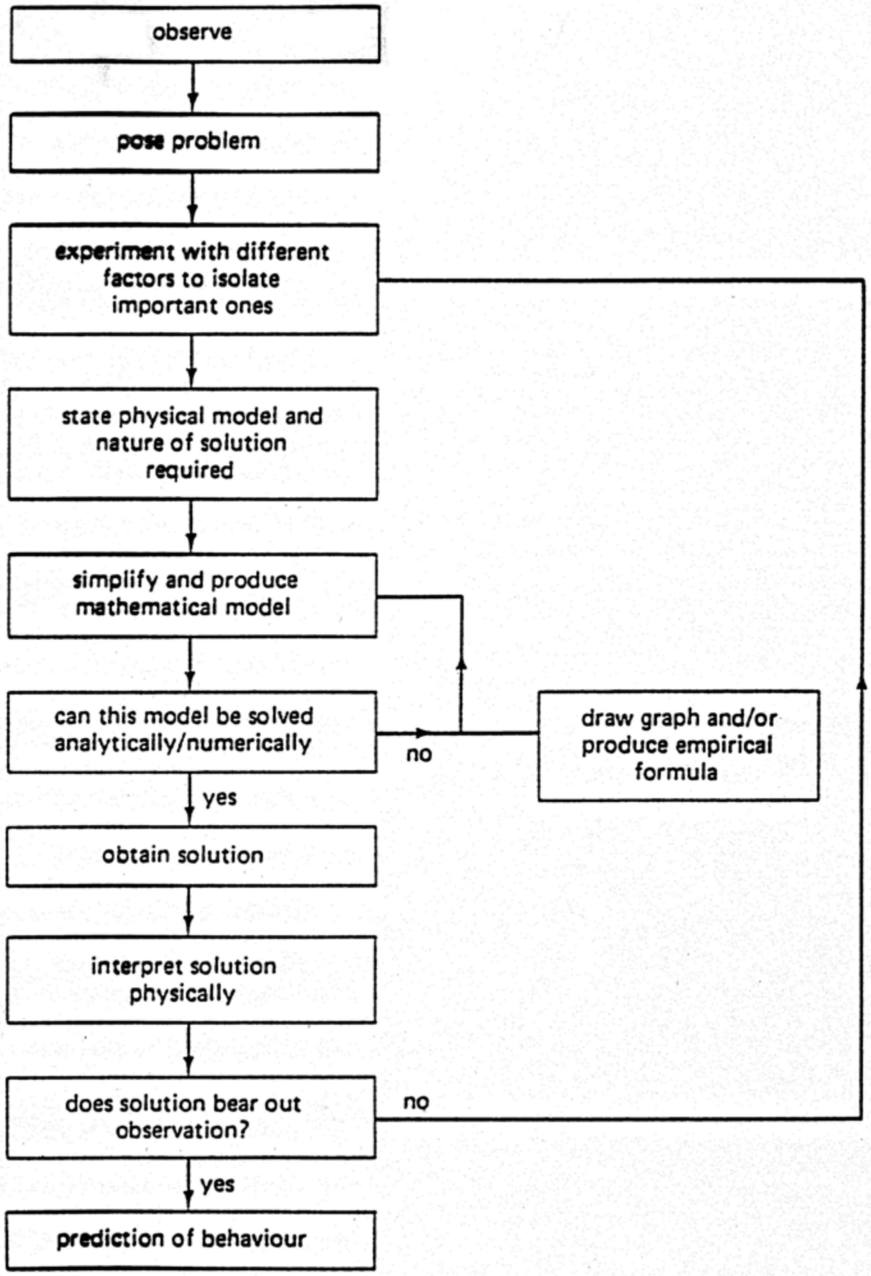


Fig. 2.1. The modelling methodology of Bajpai *et al.* (1975).

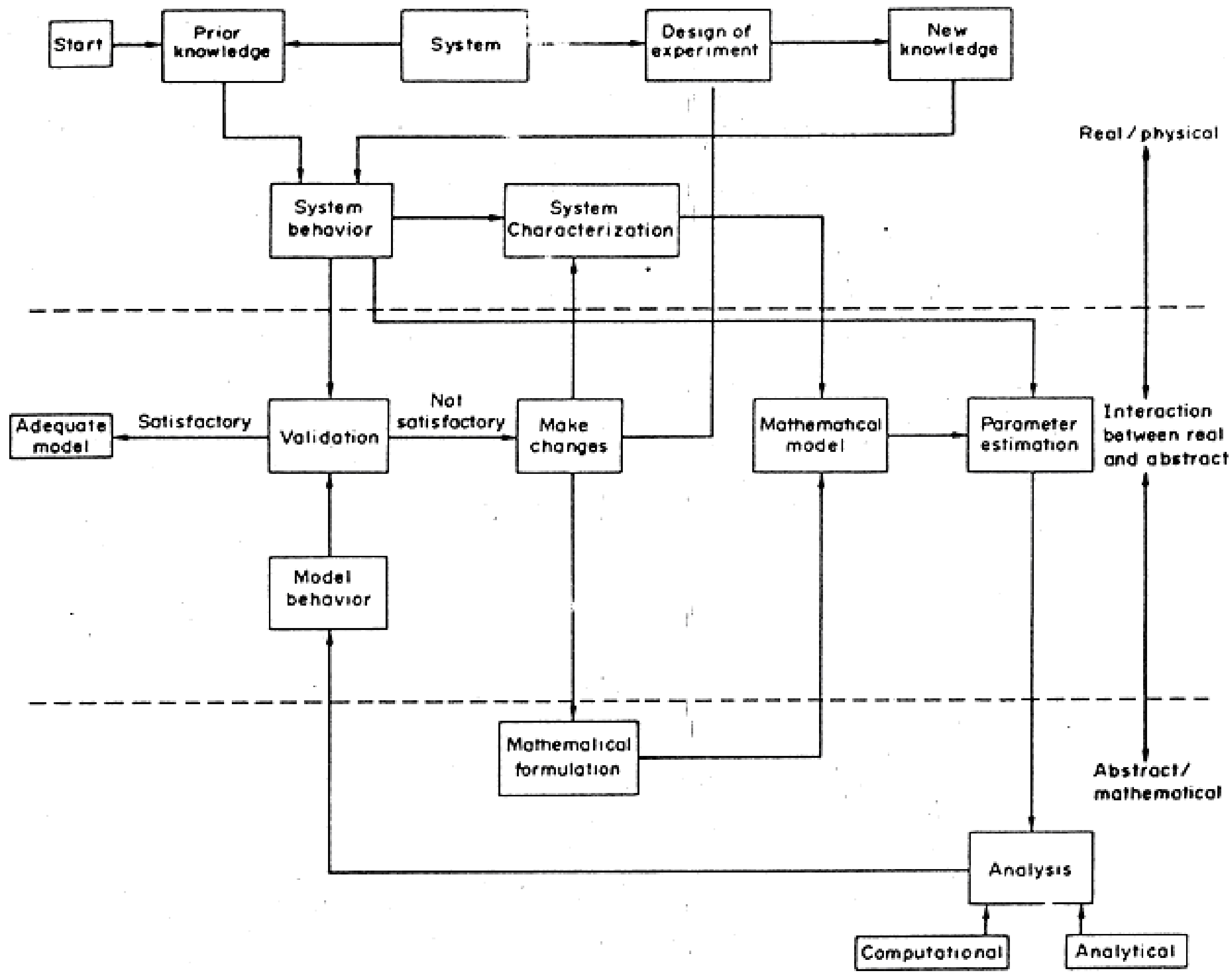
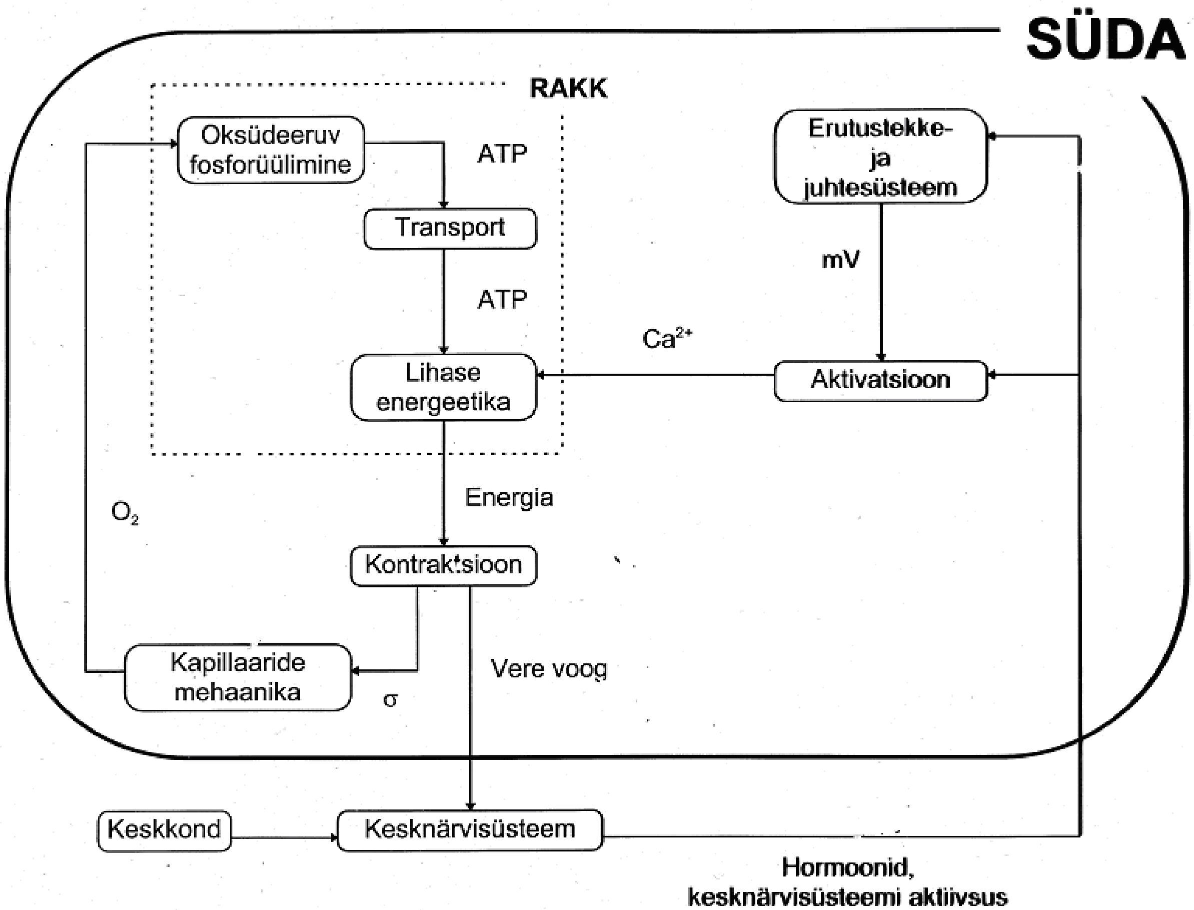
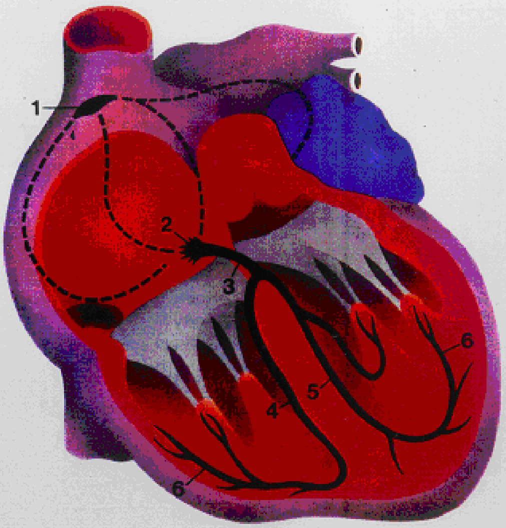


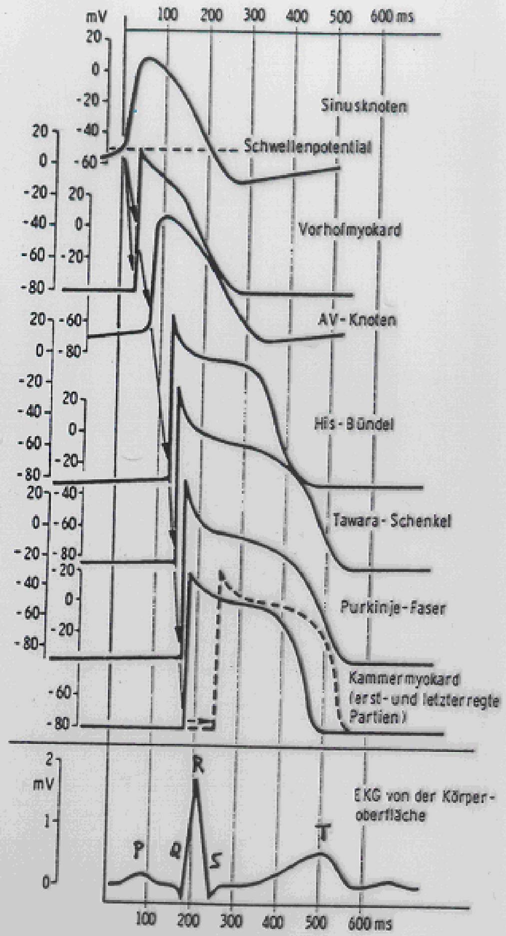
FIGURE 4.3 The model building process (detailed).

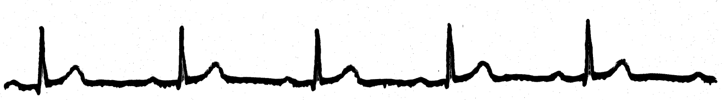
Näide: Südamerütmide erutusmehhanismi matemaatiline mudel





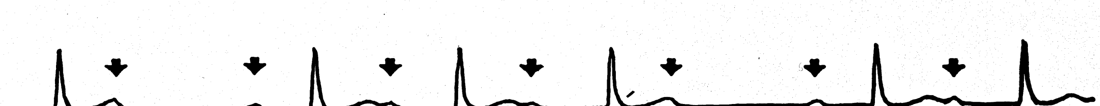
1. SA-node
2. AV-node
3. His bundle
4.–5. Bundle branches
6. Purkinje fibres



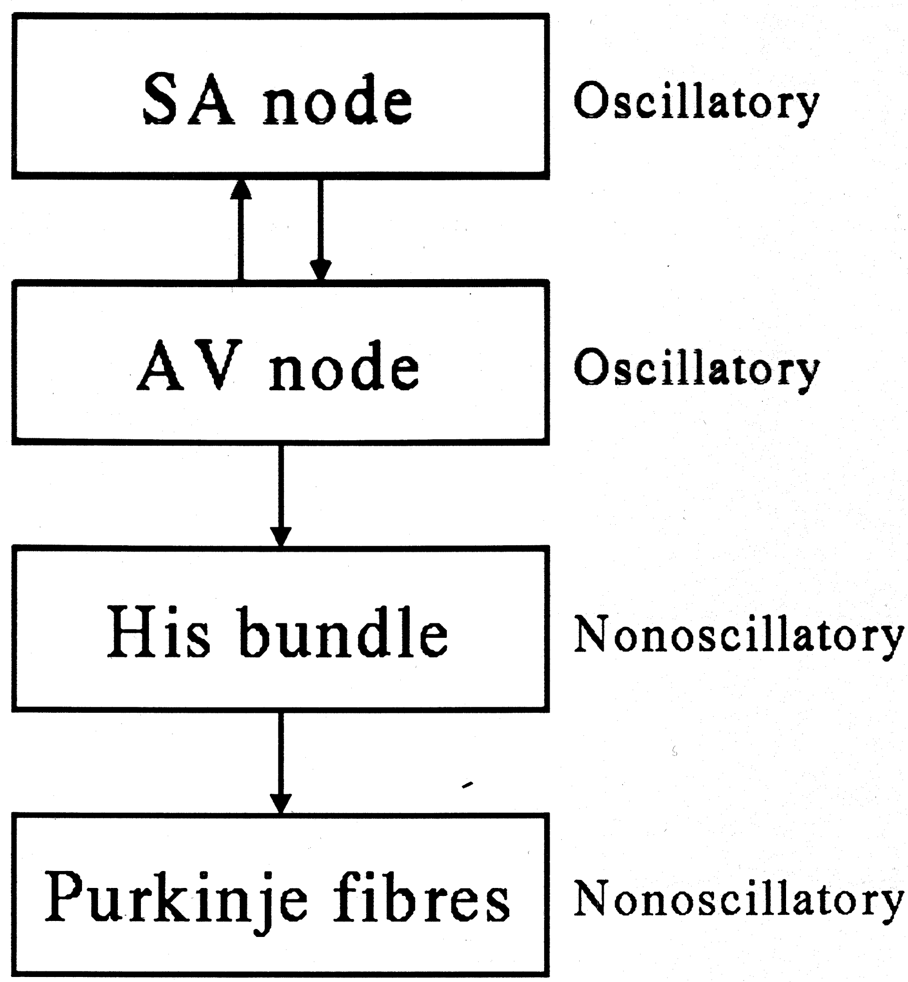


1. Normally beating heart

2. Mobitz2 — second order A-V block with constant PQ-time



3. Wenckebach — second order AV block with progressively increasing PQ-time


SA node
Oscillatory
AV node
Oscillatory
His bundle
Nonoscillatory
Purkinje fibres
Nonoscillatory

CONVENTIONAL APPROACH

$$\frac{\partial^2 v}{\partial x^2} = c r_s \frac{\partial v}{\partial \tau} + r_s I$$

parabolic

↑ nonlinearity

Hodgkin - Huxley (HH)

$$I = g_k n^4 (v - v_R - v_K) + g_{Na} m^3 h (v - v_R - v_{Na})$$

$$+ g_L (v - v_R - v_L) + C_m \partial v / \partial t$$

$$\frac{dn}{dt} = -\frac{n - n_0}{\tau_n}$$

$$\frac{dm}{dt} = -\frac{m - m_0}{\tau_m}$$

$$\frac{dh}{dt} = -\frac{h - h_0}{\tau_h}$$

FitzHugh - Nagumo (FHN)

$$I = -w - \left(v - \frac{v^3}{3} \right)$$

$$c \frac{\partial w}{\partial t} = a - v$$

EVOLUTION EQUATION APPROACH

$$\frac{\partial^2 v}{\partial X \partial \xi} + f(v)\frac{\partial v}{\partial \xi} + g(v) = 0$$

↑

FHN type recovery variable

ξ - **moving frame coordinate**

Stationary:

$$v'' + f(v)v' + \theta^{-1} g(v) = 0$$ - **LIÉNARD - TYPE**

$$(\ )' = d/d\eta, \quad \eta = X + \theta\xi$$

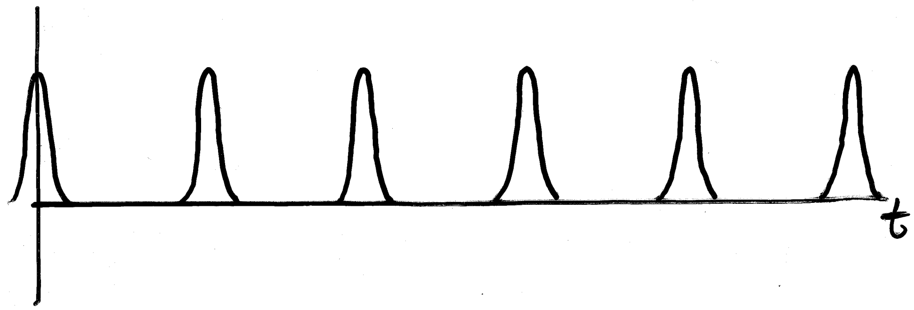
Engelbrecht, 1981
1991

GENERALIZATIONS:

- **Modifications (wave hierarchies)**
- **Driven evolution equations**
  basic for a generalized heart model
- **Thermodynamics**
  link to the theory of continua

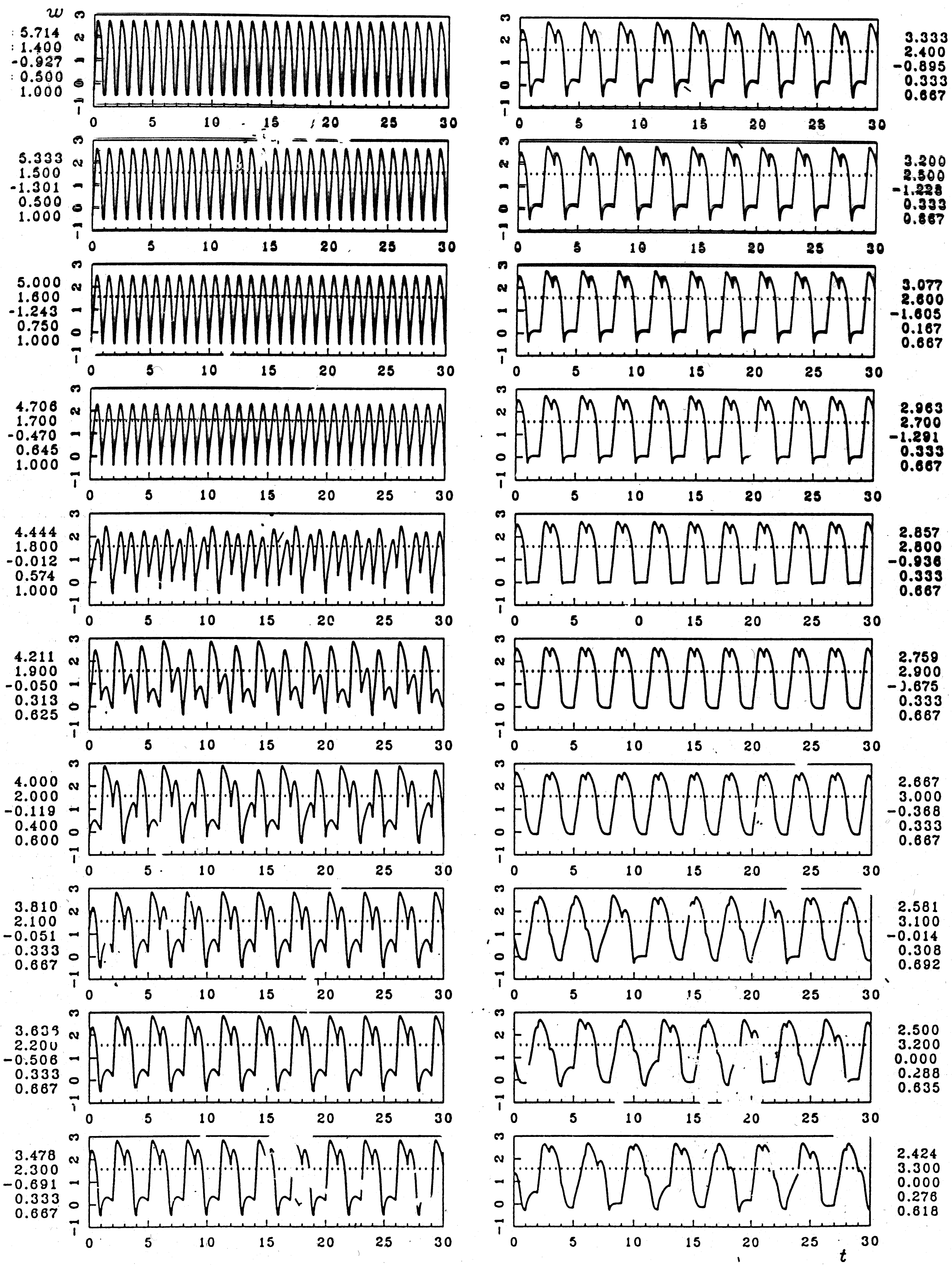
Kongas, von Hertzen, 1995
Engelbrecht, Maugin, 1994

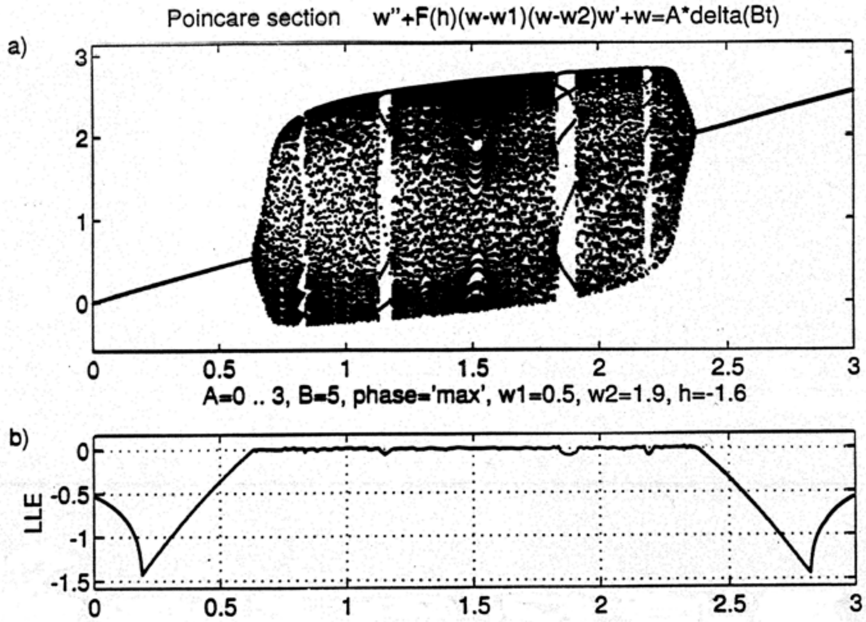
3) $w'' + f(w)w' + kw = A\sum_{n=0}^{\infty}\delta(t - nT)$

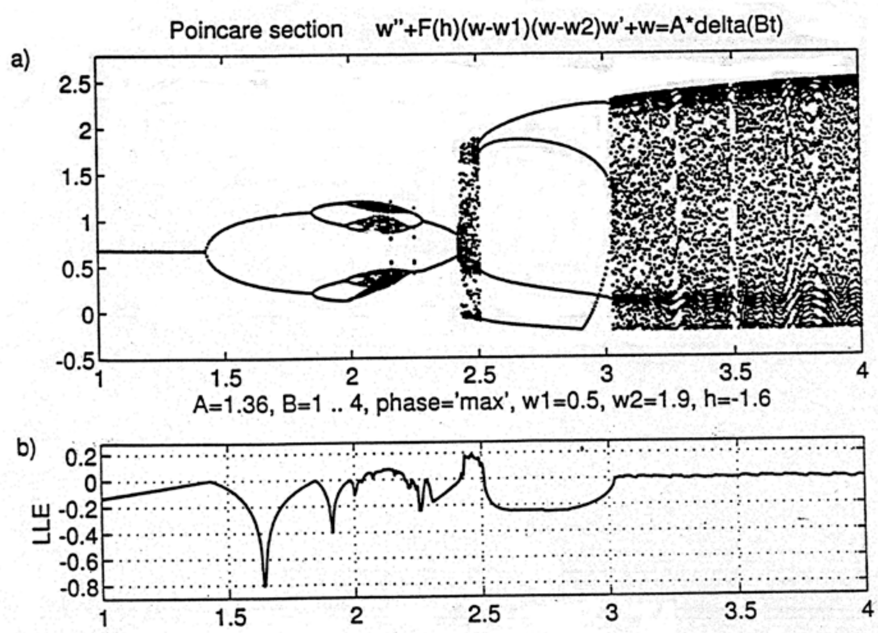


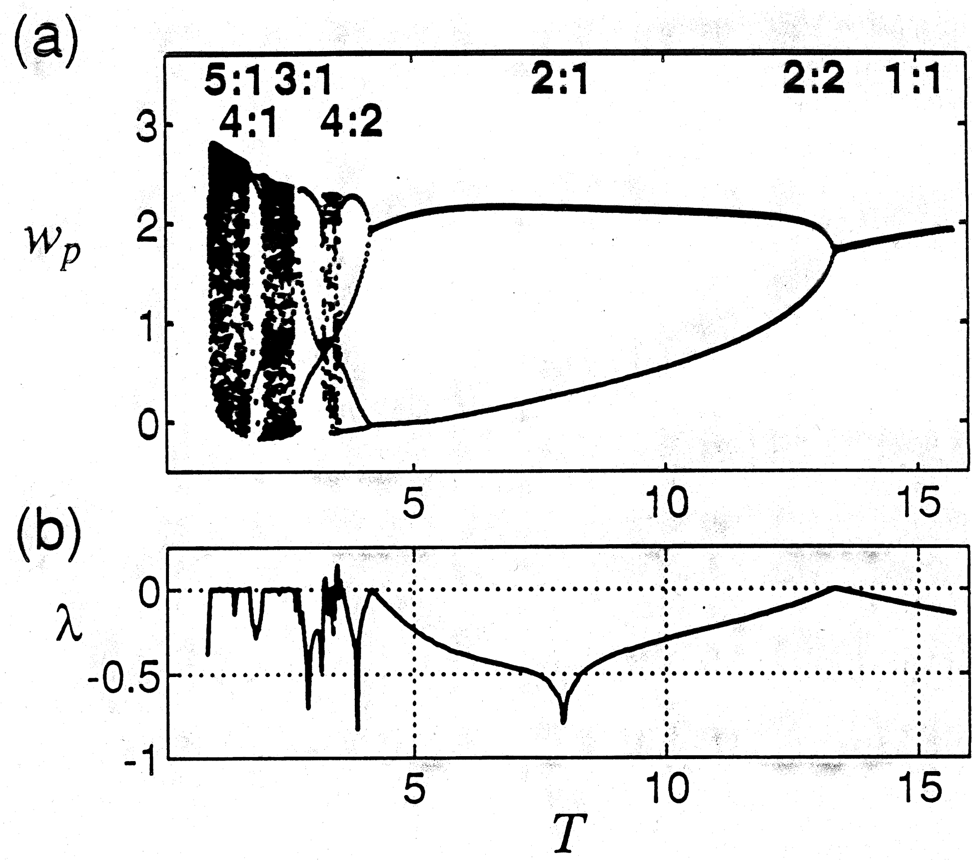
$T, \quad \omega = \frac{2\pi}{T}$

$I\,\omega = 8.00$


w
5.714
1.400
-0.927
0.500
1.000
3.333
2.400
-0.895
0.333
0.667
5.333
1.500
-1.301
0.500
1.000
3.200
2.500
-1.228
0.333
0.667
5.000
1.600
-1.243
0.750
1.000
3.077
2.600
-1.605
0.167
0.667
4.706
1.700
-0.470
0.645
1.000
2.963
2.700
-1.291
0.333
0.667
4.444
1.800
-0.012
0.574
1.000
2.857
2.800
-0.936
0.333
0.667
4.211
1.900
-0.050
0.313
0.625
2.759
2.900
-0.675
0.333
0.667
4.000
2.000
-0.119
0.400
0.600
2.667
3.000
-0.368
0.333
0.667
3.810
2.100
-0.051
0.333
0.667
2.581
3.100
-0.014
0.308
0.692
3.636
2.200
-0.506
0.333
0.667
2.500
3.200
0.000
0.288
0.635
3.478
2.300
-0.691
0.333
0.667
2.424
3.300
0.000
0.276
0.618
t

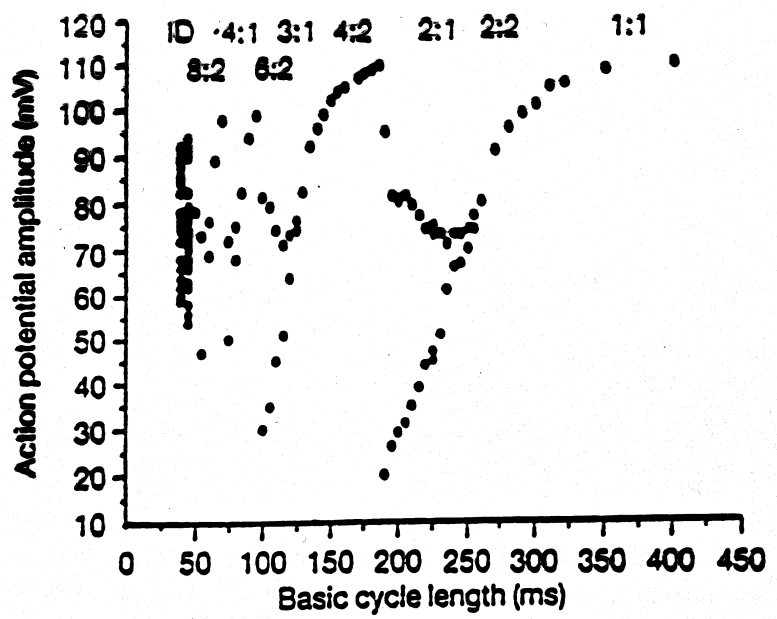

Poincare section w''+F(h)(w-w1)(w-w2)w'+w=A*delta(Bt)
a)
A=0 .. 3, B=5, phase='max', w1=0.5, w2=1.9, h=-1.6
b)
LLE


Poincare section w''+F(h)(w-w1)(w-w2)w'+w=A*delta(Bt)
a)
A=1.36, B=1 .. 4, phase='max', w1=0.5, w2=1.9, h=-1.6
b)
LLE

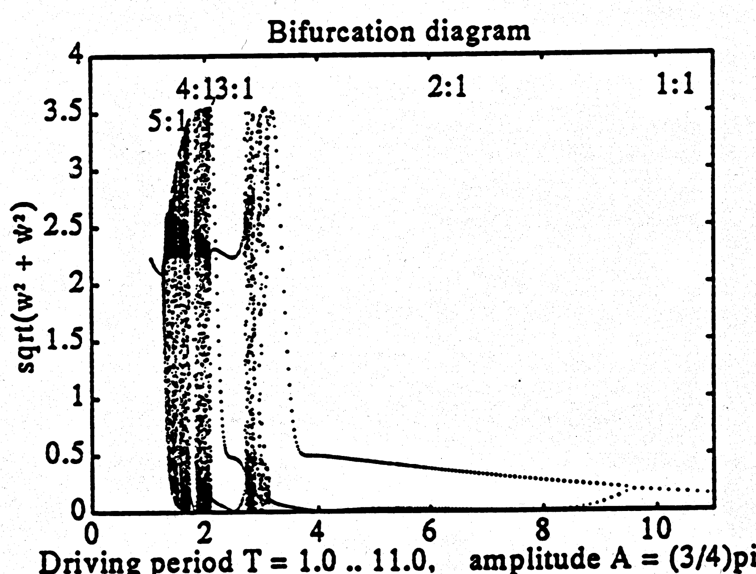


1:1 → 2:2, 2:1 → 4:2 → chaos → 3:1 →
6:2, quasiperiodic, 4:1, quasiperiodic → etc

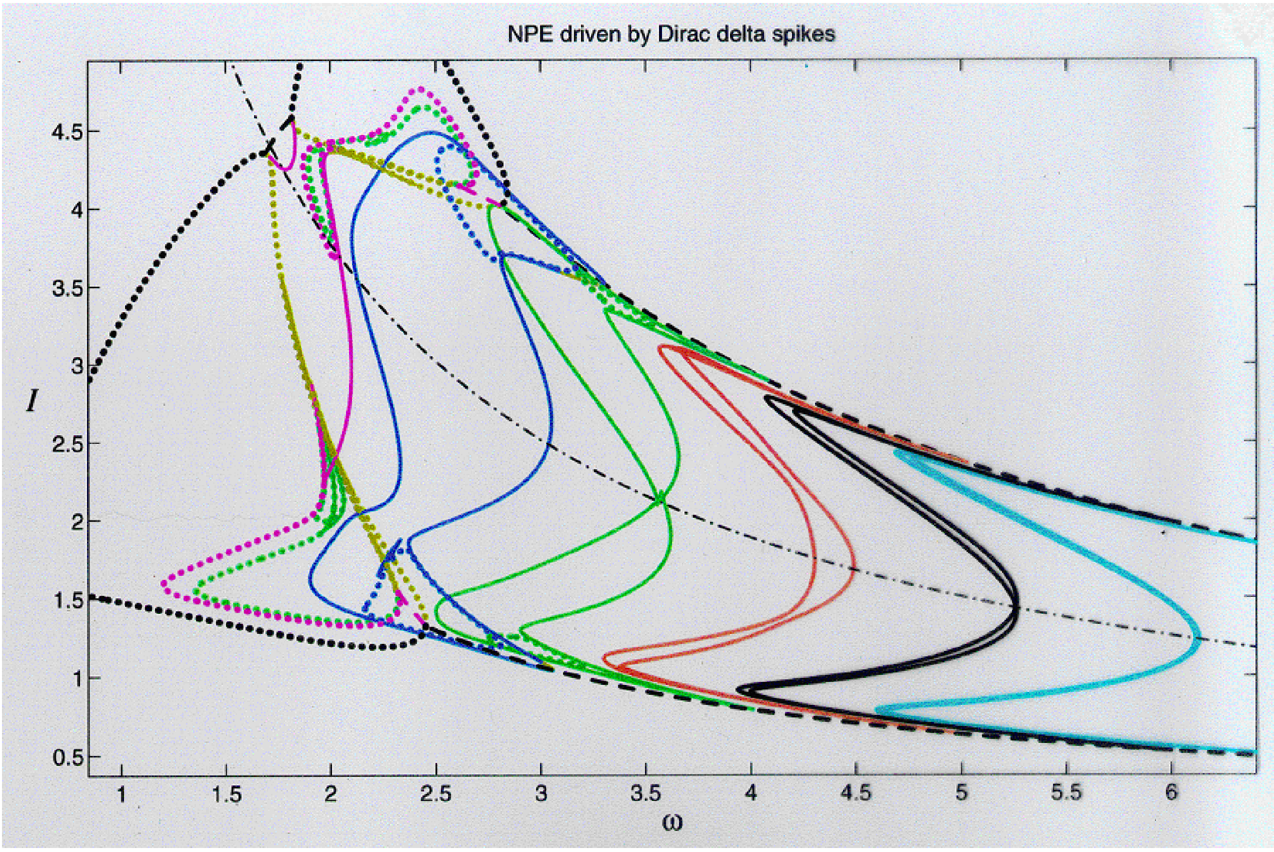
Difference: 4:1 becomes quasiperiodic
via saddle-node bifurcation
Chialvo et al., : period doubling
Here, NPE desribes earlier development



Experimental action potential from a driven sheep cardiac Purkinje fibre.



Action potential calculated from the driven nerve pulse equation.


NPE driven by Dirac delta spikes
I
ω
0.5
1
1.5
2
2.5
3
3.5
4
4.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6